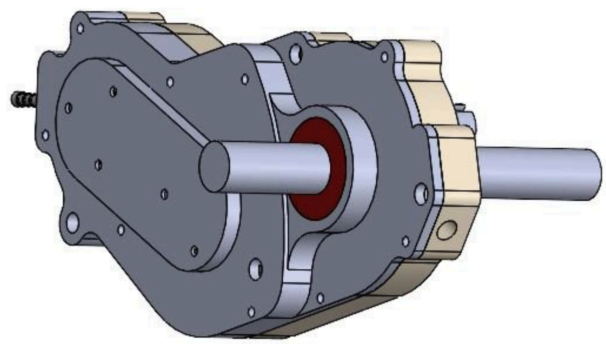
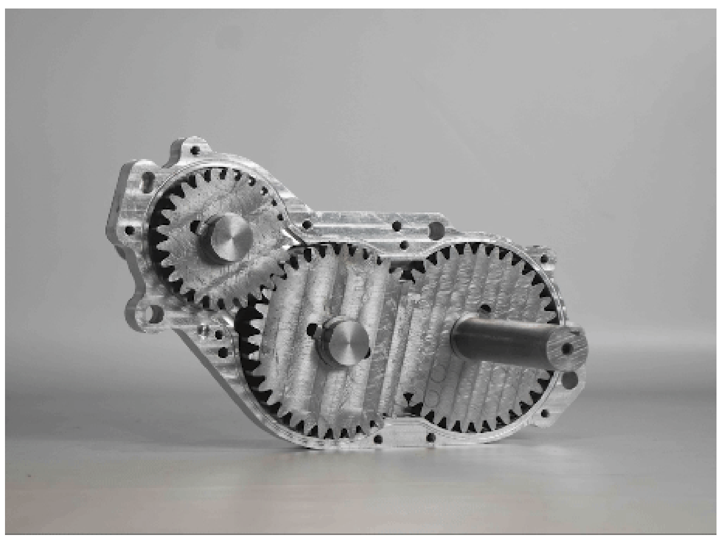
Olin Baja Gearbox Structural Integrity

by Kenta Terasaki





Abstract

The Baja SAE (Society of Automotive Engineers) competition presents a unique challenge for engineering students to design and build off-road vehicles capable of navigating rough terrains under extreme conditions. A key element in achieving high performance in such vehicles is the gearbox, responsible for efficient power transmission and the adjustment of torque-speed ratios. This study focuses on the analysis and optimization of a single-speed gearbox used in the Olin Baja SAE 2023-2024 vehicle, with particular attention to normal stress, shear stress, and bending deflections within the gearbox. The aim is to ensure that the gearbox design meets performance expectations and withstands the demanding conditions encountered in Baja SAE competitions.

Introduction

Gearboxes are essential components in the powertrain of Baja SAE vehicles, serving to transmit power from the engine to the wheels while adjusting the torque-speed ratio based on terrain demands. In this paper, we explore the design and stress analysis of a single-speed gearbox, a relatively simple yet effective configuration that offers a fixed gear ratio. Single-speed gearboxes, while limited in adaptability, provide robust solutions where simplicity and reliability are priorities.

The gearbox design process for the Olin Baja SAE team involved analyzing stresses, deflections, and safety factors using principles from Mechanics of Solids and Structures. Additionally, finite element analysis (FEA) was applied to validate hand calculations and refine the design.

Objective

The primary objective of this study is to evaluate the stresses and deflections experienced by the gearbox components under operating conditions. This analysis integrates theoretical knowledge of normal and shear stress with practical FEA simulations to optimize the gearbox design for durability and performance. The specific goals include:

- Application of the Lewis Form Equation to calculate the stress in gear teeth.
- Analysis of torque transfer through various gear ratios.
- Use of FEA to model stress distributions, deflections, and factor of safety under different load conditions.

Methodology

The methodology combines analytical and simulation-based approaches. Hand calculations were performed to estimate stress values using classical mechanics principles, particularly focusing on normal and shear stresses, and bending deflections. The Lewis Form Equation was employed to assess the stress in the gear teeth, which is critical for ensuring that the gear system can handle the high loads experienced during off-road operation.

Finite Element Analysis (FEA) was conducted to simulate the gearbox's response to applied forces, with emphasis on validating the results of hand calculations. The FEA simulations allowed for the evaluation of stress concentration areas and the overall structural integrity of the gearbox components. Additionally, the simulations provided insights into displacement and factor of safety under extreme conditions, offering a more detailed understanding of the gearbox's performance.

SAE Baja Restricted Kohler CH440							
Engine Speed (rpm)	3600	3400	3200	3000	2800	2600	2400
Torque Net Corrected (ft-lb)	13.5	14.5	15.4	16.6	17.4	18.1	18.5
Power Net Corrected (HP)	9.3	9.4	9.4	9.5	9.3	9.0	8.5

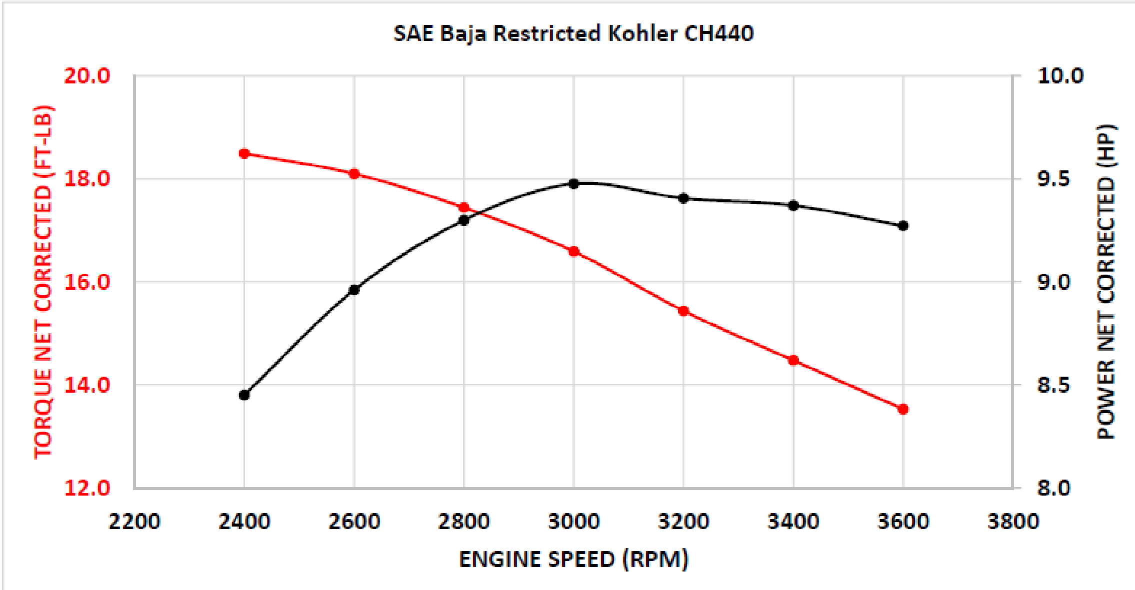


Figure 1

The gearbox is a two-stage gearbox with four gears. The stress in the gear teeth needs to be analyzed in order to ensure that the gears do not fail under max torque from the engine. *Figure 1* is the CH440 Kohler Engine Performance sheet with a max torque of 18.5 ft-lbs. Performance sheet, illustrates the engine's performance characteristics, serving as a critical input for the gearbox stress analysis. The maximum torque output from the engine dictates the upper stress limits on the gears, necessitating a robust analysis to ensure reliability and performance.

The gearbox under analysis is a two-stage configuration comprising four gears, designed to transmit power from the engine to the drivetrain. A key consideration in this design is the stress experienced by the gear teeth, which must be carefully evaluated to prevent failure under maximum torque conditions.

The engine powering the Baja SAE vehicle is coupled with a continuously variable transmission (CVT), which adjusts the gear ratio dynamically to optimize torque and speed based on driving conditions. The CVT employed in this system operates with a gear ratio range between 3.9 and 0.9, providing significant flexibility in torque transfer.

The following sections outline the foundation of the gearbox stress analysis, which is essential for validating the mechanical integrity of the design under real-world conditions.

Gear Stress Analysis and Factor of Safety Evaluation

Max torque on the input or driveshaft of the gearbox

$$\tau_{engine} * Max\ Gear\ Ratio_{CVT} = \tau_{gearbox\ input}$$
$$\tau_{gearbox\ input} = 18.5\ ft - lb * 3.9 = 72.15\ ft - lb$$

The max torque on the driveshaft of the gearbox is $72.15\ ft - lb$.

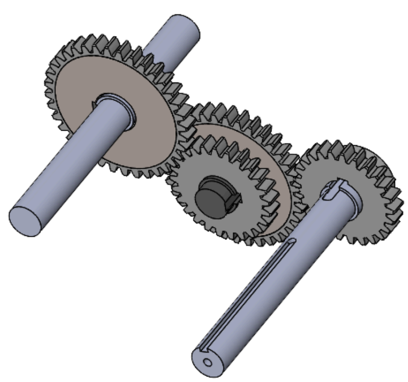
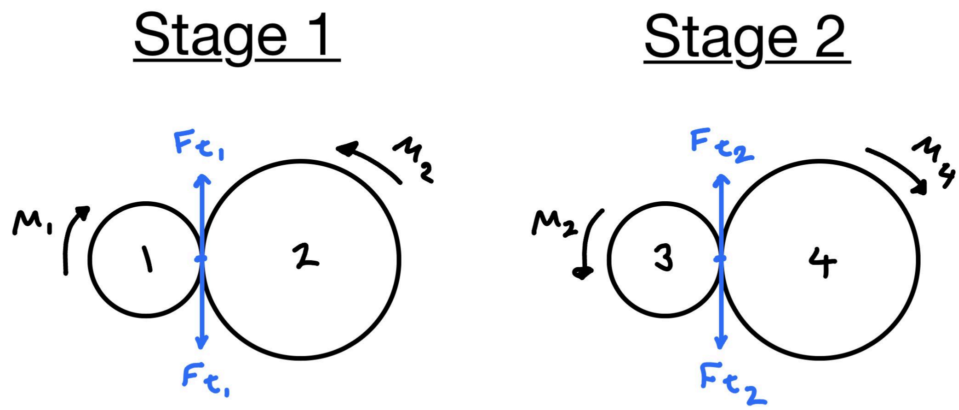


Figure 2

The gearbox has a pair of identical gears, with one pair being larger than the other. All the gears are S45c aisi 1045 carbon steel with a normal yield strength of 76,870 psi. The driving shaft holds the small gear, which is a mod 3, 25 tooth gear, pressure angle of 20°, and with a lewis form factor of 0.34. The large gear and small gear are held by the intermediate shaft. The driven shaft holds another large, which is a mod 3, 36 tooth gear, pressure angle of 20°, and with a lewis form factor of 0.377. The pitch diameter of the two gears are 0.246 ft (2.953 in) and 0.354 ft (4.252 in) respectively. Lastly, both have a diametrical pitch of 8.47 1/in. *Figure 2* displays the gears described.

Hand Calculations For Gear Stress and FoS

To ensure the mechanical integrity of the gearbox under maximum engine torque, it is essential to determine which gear(s) will experience the largest forces. In a multi-stage gearbox, the gear that undergoes the highest stress is typically the one that is subject to the greatest torque multiplication. Identifying this critical gear allows for a targeted analysis, as verifying the structural integrity of this gear can serve to confirm the reliability of the entire gear set.



The gears in stage 2 will experience the most force out of the four gears. This is because stage one has a gear ratio greater than one, so there is an increase in torque in stage 2. The forces acting on gears 3 and 4 (labeled above) will be the same due to Newton's 3rd Law. Therefore, gear 3 and 4 will equally experience the greatest force.

Because the forces acting on the teeth are the same, the tooth that will undergo more stress will come from the gear with less teeth with the same modulus because the size of the teeth are smaller. Therefore, gear 3's teeth will experience the greatest stress and is why the analysis should be done on gear 3 to ensure the rest of the gears teeth are validated.

Force between gear 1 and 2 are found:

$$F_{t1} = \frac{\tau_{gearbox\ input}}{r_1}$$

$$F_{t1} = \frac{72.15ft - lb}{0.123ft}$$

$$F_{t1} = 586.59lb$$

Moment around the 2nd gear is found. Because gear 2 and 3 are mounted on the same shaft, the moment around both will be the same.

$$M_2 = F_{t1} r_2$$
$$M_2 = 58606 lb * 2.126 in$$
$$M_2 = 1,247.1\ lb - in$$

The moment is used to find the force acting tangentially to the tooth in gear 3 and 4.

$$F_{t2} = \frac{M_2}{r_3}$$
$$F_{t2} = \frac{1,247.1\ lb - in}{1.477 in}$$
$$F_{t2} = 844.33 lb$$

The gear experiencing the largest force under maximum engine torque is analyzed using the **Lewis Form Equation** to estimate the maximum bending stress at the root of the tooth, the most critical point for potential failure. This calculation helps assess the risk of gear failure under load.

$$\sigma_3 = \frac{F_{t2} P}{W y}$$
$$\sigma_3 = \frac{844.33\ lb * 8.47\ 1/in}{0.5\ in * 0.34}$$
$$\sigma_3 = 42,047\ psi$$

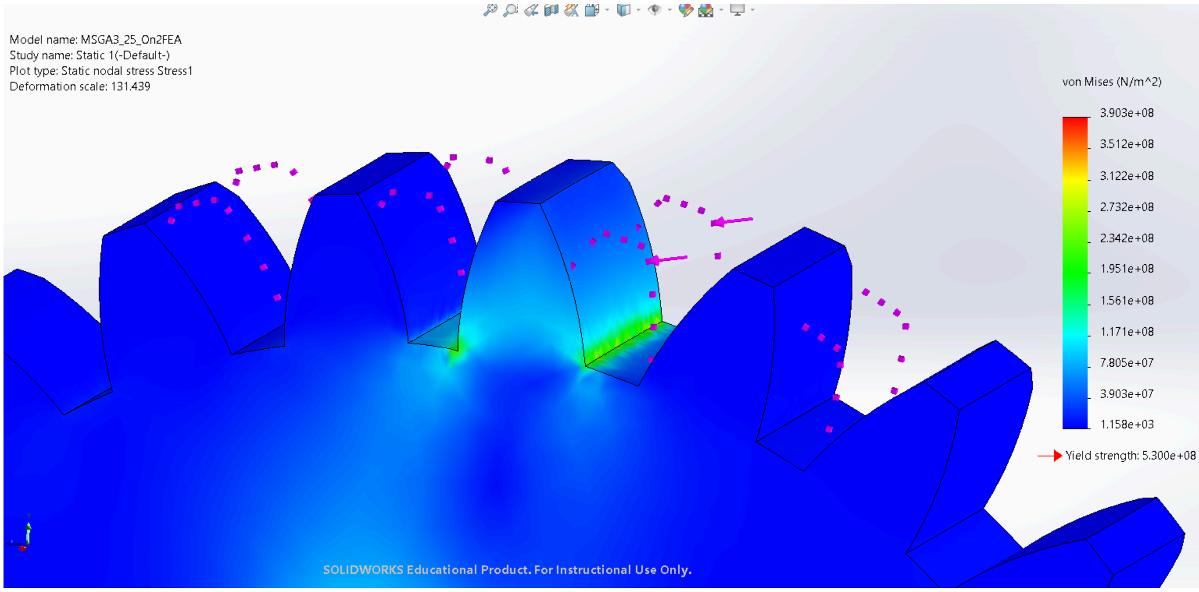
$$FoS = \frac{\sigma_{yield}}{\sigma_{max}}$$
$$FoS = \frac{76,870 psi}{42,047 psi}$$
$$FoS = 1.83$$

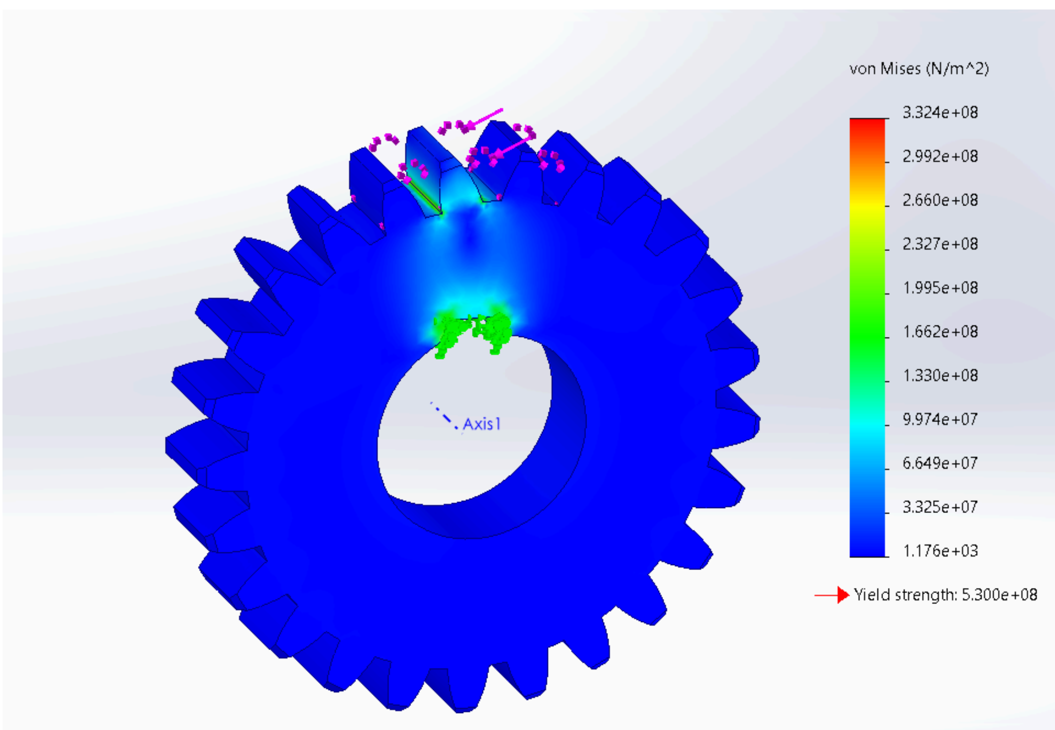
After determining the maximum stress, the **factor of safety (FoS)** is calculated to evaluate the gear's reliability under load. The FoS of 1.83 indicates a sufficient margin between the applied stress and the material's failure limit, ensuring the gear can withstand operating conditions without risk of failure.

FEA Analysis

Following the hand calculations, a Finite Element Analysis (FEA) model is developed to simulate the stress distribution and factor of safety for the gear experiencing the largest force. This computational model serves to validate the hand calculations by providing a more detailed visualization of stress concentrations and overall load-bearing performance. The comparison between the FEA results and the hand calculations offers valuable insight into the accuracy of the analytical approach.

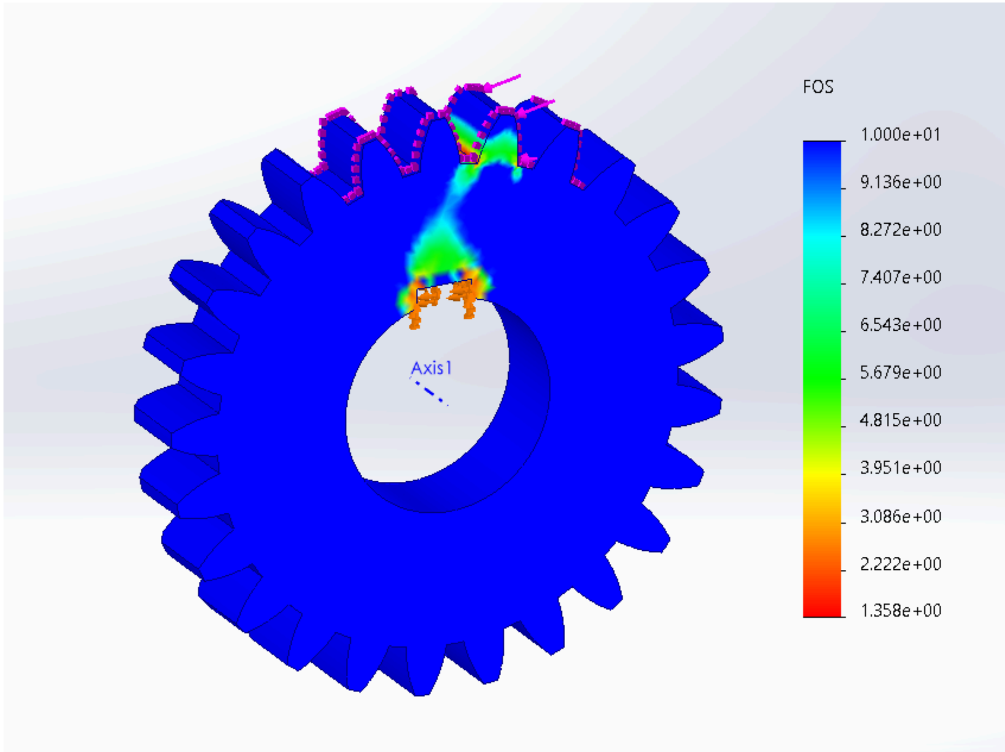
Stress Visuals:

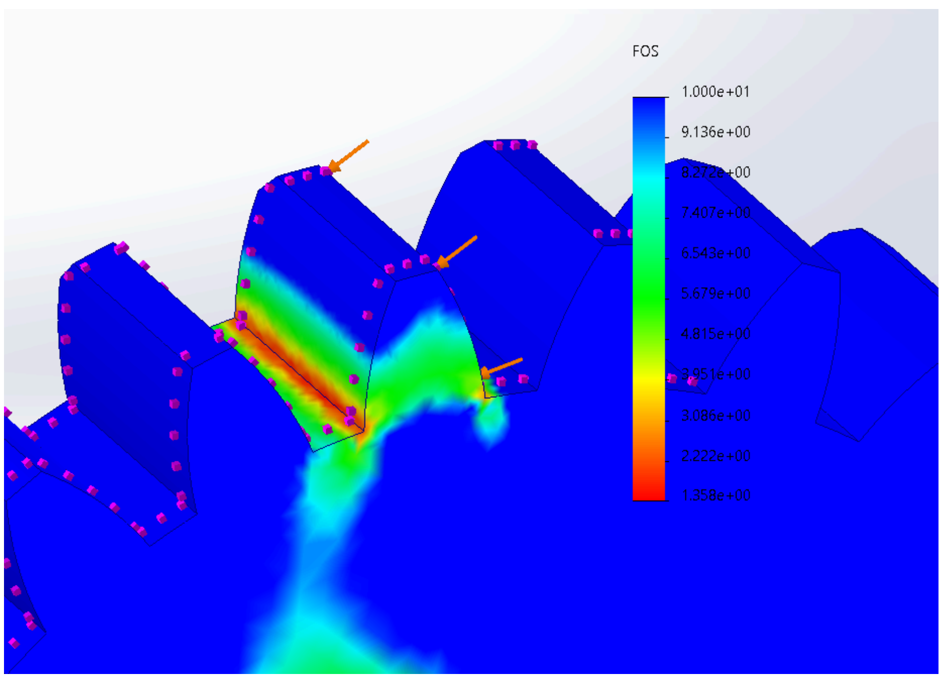




According to the FEA, the gear stress is 40,610 psi (2.8*10^8 Pa). This is accurate to the 42,047 psi calculated at the tooth from hand calculations.

Factor of Safety Visuals:





Looking at the FEA, the gear tooth factor of safety is 1.95. This is close to the FoS calculated through hand calculations of 1.83.

Validation of Hand Calculations with FEA and Assumptions

The Finite Element Analysis (FEA) results closely match the hand calculations, confirming the accuracy of the analytical methods used. Despite this alignment, several key assumptions were made during the analysis:

1. **No Frictional Losses**: The gear is assumed to experience maximum engine torque without accounting for frictional losses in the drivetrain, which could reduce the actual load.
2. **Tangential Force Distribution**: The force on the gear tooth is modeled as tangential and evenly distributed across the surface, without considering potential uneven force applications.
3. **Single Tooth Load**: The analysis assumes that only one tooth bears the entire load, whereas multiple teeth may share the load in reality, making this a conservative estimate.
4. **Keyed Slot Fixity**: The keyed slot in the gear is considered perfectly fixed, disregarding any play or deformation that could affect load transfer.
5. **Elastic Region Assumption**: All materials are assumed to remain within their elastic limits, ensuring the validity of the stress-strain relationships used.

While these assumptions simplify the analysis, the FEA validates the hand calculations, demonstrating the gearbox's ability to withstand maximum torque with sufficient reliability.

Key Shear Stress Analysis and Factor of Safety Evaluation

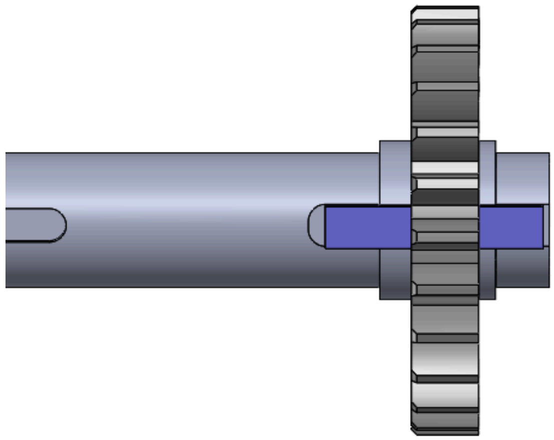


Figure 3

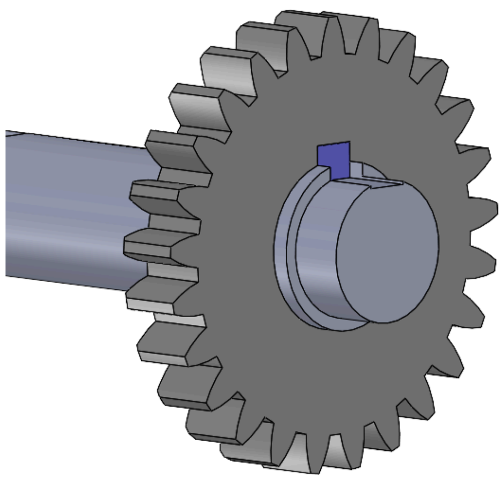
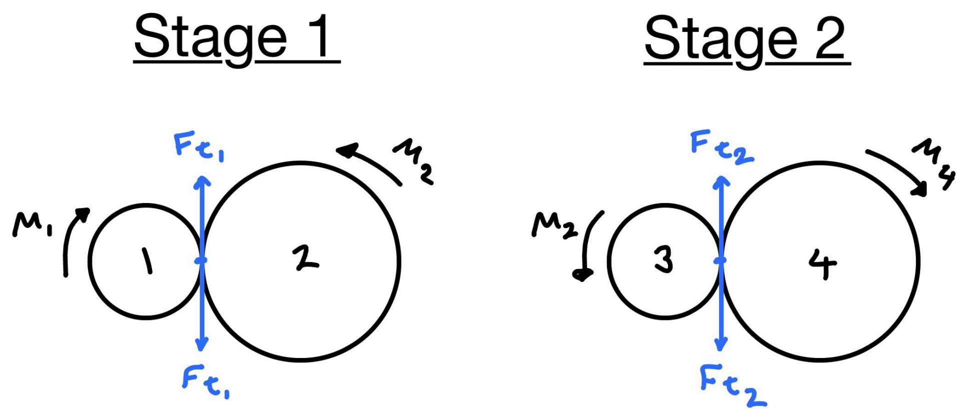


Figure 4

The key is a critical component of the gearbox as the gears are constrained from rotation about the shaft. In reality, the key is longer than the width of the gear as shown in *Figure 3*. However, for simplicity, the assumption is that the key is the width of the gear, shown in *Figure 4*. This is a fair assumption, as it makes the theoretical key calculations weaker than it is in reality. The key has a cross-sectional dimension of 0.25” by 0.25” by the width of the gear (0.5”). The key is made of 4140 alloy steel, and the yield shear stress of the key is 45,000 psi.

Hand Calculations For Shear Stress and FoS



To determine which gear key experiences the largest shear stress and force, an analysis was performed to identify the key subjected to the highest loads. Once identified, the shear stress for that key was calculated using standard shear stress formulas.

The gear key's largest force relies on the amount of force that is applied at the shear cross section of the key, which in this case is the center internal cross section of the key. Moment on the 4th gear (M_4) is larger that moment on the 3rd gear (M_3). With both key locations where the shear force and shear stress applied consistently, the force will be greatest on the 4th gear. The 4th gear will also have the highest shear stress because the area of cross section is consistent between all the keys.

$\gamma_{shear} = \frac{F}{A}$, where force is increased in the 4th gear and A is constant. Therefore, shear stress is greater in the 4th gear.

To find shear stress about gear 4, γ:
Moment of the 4th gear is calculated.

$$M_4 = F_{t2} * r_4$$
$$M_4 = 844.33lb * 2.126in$$
$$M_4 = 1795\, lb - in$$

The force in the 4th gear key is calculated.

$$F_{key} = \frac{M_4}{r_{key4}}$$
$$F_{key} = \frac{1795\,lb - in}{r_{0.5}}$$
$$F_{key} = 3590\,lb$$

Area of the 4th gear key cross section is calculated.

$$A = L * W$$
$$A = 0.25in * 0.5in$$

Shear stress is calculated using the shear stress formula.

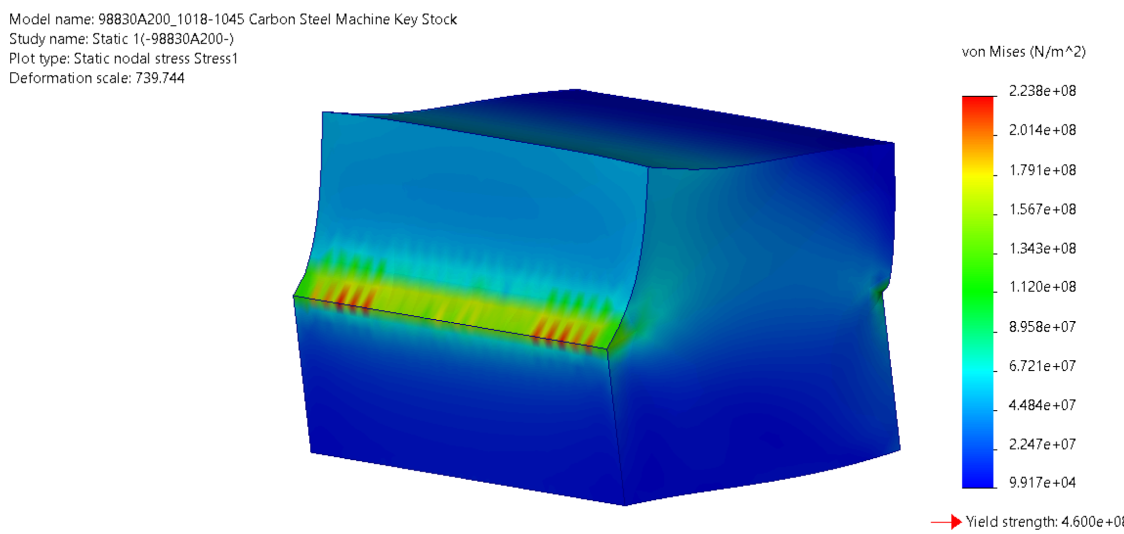
$$\gamma = \frac{F}{A}$$
$$\gamma = 28,702\,psi$$

$$FoS = \frac{\sigma_{yield}}{\sigma_{max}}$$
$$FoS = \frac{45,000psi}{28,702psi}$$
$$FoS = 1.57$$

The factor of safety (FoS) for the key with the largest force was calculated to ensure the design's robustness under operational conditions. This FoS is critical for evaluating the key's reliability when subjected to maximum load. The resulting FoS of 1.57 indicates that the key has an adequate safety margin to withstand the applied forces without failure.

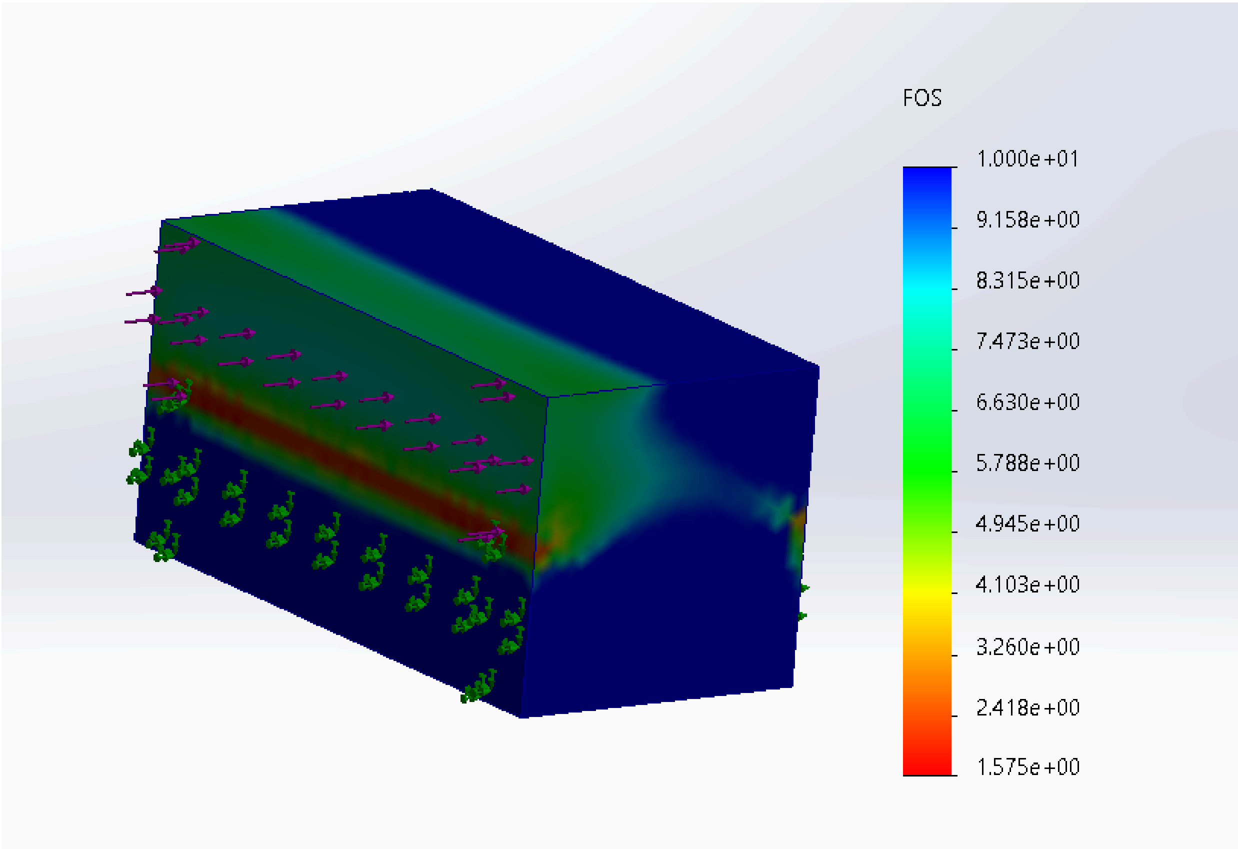
FEA Analysis

Stress Visuals:



According to the FEA, the key Stress is 29,007 psi (2.1*10^8 Pa) This is pretty accurate to the 28,702 psi calculated at the tooth from hand calculations.

Factor of Safety Visuals:



According to the FEA, the gear and shaft key factor of safety is 1.65 This is accurate to the 1.57 FoS calculated through hand calculations.

Validation of Hand Calculations with FEA and Assumptions

The Finite Element Analysis (FEA) results closely align with the hand calculations, confirming the accuracy of the analytical approach. Despite this consistency, several key assumptions were made during the analysis:

1. **Key Width Simplification**: For simplicity, the key was modeled with a width equal to that of the gear (as shown in Figure 4), whereas in reality, the key extends beyond the gear's width. This assumption makes the theoretical calculations more conservative, as the key is theoretically weaker than in actual conditions.
2. **No Frictional Losses**: It was assumed that the key experiences the maximum engine torque without accounting for any frictional losses in the system, which could reduce the actual applied load.
3. **Fixed Key Slot**: The bottom section of the key slotted into the shaft was considered perfectly fixed. In practice, there may be slight movement or play, which could influence the stress distribution.
4. **Elastic Region Assumption**: It was assumed that all materials remain within their elastic limits throughout the analysis, as the equations used apply only within this region.

These assumptions simplify the analysis, but the FEA results effectively validate the hand calculations, demonstrating that the key can withstand maximum torque with sufficient reliability.

Beam Stress Analysis and Factor of Safety Evaluation

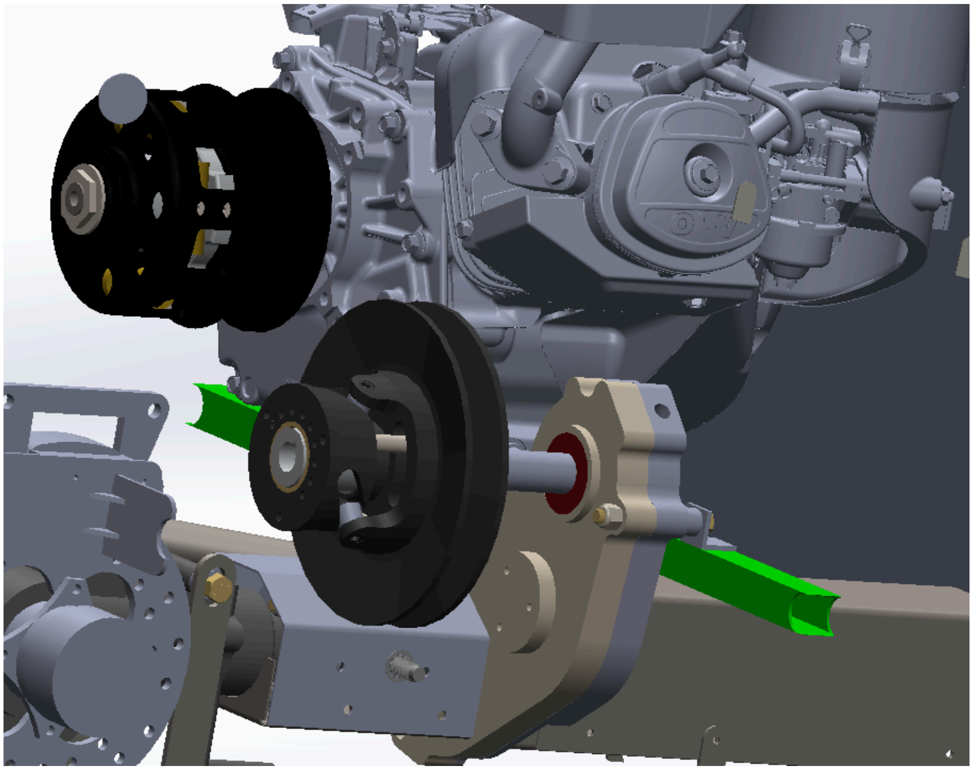


Figure 5

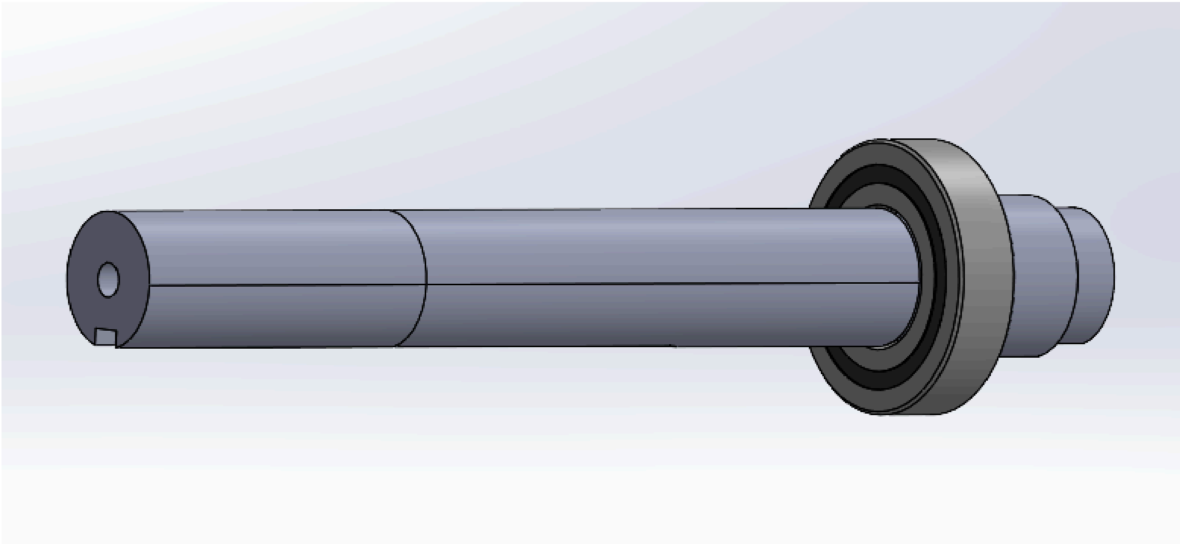


Figure 6

The input shaft of the gearbox's stress, deflection and factor of safety needs to be analyzed and verified to be safe. The input shaft is of greater concern compared to the output shaft because the input shaft experiences significantly more force and is longer. There is a force that acts on the input shaft of the gearbox by the CVT that is attached to the end. The force exerted on the input shaft comes from the tension in the belt of the CVT and is 182 lb-force. The input shaft is 7" long and output shaft is 4.6" long and both have a diameter of 1". The force acts 5.75'' away from the end of the bearing, and the bearing is treated as a fixed point on the shaft.

Hand Calculation For Deflection and FoS

Looking at a beam stress equation, in $\sigma = \frac{My}{I}$, you can see that *M* or the moment directly affects stress. This means that increasing force and length from the force to contact point will increase the stress on the shaft. The input shaft is applied with a greater force and is longer, which will make the stress greater compared to the output shaft. Therefore, if the input shaft is analyzed and verified, it is fair to assume that the output shaft will be fine.

Stress is calculated using stress for in a simple beam formula

$$\sigma = \frac{My}{I}$$

$$M = Fr$$

$$I_{hollow\ hoop} = \frac{\pi(d_o^4 - d_i^4)}{64} = 4.89 * 10^{-2} in^4$$

$$\sigma_{max} = \frac{182lb * 5.75in * 0.5in}{4.89 * 10^{-2} in^4}$$

$$\sigma_{max} = 10,707psi$$

The deflection equation for a point force in between the length of the rod is used. Noting, a is the force distance and L is the total rod distance.

$$\delta_b = \frac{Fa^2}{6EI}(3L - a)$$

$$\delta = \frac{182lb * 5.75^2 in}{6 * 29,732,736psi * 4.89 * 10^{-2} in^4}(3 * 7in - 5.75in)$$

$$\delta = 7.93 * 10^{-3} in$$

$$FoS = \frac{\sigma_{yield}}{\sigma_{max}}$$

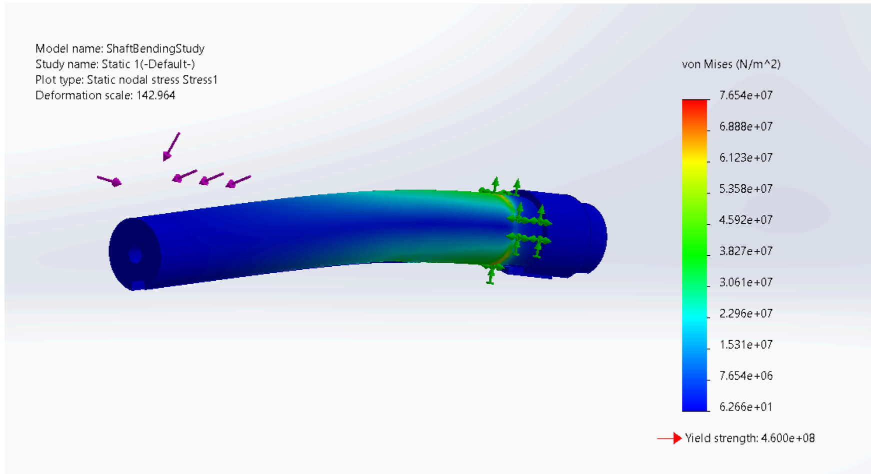
$$FoS = \frac{76,870psi}{10,707psi}$$

$$FoS = 7.1$$

An extremely sufficient factor of safety of 7.1 ensures that the input shaft will not fail under maximum operational loads.

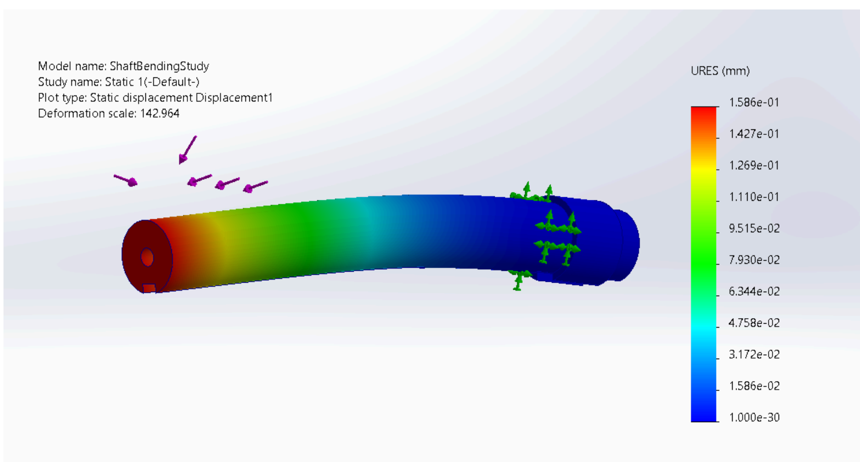
FEA Analysis

Stress Visuals:



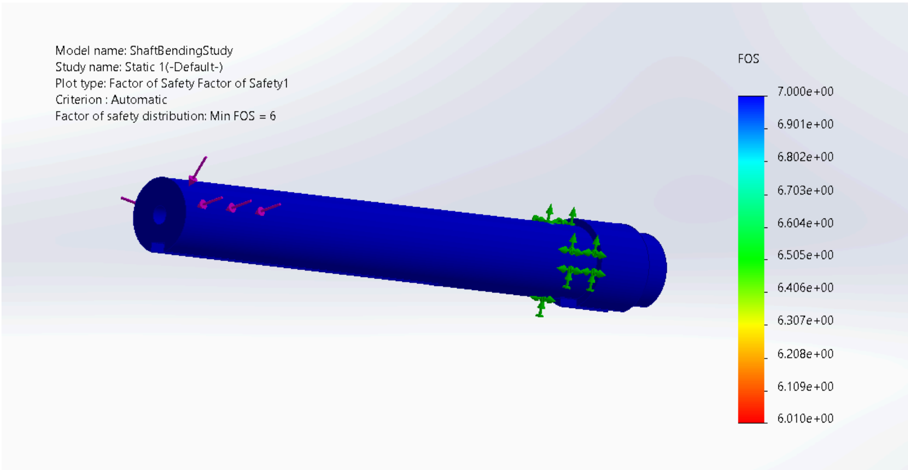
According to the FEA model, the max stress (σ_{max}) equals $11,095\ psi$ ($4.6 * 10^{8} Pa$). This is pretty close to the hand calculation max stress of $10,707\ psi$.

Deflection Visuals:



Looking at the FEA model, the deflection at the end of the beam is $6.24 * 10^{-3} in$ ($0.1586\ mm$).This compares well to the hand calculations deflection of $7.93 * 10^{-3} in$

Factor of Safety Visuals:



Looking at the FEA model, the minimum Factor of Safety is 6. This is comparable to the 7.1 from hand calculations.

Validation of Hand Calculations with FEA and Assumptions

The Finite Element Analysis (FEA) results closely align with the hand calculations, confirming the accuracy of the analytical approach. Despite this consistency, several key assumptions were made during the analysis:

1. **Uniform Shaft Geometry**: The shaft was modeled with a uniform hoop geometry, while in reality, it contains a keyway slot. This simplification overlooks stress concentrations at the slot, potentially underestimating local stress levels.
2. **Fixed Bearing Assumption**: The analysis assumed that the bearing surrounding the shaft is perfectly fixed, disregarding any potential movement or play. In practice, slight shifts in the bearing could affect the load distribution along the shaft.
3. **Elastic Region Assumption**: It was assumed that all materials remain within their elastic limits, ensuring the validity of the stress-strain relationships used in the calculations and simulations.

While these assumptions simplify the analysis, the FEA results effectively validate the hand calculations, confirming that the shaft can handle the applied loads with sufficient reliability.

Conclusion

This analysis provided an in-depth evaluation of the gearbox design for the Olin Baja SAE 2023-2024 vehicle. By combining hand calculations with Finite Element Analysis (FEA), the study verified the gearbox's reliability and durability while allowing for the practical application of theoretical concepts to a real-world engineering problem. The final gearbox design was optimized to meet the rigorous demands of the Baja SAE competition, achieving a balance between simplicity and high performance. This approach demonstrated the effectiveness of using both analytical and simulation methods in mechanical design.

List of Reference Materials:

1. Philpot Appendix G (Deflections and Slopes of Beams)
2. Kohler CH440 Engine Specs: https://www.bajasae.net/cdsweb/gen/DownloadDocument.aspx?DocumentID=b250342f-ea19-46c2-95ea-6deb728ddb7a
3. Lewis Form Bending Equation Formula and Table: https://www.engineersedge.com/gears/lewis-factor.htm