The FA2 script: a reader's companion 🤓

The goal of this Google document is to allow for students to share (suspected) typos and clarifications related to the FA2 script. Anyone with the link can edit the document. Edits are done anonymously.

Typos

- Page 84: Computation on bottom, should be t^p and not t^{p-1}.
- Page 87: Definition of uniform convexity is fishy; for all epsilon there exists a delta s.t. for all points …
- Page 89: Operator norm of J(f) should contain the sup, not inf.
- Page 97: For the case f in L^infinity, both bounds of the two norms of T(f) are missing a *norm(f).
- Page 99: The convergence of T(fk)-T(f) to 0 in L^q0 + L^q1 implies a.e. Convergence of T(fk) to T(f) a priori only along a subsequence, not the whole sq.
- Page 134: The CZ decomposition is with threshold alpha = 2^{-i}, since we need g <= alpha on the "good" set Omega^c.
- Page 140: In Remark 7.2, it should be v = -y/|y| and not v = y/|y|.
- Page 145: Definition of \hat{K}\epsilon should be lim_{r \to \infty}.
- Page 150: Should be K := inverse FT of m and not K := FT of m.
- Page 157: After "one has…", it should be = and not +.
- Page 157: In 7.76, the + should be -.
- Page 159: It should be |g_\alpha(x)| \le \alpha and not |f(x)| \le \alpha.

Clarifications

If you found yourself stuck on some detail for way too long, feel free to share how you resolved it – chances are others will struggle with the same points.

Also, if you have Qs or would like to sanity check your reasoning, feel free to mark your item with "Question". Once someone else has had a look at your item, they can do a quick write-up of their reasoning and toggle the status to "Resolved".

- Page 123: I think more care is needed in the proof of Proposition 6.32 b). If we expand the expression for the Lorentz norm of f_k - f_\infty, we don't get (f_k)_* - (f\infty)_*. So the proof in the text doesn't work. Makes sense? Resolved
  Potential solution: This makes sense, since linearity doesn't hold here. What we need is that (f_k - f_\infty)_* converges pointwise to 0, which would then allow us to invoke DCT. Inspired by gpt, a potential (maybe unnecessarily complicated) way to show this is as follows:
  1. Observe that convergence in measure to 0 implies that the decreasing arrangement converges pointwise to 0;

    2. For all s > 0, the set where |f_infty| > s has finite measure, since f_infty lies in L^{p,q} for some p < \infty;
    3. Let r > 0. If |f_k - f_infty| > r, then 2 * |f_infty| >= |f_k| + |f_infty| > r and hence |f_infty| > r/2. So the set where |f_k - f_infty| > r is contained in the set where |f_infty| > r/2 ;
    4. Since the measure of the set where |f_infty| > r/2 is finite (by the 2. point), convergence in measure is implied by almost everywhere convergence. In particular, the measure of the set where |f_k - f_infty| > r converges to 0;
    5. Since r was arbitrary, (f_k - f_infty) converges in measure to 0. By the 1. point we are done.
- Page 123: In the proof of Theorem 6.33 (2), before dividing by |f|_{L^{p,r}}^{(r-p)/r}, we need that |f|_{L^{p,r}} is finite. This follows from the second last equality and the fact that |f|_{L^{p,\infty}} is finite if |f|_{L^{p,q}} is finite.
- Page 125: In the proof of Theorem 6.35, it isn't immediate from 6.37 that this is a norm. Positivity follows from equivalence with the quasinorm proved below (or by a direct proof, noting that f_** is continuous). Homogeneity needs to be proved separately, but is not too bad.
- Page 130: "and thus, as in the proof of Proposition 6.33". They just need the result of Proposition 6.33, not the entire proof.
- Page 150: In Theorem 7.10, it should be l in N (including 0).
- Page 151: K_k := inverse FT of m_k.

Recommendations

- Basically anything by Grafakos:
    - https://link.springer.com/book/10.1007/978-1-4939-1194-3
    - https://link.springer.com/book/10.1007/978-3-031-56500-7