Assignment#1

MTH622 (SPRING 2022)

Total marks: 20

Modules#1-52

Due date: June 10, 2022

DON'T MISS these important instructions:

- Upload assignments properly through LMS.
- All students are directed to use the font and style of text as is used in this document.
- This is an individual assignment, not group assignment, so keep in mind that you are supposed to submit your own and self-made assignment even if you discuss the questions with your class fellows. All similar assignments (even with some meaningless modifications) will be awarded zero marks and no excuse will be accepted. This is your responsibility to keep your assignment safe from others.
- Solve the assignment on MS word document.

Question#1

Find a unit normal to the surface $x^2y + 2xz = 3$ **at the point** $(1,1,1)$ **.**

MARKS 10

Solution:

$$Unit\ normal\ vector = \hat{n} = \frac{\nabla\varphi}{|\nabla\varphi|}$$

$$\nabla = \frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}$$

$Given:$

$$x^2y + 2xz = 3$$

$$\varphi = x^2y + 2xz - 3$$

$$\frac{\delta\varphi}{\delta x} = \frac{\delta(x^2y + 2xz - 3)}{\delta x} = 2xy + 2z$$

$$\frac{\delta\varphi}{\delta y} = \frac{\delta(x^2y + 2xz - 3)}{\delta y} = x^2$$

$$\frac{\delta\varphi}{\delta z} = \frac{\delta(x^2y + 2xz - 3)}{\delta z} = 2x$$

$$\nabla\varphi = \frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}$$

$$\nabla\varphi = (2xy + 2z)\hat{i} + x^2\hat{j} + 2x\hat{k}$$

$At\ the\ po\text{int}\ (1,1,1)$

$$\nabla\varphi = (2(1)(1) + 2(1))\hat{i} + (1)^2\hat{j} + 2(1)\hat{k}$$

$$\nabla\varphi = 4\hat{i} + \hat{j} + 2\hat{k}$$

$$|\nabla\varphi| = \sqrt{(4)^2 + (1)^2 + (2)^2} = \sqrt{16+1+4} = \sqrt{21}$$

$$Unit\ normal\ vector = \hat{n} = \frac{\nabla\varphi}{|\nabla\varphi|} = \frac{4\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{21}} = \frac{1}{\sqrt{21}}\left(4\hat{i} + \hat{j} + 2\hat{k}\right)$$

Question#2

Find the total work done in moving a particle in a force field given by

$\vec{F} = 2xy\hat{y} — 3z\hat{\jmath} + 8x$ **Along the curve** $x = t^2,\ y = 2t^2 + 1,\ z = t^3 - 1$ **from** t **= 0 to** t**= 1. (MARKS 10)**

Solution:

$$As\ we\ know\ work\ done\ on\ a\ curve\ is:$$

$$\int_c f.d\bar{r}$$

$$d\bar{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\bar{F}.d\bar{r} = \left(2\boldsymbol{xy\hat{i}} — 3\boldsymbol{z\hat{j}} + 8\boldsymbol{x\hat{k}}\right)\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$= \left(2xydx - 3zdy + 8xdz\right)..............(1)$$

$\text{Since}:$

$$x = t^2 \quad , \frac{dx}{dt} = 2t \;\Rightarrow dx = 2tdt$$

$$y = 2t^2 + 1 \quad , \frac{dy}{dt} = 4t \Rightarrow dy = 4tdt$$

$$z = t^3 - 1 \quad , \frac{dz}{dt} = 3t^2 \;\Rightarrow dz = 3t^2 dt$$

Put All these values in (1)

$$= 2\left[\left(t^2\right)\left(2t^2 + 1\right)\left(2tdt\right)\right] - 3\left[\left(t^3 - 1\right)\left(4tdt\right)\right] + 8\left[\left(t^2\right)\left(3t^2 dt\right)\right]$$

$$= 2\left(t^2\right)\left(4t^3 + 2t\right)dt - 3\left(4t^4 - 4t\right)dt + 8\left(3t^4\right)dt$$

$$= 2\left(4t^5 + 2t^3\right)dt - 3\left(4t^4 - 4t\right)dt + 8\left(3t^4\right)dt$$

$$= 8t^5 dt + 4t^3 dt - 12t^4 dt + 12tdt + 24t^4 dt$$

$$= (8t^5 + 12t^4 + 4t^3 + ot^2 + 12t)dt$$

$$\text{work done} = \int_c f.d\overline{r} = \int_0^1 (8t^5 + 12t^4 + 4t^3 + ot^2 + 12t)dt$$

$$= \left| \frac{8t^6}{6} + \frac{12t^5}{5} + \frac{4t^4}{4} + \frac{0t^3}{3} + \frac{12t^2}{2} \right|_0^1$$

$$= \left(\frac{8(1)^6}{6} + \frac{12(1)^5}{5} + \frac{4(1)^4}{4} + \frac{0(1)^3}{3} + \frac{12(1)^2}{2} \right) - \left(\frac{8(0)^6}{6} + \frac{12(0)^5}{5} + \frac{4(0)^4}{4} + \frac{0(0)^3}{3} + \frac{12(0)^2}{2} \right)$$

$$= \left(\frac{4}{3} + \frac{12}{5} + 1 + 6 \right)$$

$$= \frac{4}{3} + \frac{12}{5} + 7$$

$$= \frac{56}{15} + 7$$

$$= \frac{56+105}{15} \Rightarrow \frac{161}{15}$$

$$= 63$$

Hence the work done is 63