1.

a. For the complex number $z = -2 + 5\mathrm{i}$, find:

i. $|z|$

(1)

ii. $\arg(z)$, where $-\pi < \arg(z) \leqslant \pi$

(1)

b. State the modulus and argument of z^*

(2)

2.

On an Argand diagram, illustrate the locus of points z that satisfy the inequality

$$3 < |z - 3 + 4\mathrm{i}| \leqslant 5$$

(4)

3.

On an Argand diagram, represent the region defined by the inequalities

$$0 < \arg(z - 2\mathrm{i}) \leqslant \frac{3\pi}{4} \text{ and } |z - 2\mathrm{i}| \geqslant 2$$

(4)

4.

The complex number $3 - 3\mathrm{i}$ is denoted by a

a. Find $|a|$ and $\arg(a)$

(2)

b. Sketch on a single Argand diagram the loci given by

i. $|z - a| = 3\sqrt{2}$

ii. $\arg(z - a) = \frac{\pi}{4}$

(2)

c. Indicate, by shading, the region of the Argand diagram for which

$|z - a| \geqslant 3\sqrt{2}$ and $0 \leqslant \arg(z - a) \leqslant \frac{\pi}{4}$

(1)

5.

Let $z_1 = \dfrac{\sqrt{6} + \mathrm{i}\sqrt{2}}{2}$, and $z_2 = 1 - \mathrm{i}$

a. Find the value of $\dfrac{z_1}{z_2}$ in the form $a + b\mathrm{i}$, where a and b are to be determined exactly in surd form.

(2)

b. Show that $\dfrac{z_1}{z_2} = \cos\left(\dfrac{5\pi}{12}\right) + \mathrm{i}\sin\left(\dfrac{5\pi}{12}\right)$

(2)

6.

By considering the product $(2 + \mathrm{i})(3 + \mathrm{i})$, show that $\arctan\dfrac{1}{2} + \arctan\dfrac{1}{3} = \dfrac{\pi}{4}$

(4)

7.

Complex number $z \text{ and } w$ are given by

$$z = 2\left(\cos\frac{\pi}{3} - \mathrm{i}\sin\frac{\pi}{3}\right), \qquad w = \sqrt{3}\left(\cos\frac{5\pi}{6} - \mathrm{i}\sin\frac{5\pi}{6}\right)$$

a. Find the value for $\arg\dfrac{z}{w}$

(1)

b. Sketch on the same Argand diagram for $z,\ w \text{ and } z - w$

(3)

c. Show that $|z - w| = \sqrt{7}$

(2)

8.

a. Find the Cartesian equation of:

i. the locus of points representing $|z - 3 + \mathrm{i}| = |z - 1 - \mathrm{i}|$

ii. the locus of points representing $|z - 2| = 2\sqrt{2}$

(6)

b. Find the two values of *z* that satisfy both $|z - 3 + i| = |z - 1 - \mathrm{i}| \text{ and } |z - 2| = 2\sqrt{2}$

(2)

The region R is defined by the inequalities $|z - 3 + i| \geqslant |z - 1 - \mathrm{i}| \text{ and } |z - 2| \leqslant 2\sqrt{2}$

c. Show the region R on an Argand diagram.

(4)
