

YEAR 12 - MATHEMATICS

HSC Topic 10 – The Anti-Derivative and Indefinite Integral MA-C4.1

MATHEMATICS ADVANCED

LEARNING PLAN

Learning Intentions Student is able to:	Learning Experiences Implications, considerations and implementations:	Success Criteria I can:	Resources
1. Define anti-differentiation as the reverse of differentiation and use the notation $\int f(x)dx$ for anti-derivatives or indefinite integrals	Define the primitive (anti-derivative) as the reverse process of differentiation recognise that any two anti-derivatives of $f(x)$ differ by a constant and hence the primitive should have a constant c.	1. Find a primitive function. 2. Explain the geometrical interpretation of a series of primitive functions. 3. Answer questions involving primitive functions.	
2. Establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$, for $n \neq -1$	Include examples where the student must write the expression in index for first. $\int (2x+1)^2 dx$, $\int x\sqrt{x} dx$, $\int (\sqrt{x}-1)^2 dx$, $\int (3x-2)(4x-3) dx$.	Use the rules for integration	

3. establish and use the formula $\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$ where $n \neq -1$ (the reverse chain rule)	Integrals such as $\frac{x^3+5}{x^2}$, $3x + \frac{\sqrt{x}}{x}$ which initially need to be manipulated algebraically.		
	HSC QUESTIONS 17 11b Find $\int (2x+1)^4 dx$ 1 Solution 10 2e Given that $\int_0^6 (x+k) dx = 30$ and k is a constant, find the value of k. 2 Solution 10 2d Find $\int \sqrt{5x+1} dx$ 2 Solution 09 2b Find $\int \frac{3}{(x-6)^2} dx$ 2 Solution 11 2e Find $\int \frac{1}{3x^2} dx$ 2 Solution 09 2b Find $\int_1^4 x^2 + \sqrt{x} dx$ 3 Solution 16 9 What is the value of $\int_{-3}^2 x+1 dx$? 1 Solution		

	(A) $\frac{5}{2}$ (B) $\frac{11}{2}$ (C) $\frac{13}{2}$ (D) $\frac{17}{2}$ 16 11d Evaluate $\int_0^1 (2x+1)^3 dx$ 2 Solution 14 11d Find $\int \frac{1}{(x+3)^2} dx$ 2 Solution		
4. Establish and use the formulae for the anti-derivatives of $\sin(ax + b)$, $\cos(ax + b)$ and $\sec^2(ax + b)$.			Integration of Trig Fns Integration Practice - Khan Academy
5. Establish and use the formulae $\int e^x dx = e^x + c$ and $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$			
6. Establish and use the formulae: $\int \frac{1}{x} dx = \ln x + c$ and $\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$ for $x \neq 0$, $f(x) \neq 0$, respectively.	1. Find the integrals that initially require a division operation, eg $\frac{2x}{x+1} = 2 - \frac{2}{x+1}$ Show that and hence find $\int \frac{2x}{x+1} dx$.	Find and use the derivatives of functions to help to find more difficult integrals.	

	2. Use logarithms to find integrals of $\tan ax$, $\cot ax$ 3. Find the integrals that link differentiation and integration as inverse operations, eg Differentiate $\ln(\sec x + \tan x)$ hence evaluate $\int_0^{\frac{\pi}{4}} \sec x \, dx$.	(f) e.g. Differentiate $x \tan x$ and hence evaluate $\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx$	
7. Establish and use the formulae $\int a^x dx = \frac{a^x}{\ln a} + c$	$\int 2^x dx = \frac{1}{\ln 2} 2^x + c$		
8. Determine indefinite integrals of the form $\int f(ax + b)dx$.	Integrals of the form $\int (ax + b)^n dx$, including fractional values of n such as $\int \sqrt{ax + b} dx$, $\int \frac{1}{(ax + b)^n} dx$, etc	(i) Differentiate $y = \sqrt{9 - x^2}$ with respect to x. (ii) Hence, or otherwise, find $\int \frac{6x}{\sqrt{9 - x^2}} dx$.	
9. Determine $f(x)$, given $f'(x)$ and an initial condition $f(a) = b$ in a range of practical and abstract applications including coordinate geometry, business and science.	e.g. 1. The gradient function of a curve is given as $\frac{dy}{dx} = 3x^2 - 1$ and the curve passes through the point (4,1).	Sketch the primitive function given the graph of $\frac{dy}{dx} = f(x)$.	

	Find the equation of the curve.		
	2. Sketch $y=f(x)$ given graph of $y=f'(x)$.		

Established Goals(Syllabus Outcomes): MA12 – 3, 7, 9, 10