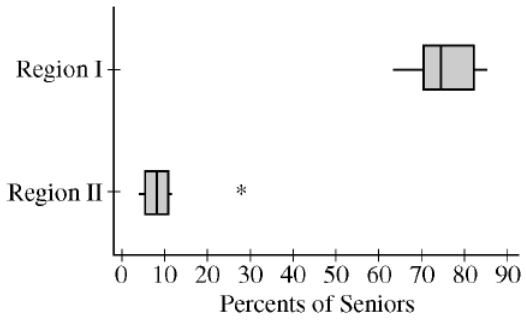
| AP Stat- Final FR- 2016/17 | Name: KEY |
|---|---|
| SECTION II (1-5): spend approx. 65 mins | Date: Pd: |

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

---

1. A university researcher is interested in comparing the percentages of high school seniors who took a college entrance exam for two different regions of the country. Region I and Region II. The researchers recorded the percent of seniors taking the exam for each high school within the two regions. The boxplots of the distributions of the percent of seniors who took the college entrance exam are shown below.




a) Compare the distributions of percent of seniors who took the college entrance exam for the two regions.

**Solution**

**Part (a):**

The median of the percentages of seniors who took the college entrance exam is more than 60 percentage points higher in Region I than in Region II. In fact, there is no overlap between the two distributions. A gap exists between the two distributions stretching from about 28% to about 63%, with Region I having the higher values. There is more variability in the percentages for Region I as measured by the IQR, but due to one large outlier in Region II, the range for Region II is slightly larger than the range for Region I.

Continue to next page...

**Scoring**

The question is scored in four sections, at least one of which must be in context. Section 1 consists of a comparison of center in part (a), section 2 consists of a comparison of spread in part (a), section 3 consists of a comment on the gap in part (b), and section 4 consists of a description of two clusters in part (b). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I). Section 4 is scored as essentially correct (E) or incorrect (I).

**Section 1** is scored as follows:

Essentially correct (E) if the student correctly *compares* center (or location) of the two distributions. If a comparison is made, no numerical values need to be given.

Partially correct (P) if the student correctly gives the medians of the two distributions, but does not compare them or compares incorrectly.

Incorrect (I) if the response does not meet the criteria for E or P.

**Section 2** is scored as follows:

Essentially correct (E) if the student correctly *compares* interquartile range *OR* range of the two distributions. If a correct comparison is made, no numerical values need to be given. Such comparisons include:

- IQR of Region I is larger than that of Region II, or
- Range of Region II is slightly larger than that of Region I, or
- Range (or spread) of Region I is larger than that of Region II without the outlier.

Partially correct (P) if the student correctly gives the ranges or interquartile ranges of the two distributions, but does not compare them or compares them incorrectly;
*OR*
if the comparison is made using only a general term such as *spread* or *variability* without defining it to claim that Region I has more spread than Region II;
*OR*
if the student makes a correct comparison based on stated but incorrect values for the range or the IQR.

Incorrect (I) if the response does not meet the criteria for E or P.

**Section 3** is scored as follows:

Essentially correct (E) if the student correctly refers to the gap.

Partially correct (P) if the student refers to the gap only in part (a);
*OR*
says that no values occur between about 12% and 63%;
*OR*
says in part (b) that there is "no overlap" between the two distributions.

Incorrect (I) if the student does not mention the complete separation between the two clusters in either part (a) or in part (b).

Continue to next page...

b) In writing a report, the researchers produced a single histogram of the combined data for Region I and Region 2. Describe the shape of the histogram for the combined data

**Part (b):**

The histogram will have two clusters with one centered around 8% and another around 74%, and a gap between the values of about 12% and 63% except for a solitary value around 28%.

**Section 4** is scored as follows:

Essentially correct (E) if the student notes in part (b) that the combined distribution consists of two clusters. The locations of the two clusters do not need to be stated.

Incorrect (I) if the response does not meet the criteria for E.

Each essentially correct (E) section counts as 1 point. Each partially correct (P) section counts as ½ point.

Each essentially correct (E) section counts as 1 point. Each partially correct (P) section counts as ½ point.

**4** **Complete Response**

**3** **Substantial Response**

**2** **Developing Response**

**1** **Minimal Response**

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

*Note:* In at least one of the four sections, the response must refer to the variable of percentage of students in the high school taking the exam in addition to referring to Region I and Region II. If no reference is made, the response cannot earn a score of 4.

Continue to next page...

2. .Swedish researchers investigated the relationship between chocolate consumption and stroke. The researchers gave a questionnaire about eating habits to a randomly selected sample of Swedish men. Based on the responses to the questionnaire, the men were classified into two groups. Group A consisted of the 9,250 men who ate the most chocolate per week, and group B consisted of the 9.250 men who ate the least chocolate per week. The researchers tracked the men's health for ten years. During that time, there were 458 cases of stroke among the men in group A and 543 cases of stroke among the men in group B.

a) Do the data provide convincing statistical evidence that Swedish men who would be classified into group A have a lower probability of stroke than Swedish men who would be classified into group B.

**Solution**

**Part (a):**

Step 1: States a correct pair of hypotheses.

Let $p_A$ and $p_B$ represent the probability of a stroke during the 10 years of the study for the populations of Swedish men who would be classified into group A and group B, respectively.

The hypotheses to be tested are $H_0 : p_A = p_B$ versus $H_a : p_A < p_B$, or equivalently, $H_0 : p_A - p_B = 0$ versus $H_a : p_A - p_B < 0$.

Step 2: Identifies a correct test procedure (by name or by formula) and checks appropriate conditions.

The appropriate procedure is a two-sample $z$-test for comparing proportions.

The men in the study were randomly sampled from the population of Swedish men.

The second condition is that the sample sizes are large relative to the proportions involved. This condition is satisfied because all sample counts are larger than standard thresholds such as 5 and 10. There were 458 cases with strokes and 8,792 cases without strokes in group A, and 543 cases with strokes and 8,707 cases without strokes in group B.

An additional condition may be checked: The population sizes are much larger than 10 (or 20) times the sample sizes.

Continue to next page...

Step 3: Correct mechanics, including the value of the test statistic and $p$-value.

The sample proportions who had strokes are $\hat{p}_A = \frac{458}{9250} = 0.0495$ and $\hat{p}_B = \frac{543}{9250} = 0.0587$.

The combined proportion who had a stroke is $\hat{p}_{combined} = \frac{458 + 543}{9250 + 9250} = 0.0541$.

The test statistic is $z = \frac{0.0495 - 0.0587}{\sqrt{0.0541(1 - 0.0541)\left(\frac{1}{9250} + \frac{1}{9250}\right)}} \approx -2.77$.

The $p$-value is $P(Z \leq -2.77) = 0.003$, where $Z$ has a standard normal distribution.

Step 4: State a correct conclusion in the context of the study, using the result of the statistical test.

Because the $p$-value is very small (for instance, less than 0.05), we reject the null hypothesis and conclude that the data provide convincing statistical evidence that the Swedish men who would be classified into group A have a lower probability of stroke than the Swedish men who would be classified into group B.

**Scoring**

The scoring has 4 sections. Section 1 consists of part (a) steps 1 and 3, section 2 consists of part (a) step 2, section 3 consists of part (a) step 4, and section 4 consists of part (b). Sections 1, 2, 3, and 4 are scored as essentially correct (E), partially correct (P), or incorrect (I).

**Section 1** is scored as follows:

Essentially correct (E) if the response includes the following components:
1. Identifies correct parameters;
2. Includes both hypotheses with labels and stating the correct relationships between the parameters;
3. Correctly calculates or states the value of the test statistic, and;
4. Provides a correct $p$-value.

Partially correct (P) if the response includes only two or three of the components necessary for E.

Incorrect (I) if the response includes at most one of the components necessary for E.

*Notes*:
- For step 1, either defining the parameter symbols in context, or simply using common parameter notation, such as $p_A$ and $p_B$, is sufficient. However, if parameter symbols are defined they must be done correctly, and must implicitly or explicitly refer to the population and not the samples.
- For step 3, the $p$-value is considered correct if it is consistent with the alternative hypothesis stated in the response and the calculated test statistic, even if those are incorrect.

Continue to next page...

**Section 2** is scored as follows:

Essentially correct (E) if the response correctly includes the following three components:

1. Identifies the correct test procedure (by name or by formula);
2. Notes the use of a random sample, and;
3. Checks for approximate normality by citing that all four counts are larger than some standard criterion such as 5 or 10, or by stating that all four counts are clearly greater than standard criteria.

Partially correct (P) if the response correctly includes only two of the three components required for E.

Incorrect (I) if the response correctly includes one or none of the three components required for E.

*Note:* Correctly using the combined proportion is sufficient for checking approximate normality.

**Section 3** is scored as follows:

Essentially correct (E) if the response correctly provides a conclusion in context and in terms of the alternative hypothesis AND correctly provides a justification based on linkage between the *p*-value and the conclusion. (No value of alpha was provided, so it is not necessary to state a specific alpha for the linkage.)

Partially correct (P) if the response provides a correct decision to reject $H_0$ but with an incorrect conclusion, AND with either correct linkage, or context, or both;

*OR*

if the response provides a correct conclusion in terms of the alternative hypothesis but with only correct linkage or only context, but not both.

Incorrect (I) if the response does not meet the criteria for E or P.

*Notes:*

- If the conclusion is consistent with an incorrect *p*-value from step 3, and also in context with justification based on linkage to the *p*-value, then section 3 is scored as E, unless the conclusion is stated as *accepting* the null hypothesis.
- If an appropriate value for $\alpha$ and the *p*-value are stated together, linkage is implied.
- If an interpretation of the *p*-value is included, it must be correct in order to receive credit for the conclusion.
- *For sections 1 to 3:* If the response includes a correct chi-squared test for homogeneity of proportions, then the response should be scored no higher than 3. If the chi-squared test *p*-value is halved to reflect the one-sided alternative hypothesis, the response should still be scored no higher than 3.

Continue to next page...

b) A report in a newspaper concluded that Swedish men can reduce their probability of stroke by eating more chocolate. Based on the description of the investigation, was the conclusion appropriate? Justify your answer.

**Part (b):**

The conclusion in the report was not appropriate. The report implied that eating more chocolate would *cause* the probability of having a stroke to go down. But the results were based on an observational study, not a randomized experiment. So a cause and effect conclusion is not justified

**Section 4** is scored as follows:

Essentially correct (E) if the response states that the newspaper conclusion was not appropriate, and provides justification based on a specific characteristic of the study, such as being an observational study, not using random assignment, not being an experiment, or not *controlling* for confounding variables.

Partially correct (P) if the response states that the newspaper conclusion was not appropriate, and gives a reasonable but generic justification such as association doesn't imply causation or a vague reference to the existence of confounding variables.

Incorrect (I) if the response does not meet the criteria for E or P.

Each essentially correct (E) section counts as 1 point, and a partially correct (P) section counts as ½ point.

**4 Complete Response**

**3 Substantial Response**

**2 Developing Response**

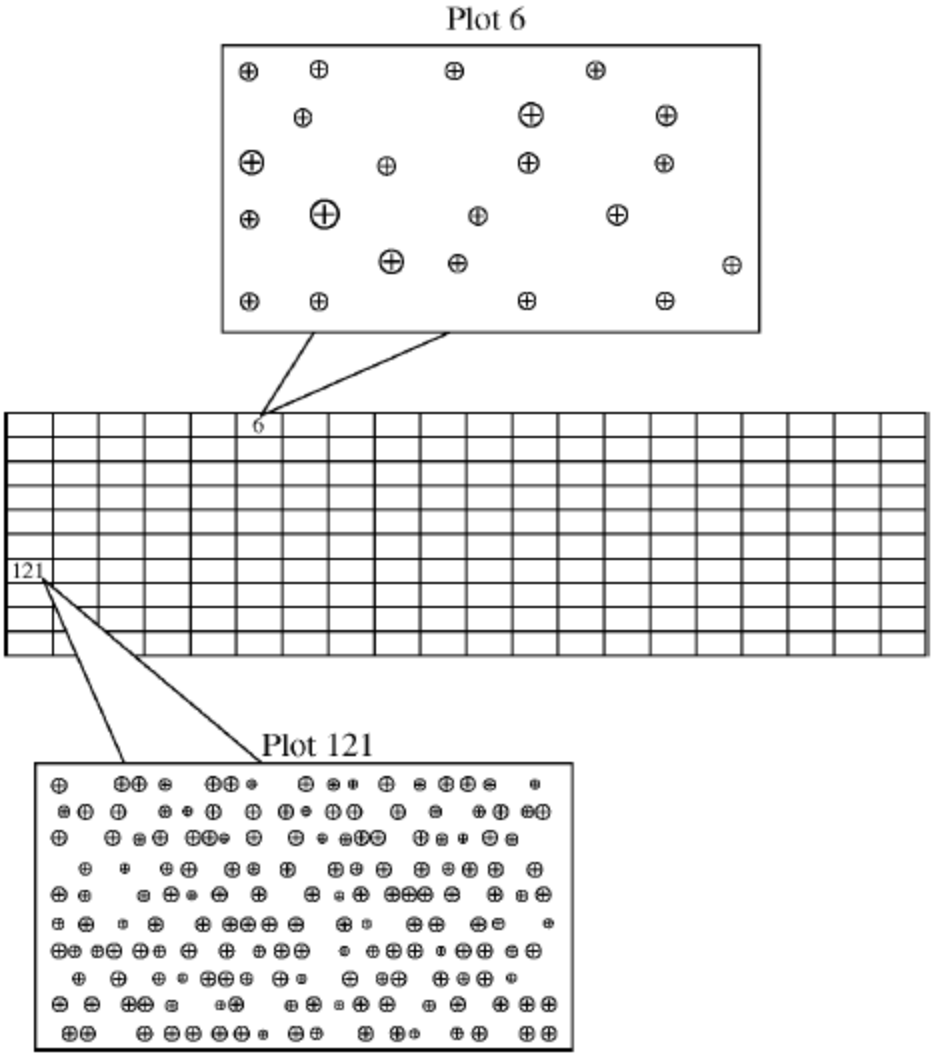
**1 Minimal Response**

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and communication.

Continue to next page...

3. Recently, a company acquired the rights to use a forest to harvest trees to produce lumber. The company wants to conduct a study to estimate the mean trunk diameter of the trees from the forest by taking a random sample of approximately 5 percent of the trees from the forest. For the study, the company divides the forest into 200 equally sized plots of approximately one acre each, as shown in the figure below.

Because of previous logging practices and growth patterns, plots with older trees, such as Plot 6, tend to have fewer trees but with larger trunk diameters, and plots with younger trees, such as Plot 121, tend to have more trees but with smaller trunk diameters. This is illustrated in the two figures of Plots 6 and 121 by the varying number and sizes of the symbol ⊕.




Continue to next page...

a) Describe a procedure for using cluster sampling to obtain a random sample of approximately 5 percent of the trees from the forest, using the plots as clusters.

**Solution**

**Part (a):**

To obtain a random sample using cluster sampling, number the plots from 1 to 200 (or use the numbers implied in the stem of the problem). Using a random number generator, such as a random number table or a random number generator from a calculator or computer, generate 10 unique random integers from 1 to 200. Select the plots corresponding to the 10 integers. Measure the diameters of *all* trees in the 10 selected plots.

**Section 1** is scored as follows:

Essentially correct (E) if the response describes a correct sampling procedure for a cluster sample that contains the following three components:

1. Samples about 5% of the trees (by sampling 5% of the plots).
2. Indicates that *all* trees in the selected clusters will be included in the sample.
3. Provides sufficient detail about how clusters will be selected.

*Note*: To satisfy component 3, it is not enough to just say that 10 plots are selected at random.

Partially correct (P) if the response satisfies only two of the three components *OR* satisfies only component 2 *OR* satisfies only component 3.

Incorrect (I) if the response does not meet the criteria for E or P.

*Note*: The statement of the problem in part (a) specifies "using the plots as clusters." If the response defines different clusters, section 1 cannot be scored as E. To earn a score of P, the response must also:

- indicate that *all* units (trees, plots, etc.) in the randomly selected clusters will be included in the sample *AND*
- provide sufficient detail about how the clusters will be selected.

b) Describe a procedure for using stratified sampling to obtain a random sample of approximately 5 percent of the trees from the forest, using the plots as strata.

**Part (b):**

In each of the 200 plots, number each of the trees from 1 to $n$, where $n$ is the number of trees in that plot. Within each plot, obtain a simple random sample of 5 percent of the trees by using a random number generator to generate unique random integers from 1 to $n$. Select the trees corresponding to the integers and measure the tree diameters.

Continue to next page...

**Section 2** is scored as follows:

Essentially correct (E) if the response describes a correct sampling procedure for a stratified random sample that contains the following three components:

1. Samples about 5% of the trees.
2. Indicates that the trees will be randomly selected from each stratum.
3. Provides sufficient detail about how trees will be selected from each stratum.

*Note*: To satisfy component 3, it is not enough to just say the trees are selected at random. More detail on how the selection will occur should be included.

Partially correct (P) if the response does not satisfy the three components but does satisfy component 2.

*Note*: The statement of the problem in part (b) specifies "using the plots as strata". If the response defines different strata, section 2 cannot be scored as E. To earn a score of P, the response must also:

- clearly indicate that a random sample will be selected from *each* stratum *AND*
- provide sufficient detail about how the units that make up the strata will be selected.

Incorrect (I) if the response does not meet the criteria for E or P.

c) For the study, give one advantage of using cluster sampling as described in part (a) over stratified sampling as described in part (b).

**Part (c):**

An advantage of using the cluster sample instead of the stratified sample is that the cluster sample is much easier to obtain. For the cluster sample, only 10 plots must be visited and the trees do not need to be individually numbered.

d) For the study, give one advantage of using stratified sampling as described in part (b) over cluster sampling as described in part (a).

**Part (d):**

If the distribution of tree diameters is different in different parts of the forest, an advantage of using stratified random sampling instead of cluster sampling is that the stratified sampling is more likely to result in a sample that is representative of the population of all tree diameters. Cluster sampling is more likely to yield a sample in which trees with large diameters or trees with small diameters are over-represented simply by chance.

Continue to next page...

**Section 3** is scored as follows:

Essentially correct (E) if the response provides the following two components:

1. A reasonable advantage of cluster sampling that is not also true of stratified sampling, with justification and context.
2. A reasonable advantage of stratified sampling that is not also true of cluster sampling, with justification and context.

Partially correct (P) if the response provides only one of the two components;
*OR*
if the response provides a reasonable advantage for both components, but has no context and/or no justification.

Incorrect (I) if the response does not meet the criteria for E or P.

**4 Complete Response**

All three sections essentially correct

**3 Substantial Response**

Two sections essentially correct and one section partially correct

**2 Developing Response**

Two sections essentially correct and one section incorrect

*OR*

One section essentially correct and one or two sections partially correct

*OR*

Three sections partially correct

**1 Minimal Response**

One section essentially correct and two sections incorrect

*OR*

Two sections partially correct and one section incorrect

Continue to next page...

4. A pinata is a container filled with toys and candy and is broken open by hitting it wth a stick. Sophia is trying to break open a pinata. The probability that she will break the pinata on the first hit is 0.7. She will continue to hit the pinata until it breaks. If she does not break the pinata on a particular hit, the pinata is weakened and the probability that she will break it on the next hit is 0.1 greater than the probability on the previous hit. For example, if the pinata does not break on the first hit, the probability that it will break on the second hit is 0.8.

a) Calculate the probability that Sophia **does not** break the pinata on the first hit and **does** break the pinata on the second hit.

**Solution**

**Part (a):**

The probability that the piñata does not break on the first hit *and* breaks on the second hit is equal to the probability that the piñata does not break on the first hit $(1 - 0.7 = 0.3)$ times the probability that the piñata does break on the second hit given that it did not break on the first hit (0.8)
Thus, $0.3 \times 0.8 = 0.24$.

**Scoring**

Part (a), (b) and (c) were scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the response correctly performs the calculation *AND* shows a correct method of solution.

Partially correct (P) if the response indicates knowledge to use the multiplication rule but multiplies two wrong, but reasonable, values based on information from the stem of the problem (for example, $0.7 \times 0.8$, or $0.7 \times 0.1$, or $0.3 \times 0.1$, or $0.3 \times 0.7$).

Incorrect (I) if the response does not meet the criteria for an E or a P, including if the response gives the correct answer with no work shown.

Let the random variable X represent the number of hits required for Sophia to break the pinata.
b) Complete the probability distribution of X in the table below.

| X | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Probability of X | | | | |

Continue to next page...

**Part (b):**

$$P(X = 2) = 0.3 \times 0.8 = 0.24$$

$$\begin{aligned} P(X = 3) &= P(\text{no break on first hit and no break on second hit and break on third hit}) \\ &= P(\text{no break on first hit}) \times P(\text{no break on second hit} \mid \text{no break on first hit}) \\ &\times P(\text{break on third hit} \mid \text{no break on first hit and no break on second hit}) \\ &= 0.3 \times 0.2 \times 0.9 = 0.054 \end{aligned}$$

$$\begin{aligned} P(X = 4) &= 1 - P(X = 1) - P(X = 2) - P(X = 3) \\ &= 1 - 0.7 - 0.24 - 0.054 = 0.006 \end{aligned}$$

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Probability of $x$ | 0.7 | 0.24 | 0.054 | 0.006 |

**Part (b)** is scored as follows:

Essentially correct (E) if the response correctly calculates all three probabilities *AND* shows sufficient work for at least two of the probabilities, which can include work coming from part (a) or part (c).

*Note*: If the answer to part (a) is not correct and is carried forward to part b), then the response to part (b) can earn an E if one additional probability is calculated correctly with supporting work and the four probabilities sum to 1.

Partially correct (P) if the response correctly calculates at least two of the three probabilities but shows sufficient work for only one of the probabilities, which can include work coming from part (a) or part (c);
*OR*
if the response correctly calculates only one of the three probabilities *AND* shows sufficient work, which can include work coming from part (a) or part (c) *AND* reports probabilities that sum to 1;
*OR*
if the responses calculates geometric probabilities *AND* shows work *AND* makes the probabilities sum to 1.

*Note*: The response in part b) can receive a score of P if the response correctly calculates all three probabilities with no supporting work shown.

Incorrect (I) if the response does not meet the criteria for an E or a P.

c) Calculate and interpret the expected value of X.

Calculate:

Interpretation:

Continue to next page...

**Part (c):**

$E(X) = 1(0.7) + 2(0.24) + 3(0.054) + 4(0.006) = 1.366$

Interpretation: If Sophia repeats this process for a very large number of piñatas, the average number of hits required to break the piñata will be very close to 1.366 hits per piñata.

**Part (c)** is scored as follows:

Essentially correct (E) if the response calculates the expected value correctly *AND* provides a correct interpretation that refers to "long run" or "average value" and context.

*Notes*:

1. An expected value calculation that is consistent with probabilities reported in (b) gets credit, even if the result is not a reasonable value.
2. If the expected value calculation is rounded to an integer, the response does not receive credit for the calculation.

Partially correct (P) if the response includes only one of the two aspects (calculation, interpretation) correctly.

Incorrect (I) if the response includes neither of the two aspects (calculation, interpretation) correctly

**4 Complete Response**

All three parts essentially correct

**3 Substantial Response**

Two parts essentially correct and one part partially correct

**2 Developing Response**

Two parts essentially correct and one part incorrect

*OR*

One part essentially correct and one or two parts partially correct

*OR*

Three parts partially correct

**1 Minimal Response**

One part essentially correct and two parts incorrect

*OR*

Two parts partially correct and one part incorrect

Continue to next page...

5. The weights of red delicious apples are approximately normally distributed with a mean of 9 ounces and a standard deviation of 0.75 ounce. An online gift store sells gift boxes contained 5 red delicious apples. At the time of packaging, 5 red delicious apples are randomly selected and packaged in a box.

a) Describe the distribution of the total weight of the 5 randomly selected apples.

**Solution**

**Part (a):**

Let $T$ denote the total weight of 5 randomly selected red delicious apples. Because the apples are randomly selected, and each apple's weight is approximately normally distributed, $T$ is also approximately normal with mean equal to the sum of the means of the distribution of each of the 5 apple's weights, $E(T) = 9 + 9 + 9 + 9 + 9 = 5(9) = 45$ ounces, and a variance equal to the sum of the variances of the five apples, $Var(T) = (0.75)^2 + (0.75)^2 + (0.75)^2 + (0.75)^2 + (0.75)^2 = 5(0.75)^2 = 2.8125$ ounces squared. The standard deviation is $\sigma_T = \sqrt{2.8125} \approx 1.677$ ounces.

**Part (a)** is scored as follows:

Essentially correct (E) if the response correctly provides all three components of the distribution of the total weight, with appropriate justification: shape (approximately normal), center (mean is 45 ounces), and spread (standard deviation is 1.677 ounces).

Partially correct (P) if the response correctly gives only two of the three components with appropriate justification.

Incorrect (I) if the response does not meet the criteria for E or P.

.

b) What is the probability that the total weight of the 5 randomly selected apples will be less than 42 ounces?

**Part (b):**

Using the parameters from part (a), the appropriate normal probability can be calculated as follows:

$$P(T < 42) = P\left(Z < \frac{42 - 45}{1.677}\right) = P(Z < -1.79) = 0.0368.$$

Continue to next page...

**Part (b)** is scored as follows:

Essentially correct (E) if the response shows the correct probability with a correct normal probability calculation method;
*OR*
if the response shows the correct probability with a well-labeled sketch.

Partially correct (P) if the response sets up a correct normal probability calculation but does not carry it through correctly;
*OR*
if the response uses the standard deviation given in the stem of the problem and carries out a correct normal probability calculation.

Incorrect (I) if the response does not meet the criteria for E or P.

*Notes*:

- If the normal probability calculation in part (b) includes an incorrect value for the mean or standard deviation as obtained in part (a), part (b) should be scored as E if the probability calculation is carried out correctly.
- A response that arrives at the correct answer in the context of a significance test lowers the score by one level (that is, from E to P, or P to I).
- A response that includes an incorrect mathematical statement lowers the score by one level (that is, from E to P, or P to I).
- Because the probability distribution and its parameter values were asked for in part (a), the response does not have to repeat that information in part (b) to earn an E.

c) The combined weight of the packing material and box in which the apples will be shipped is always 10 ounces. Let W represent the weight of a complete packaged gift box, which consists of the packing material, box, and 5 randomly selected apples. What are the mean and standard deviation of W?

**Part (c):**

Let $W$ denote the weight for the packaged gift box, so $W = 10 + T$. The expected value (mean) of $W$ is $E(W) = E(10 + T) = E(10) + E(T) = 10 + 45 = 55$ ounces. The variance of $W$ is $Var(W) = Var(10 + T) = Var(10) + Var(T) = 0 + 2.8125 = 2.8125$ ounces squared. Thus, $W$ has a standard deviation of $\sigma_W = \sqrt{2.8125} \approx 1.677$ ounces, because adding a constant to a random variable does not change its variance or standard deviation.

Continue to next page...

**Part (c)** is scored as follows:

Essentially correct (E) if the response provides the correct values for the two parameters (mean and standard deviation) with appropriate justification for each.

Partially correct (P) if the response provides the correct values, with appropriate justification, for only one of the two parameters.

Incorrect (I) if the response does not meet the criteria for E or P.

*Notes*:

- An incorrect mean and/or standard deviation carried through into part (c) and used in calculating the mean and standard deviation, with appropriate justification, is acceptable for an E.
- Appropriate justification may be provided through a correct calculation or written explanation.

**4 Complete Response**

All three parts essentially correct

**3 Substantial Response**

Two parts essentially correct and one part partially correct

**2 Developing Response**

Two parts essentially correct and one part incorrect

*OR*

One part essentially correct and one or two parts partially correct

*OR*

Three parts partially correct

**1 Minimal Response**

One part essentially correct and no parts partially correct

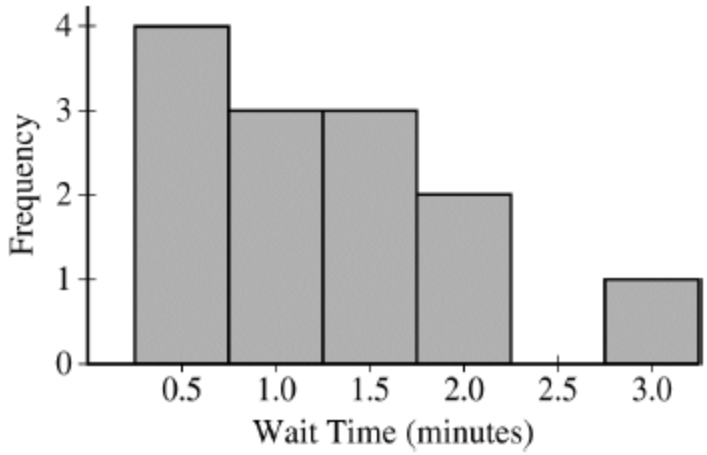
*OR*

No parts essentially correct and two parts partially correct

Continue to next page...

| AP Stat- Final FR- 2016/17 | Name: |
|---|---|
| SECTION II (6): spend approx. 25 mins | Date: Pd: |

6. Phone callers to a bank's customer service center must wait until a service representative is available to answer the phone call. The bank manager is interested in estimating the mean customer wait time. Thirteen calls were selected at random. A histogram of the 13 wait times, in minutes, is shown below.




a) Based on the histogram, explain why it might not be appropriate to use a one-sample t-interval to estimate the mean wait time for all customers.

**Solution**

**Part (a):**

Using a one-sample $t$-procedure to produce a confidence interval for the population mean would not be appropriate because the distribution of wait times is clearly skewed to the right, and the sample size ($n = 13$) is not large.

**Section 1** is scored as follows:

Essentially correct (E) if the response:

1. Comments that the distribution of the sample data is skewed (not symmetric).
2. Comments that the sample size is small.

Partially correct (P) if the response correctly includes only one of the two components.

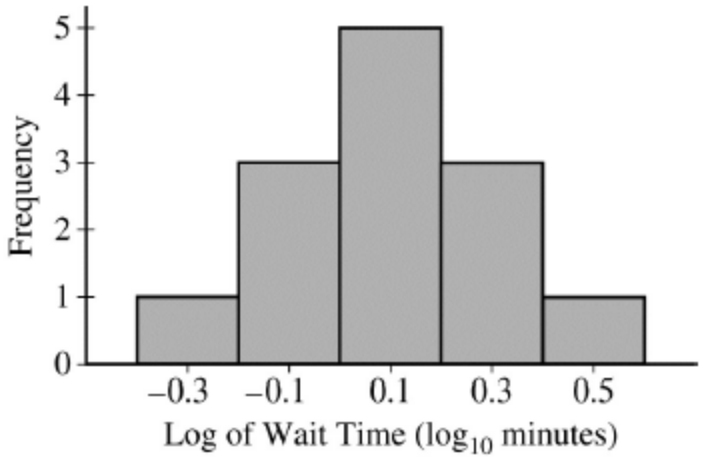
Incorrect (I) if the response correctly includes neither of the two components.

Continue to next page...

A logarithmic transformation is often used to transform data as wait times. Let x represent a customer's wait time. The log transformation of the customer's wait time is given by $log_{10} x$. The table below shows the original 13 wait times, the respective log-transformed wait times, and the corresponding means, medians, and standard deviations.

| | Wait Time, $x$ | $\log_{10} x$ |
|---|---|---|
| | 0.40 | −0.3979 |
| | 0.66 | −0.1805 |
| | 0.71 | −0.1487 |
| | 0.71 | −0.1487 |
| | 1.10 | 0.0414 |
| | 1.16 | 0.0645 |
| | 1.20 | 0.0792 |
| | 1.29 | 0.1106 |
| | 1.29 | 0.1106 |
| | 1.70 | 0.2304 |
| | 1.90 | 0.2788 |
| | 2.15 | 0.3324 |
| | 2.82 | 0.4502 |
| Mean | 1.31 | 0.0632 |
| Median | 1.20 | 0.0792 |
| Standard deviation | 0.679 | 0.235 |

A histogram of the 13 log-transformed data values is shown below.




Continue to next page...

The summary statistics for x and $log_{10} x$ are represented below.

| | Wait Time, $x$ | $\log_{10} x$ |
|---|---|---|
| Mean | 1.31 | 0.0632 |
| Median | 1.20 | 0.0792 |
| Standard deviation | 0.679 | 0.235 |

b) Based on the histogram, the conditions for inference have been met for the log-transformed data. Construct and interpret a 95% confidence interval for the population mean μ of the log of the wait times.

**Part (b):**

The interval is found as $\bar{x} \pm t_{(12,\ 0.975)} \frac{s}{\sqrt{n}}$ or $0.063 \pm 2.179\left(\frac{0.235}{\sqrt{13}}\right) = 0.063 \pm 0.142$, giving the interval $(-0.079, 0.205)$. We can be 95% confident that the mean of the population of log wait times is between −0.079 log minutes and 0.205 log minutes.

c) The mean of the log-transformed data is 0.0632 $log_{10} minutes$, which can be converted back to 1.157 minutes by calculating $10^{0.0632}$. Convert the endpoints of your interval in part (b) back to minutes and write the resulting interval.

**Part (c):**

The endpoints are $10^{-0.079}$ and $10^{0.205}$, so the interval is 0.834 minutes to 1.604 minutes.

**Section 2** is scored as follows:

Essentially correct (E) if the response correctly includes the following three components:

1. In part (b), reports the interval correctly.
2. In part (b), interprets the interval correctly including that it is for the mean of the log of the population of wait times.
3. In part (c), correctly computes both endpoints.

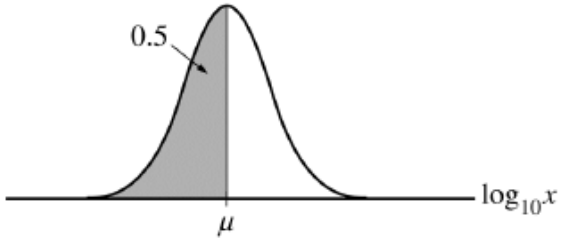
Partially correct (P) if the response correctly includes only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

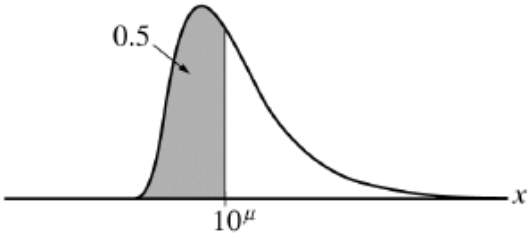
Continue to next page...

Graph 1 below shows a population distribution of the log wait times, in $log_{10}\, minutes$, which is normal with mean $\mu$. Graph 2 shows the result of converting the population distribution in Graph 1 back to the population distribution of wait times, in minutes. The lower 50 percent of the distribution is shaded in each graph.

Graph 1
Population Distribution of Log of Wait Time




Graph 2
Population Distribution of Wait Time




d) Consider the parameter $10^{\mu}$ in Graph 2.

i) How does the parameter $10^{\mu}$ compare with the median of the population distribution of wait times?

ii) How does the parameter $10^{\mu}$ compare with the mean of the population distribution of wait times?

**Part (d):**

(i) The parameter $10^{\mu}$ is equal to the median of the population of wait times.

(ii) The parameter $10^{\mu}$ is less than the mean of the population of wait times.

**Section 3** is scored as follows:

Essentially correct (E) if the response:

1. States that the parameter $10^{\mu}$ is equal to the median of the population of wait times.
2. States that the parameter $10^{\mu}$ is less than the mean of the population of wait times.

Partially correct (P) if the response correctly includes only one of the two components.

Incorrect (I) if the response correctly includes neither of the two components.

Continue to next page...

e) Write an interpretation of the interval you constructed in part (c).

**Part (e):**

The interval in part (c) is a 95% confidence interval for the population *median* of the wait times. Therefore, we are 95% confident that the median of the population of wait times is between 0.834 minutes and 1.604 minutes.

**Section 4** is scored as follows:

Essentially correct (E) if the response correctly states that the interval is a confidence interval for the population median wait time AND gives a correct interpretation of a 95 percent confidence interval;
*OR*
if the response states that the interval is a confidence interval for $10^{\mu}$ *AND* gives a correct interpretation of a 95% confidence interval.

Partially correct (P) if the response correctly states that the interval is a confidence interval for the population median of the wait times *OR* for $10^{\mu}$, but does not give a correct confidence interval interpretation;

Incorrect (I) if the response states that the confidence interval is for some other parameter, such as the mean of the population of wait times, or the median of the log wait time, and then gives a correct confidence interval interpretation based on that parameter;
*OR*
if the response otherwise does not meet the criteria for E or P.

Each essentially correct (E) section counts as 1 point, and a partially correct (P) section counts as ½ point.

**4** **Complete Response**

**3** **Substantial Response**

**2** **Developing Response**

**1** **Minimal Response**

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and communication.

Continue to next page...