6-3 Parametric Equations and Motion Day 1

Imagine that a rock is dropped from a 420-ft tower. The rock's height y (in feet) above the ground seconds later (ignoring air resistance) is modeled by $y = -16t^2 + 420$. The rock actually falls in a straight line, so we can graph 2 equations to simulate the fall.

$x = 3$

$y = -16t^2 + 420$ when t = 0, 1, 2, 3, 4, 5

These 2 equations are an example of parametric equations with a parameter t which represents time.

On your calculator:

Mode: Parametric

y = x =

y =

window: t as it is described

x

y

Definition:

The graph of the ordered pairs (x,y) where

$x = f(x)$

$y = g(x)$

are functions defined on the interval *I* of t-values is a **parametric curve**.

The equations are **parametric equations** for the curve, the variable t is a **parameter** and *I* is the **parameter interval**.

Example 1: Graphing Parametric Equations:

For the given parameter interval, graph the parametric equations:

$x = t^2$

$y = 3t$

a. $-3 \leq t \leq 1$ b. $-2 \leq t \leq 3$ c. $-3 \leq t \leq 3$

Example 2: Eliminating a Parameter:

Eliminate the parameter and identify the graph of the parametric curve.

x = 1 - 2t

y = 2 - t where $-\infty \leq t \leq \infty$

Example 3: Eliminating a Parameter:

Eliminate the parameter and identify the graph of the parametric curve.

x = t^2 - 2

y = 3t

Example 4: Eliminating a Parameter:

Eliminate the parameter and identify the graph of the parametric curve.

x = 2 cos t

y = 2 sin t

Facts about the parameter with Trig functions:

1. The parameter t is the central angle, so the condition $0 \leq t \leq 2\pi$ causes the grapher to trace the circle completely starting at (1,0).
2. Changing the endpoints of the parameter interval moves the starting and the final points for the grapher.
3. If the length of the parameter interval is greater than 2π then the grapher traces a part of the circle more than once.
4. If the length of the parameter interval is less than 2π then the grapher traces a portion of the circle.

Example 5: Finding Parametric Equations for a Line

Find the parametrization of the line through the points A = (-2,3) and B = (3,6).

Example 6: Finding Parametric Equations for a Line Segment

Find the parametrization of the segment through the points A = (-2,3) and B = (3,6).

Practice:
Eliminate the parameter and identify the curve:

1. $x = 2-t$
   $y = 2t - 1$

2. $x = t$
   $y = 1 - t^2$

3. $x = t^2$
   $y = 2 - t$

4. $x = 6 \cos t$
   $y = 6 \sin t \quad -\pi \leq t \leq \pi$

5. $x=7 \cos t$
   $y = 7 \sin t \quad -2\pi \leq t \leq 2\pi$

6. Find the parametrization of the line through A (2,1) and B (6, 1).

7. Find the parametrization of the segment through A(1,-1) and B (-2, 7).