

Teaching Numeracy:

Problem One: Teaching Subtraction

Year group: 6

NZC level: Level 3

- Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages.
- Understand addition and subtraction of fraction, decimals, and integers

Strand: Set stage 6

Achievement Objective: Advanced additive - Early multiplicative

Know how many 2s, 3s, 5s, and 10s in numbers to 100 and any remainders. Round whole numbers to the nearest 10, 100, 1000.

Problem: Hemi has 263 kumara in his kete. Aroha has 147. How much more kumara does Hemi have than Aroha?

Strategy to solve the problem: Equal Adjustment Strategy

I know that my future class will have mixed-ability learners; therefore, this lesson will be planned for the more advanced students with a sound understanding of subtraction given the higher value numbers in the worded problem. The problem should be solvable, although it will require the students to extend and challenge their current thinking, which is essential for mathematical growth. I will work alongside the group to manage and monitor student participation and encourage students to listen and learn from each other when evaluating possible solutions to the problem (Anthony & Walshaw, 2009). This is important as it allows students to work collectively and consider different approaches to solving the equation, all the while ensuring they understand the proposed strategy I would model.

I need to start the lesson with the learning intention for this particular session which is working out the difference between 263 and 147. For this lesson to benefit my students, it must be relevant, exciting, and engaging. As a teacher I must consider how well the strategy provides the opportunity for students to investigate the mathematics content in an open but structured way, connecting them with current knowledge while pushing them deeper (Breyfogle & Williams, 2009). To achieve overall success within the group instruction, I will use students' names and possible real-life scenarios to model the problem, personalising it, therefore, making it meaningful and relevant. The equal adjustment strategy will be used, as it best suits this particular problem and can be explained to students without becoming complex and overwhelming. I will use counting cubes as a manipulator to aid visual learners and provide hands-on thinking while solving this problem.

Having given students the opportunity to first try and solve the problem unaided I would then introduce my equal adjustment strategy. To solve the problem, students must first break down the scenario into its numeral components. What are we trying to solve? In this case, we are

trying to determine the difference between 263 and 147. So we know Hemi has 263 kumara and Aroha has 147; therefore, the solvable equation is $263 - 147$ is equivalent to? But how will we work this out? We first model our numbers using the counting cubes as students draw connections between numbers, symbols and manipulators when understanding a strategy (Yurekli et al., 2020) therefore students will form groups of 263 and 147 to represent both Hemi's and Aroha's kumara. Now what if we round the smaller number 147 to the nearest tidy 10, which would mean we need to add 3 cubes to that pile which would now make our total number of cubes on that pile 150. For this strategy to work, we need to add the equivalent number of extra cubes to the other pile, so 3 more cubes need to be added to 263, bringing the total number of that pile to 266. Now that we have changed the values of both piles, we need to rewrite the equation as $266 - 150$ is equivalent to? Rounding the smaller number to a tidy 10 helps us mentally count backwards as this year and the stage group should be confident skip counting in tens therefore they should be able to identify that the difference will be 116 as $200 - 100$ is 100 and $66 - 50$ is 16. I would run through this strategy a couple of times to ensure the task is understood, once I am satisfied they have grasped the concept I would supply students with a worksheet of further problems to solve including higher value numbers to stretch students' current understanding.

Problem two: Teaching Division

Year group: 6

NZC level: Level 3

- Use simple additive strategies with whole numbers and fractions.

Strand: Set stage 6

Achievement Objective: Advanced additive - early multiplicative

Knows how many 2s, 3s, 5s. And 10s in numbers to 100 and any remainders

Problem one: Hemi has 40 kina. He needs to share them equally with his 5 aunties. How many kina will each aunty get? (partitive division)

Problem two: Aroha has 40 kina. She put the kinas into bags of 5 (5 kina in each bag) . How many bags of kinas did she make? (quotitive division)

Strategy to solve the problem: Partitive and Quotitive with multiplicative facts

Before modelling the strategies to solve the above problems, I need to ensure my students understand the mathematical language associated with division so they can identify the different components used within it as Ann Downton stated “to understand division requires more than knowledge of sharing out a collection equally” (Downton, 2013). To check for understanding I need to explain to my students the different strategies of partitive & quotitive division and communicate the importance around the words dividend and divisor as they will see these words used frequently when solving division equations and how they link to multiplicative facts and repetitive subtraction.

So what is a dividend and a divider? A simple way to teach students the difference is that the dividend is the number that we are dividing, in this case 40 whereas the divider is the number by which we divide, in this case the 5. To reword it we have 40 kina (the divided) and we are sharing them equally 5 ways (the divisor). My students will also need know the difference between the two strategies that will be modelled, partitive more commonly known as the sharing model where we know how many groups there are, and need to solve how many objects go into each group, and the quotitive model where we know how many will be in each group but need to solve how many groups there will be. For this year and stage group, manipulators will be necessary in order to gain the required knowledge and understanding around groupings using both models. I would also use a warm up exercise of skip counting in 5s as this is relevant to the solvable problem.

Downton States “the notion that division of whole numbers can be interpreted in two different ways reflects its relationship to multiplication, namely division by partitive model and division by the multiplicand quotitive model” (Downton, 2013). I believe that manipulators that are hands-on will be most beneficial for this particular strategy, in this case I would use counting beans to represent the kina. In a small instructional group I will ask students to place 40 counting beans on the table that will represent our kina that we are trying to distribute and group evenly. Looking at the partitive model first I would then read the problem aloud which will be displayed on a whiteboard accessible for the group to see. I would then underline the numerical components we are trying to solve which are the dividend of 40 and the divider of 5. I would then say to my students we know we have 5 aunties, how will we share the kina so that each aunty gets the same amount? At this stage students should be able to link the problem to a multiplication equation as they should be confident with their $\times 5$ facts therefore counting in 5s up to 40 would uncover the value of 8 ($5 \times 8 = 40$). Using the manipulator I would then get students to model the equation by grouping the counting beans into 5 equal groups representing the kina which would reveal 8 therefore each aunty gets 8 kina this allows a visual representation of the partitive problem. To solve the quotitive problem, we could use repetitive subtraction. We have 40 kina (the divided) and we need to put 5 kina into each bag (the divider) so how many bags of kina will we have? We can start with 40 and use our subtractive knowledge minus 5 (as that is the divider) bringing us to 35 then minus 5 again and so on until we get to 0. As we write the equation each time and model with the beans as we subtract, it will show us that we subtracted 5 kina 8 times therefore we would need 8 bags.

As Brandberg and Sharp quotes “It is most effective to plan day by day so that reflections made from one lesson can be used to inform the next” (Brandberg & Sharp, 2020) therefore, I must ensure that my students understand the concept given before planning the next step, which may mean revisiting the current problem of 40 divided by 5 the following day until I have confirmation that students have mastered it through observation assessment. Once mastered more problems can be introduced where more challenging equations are given to strengthen other areas of multiplication which are used side by side with division.

Problem Three: Teaching addition of fractions

Year group: 7

NZC level: Level 4

- Understand addition and subtraction of fractions, decimals, and integers.
- Find fractions, decimals and percentages of amounts expressed as whole numbers, simple fractions, and decimals.

Strand: Set stage 7

Achievement Objective: Advanced multiplicative - early proportional

Problem: Hemi ate $\frac{3}{4}$ slices of pizza. Aroha ate $\frac{7}{8}$ pieces of pizza. How many pieces of pizza did they eat together?

Strategy to solve the problem: Adding Relatable and Compatible Numbers

Teaching the addition of fractions may seem daunting to students as fractions can seem complex, and therefore mindsets may already be programmed in negative modes. As a teacher, my role is to create lesson pedagogy that is engaging, interactive and relevant in order for my students to achieve. Understanding the language of fractions is necessary for comprehension regarding the lesson instruction. Students need to be aware that the denominator (the number below the line) tells you how many parts the whole has been broken up into, and the numerator (number above the line) tells you how many of these parts to take away or shade in (Clarke et al., 2008). Understanding a part-whole is also needed as its interpretation depends on the ability to partition either a continuous quantity including area, length, and volume model or a set of discrete objects into equal size subparts or sets (Clark et al., 2008). Therefore the start of my lesson will be around clarity and understanding of what we are trying to achieve and how we will achieve it using models and new and existing knowledge around fractions.

In order to solve this problem we will use a manipulator of paper and colour pencil to represent our whole shape and the subparts that we are taking away. This particular manipulator is a visual tool that students can use to identify the whole and subparts to the problem aiding in overall understanding.

For this particular problem, we are trying to figure out how many pieces of pizza Hemi and Aroha ate together. I suggest that my group of students start by drawing a circle shape on a piece of paper and cutting it out so we can identify that the circle shapes represent a pizza. Let's say we fold the paper into 4 equal parts as we know the denominator is 4, and then shade 3 parts as the numerator in this fraction represents 3 subparts. The next instruction would be to fold the paper back up; this time, we will fold it again. When we open it up, we now should have 8 parts to the whole and 6 parts shaded, representing how many slices Hemi ate. As the problem states that Aroha ate $\frac{7}{8}$ of a pizza, we know that we need to draw another whole pizza as $\frac{7}{8}$ is greater than $\frac{2}{8}$, as represented as remaining on the first pizza drawn. From this fraction, we know that we need to fold the paper into 8 parts as there are 8 parts to the denominator, and we need to shade in 7 subparts as that represents the slices Aroha has eaten.

So now we have two whole parts (two circles representing the pizzas), and if we count all the shaded parts, we will get 13, so our fraction would be $\frac{13}{8}$, which is an improper fraction due to the numerator being larger than the denominator. So therefore, we could rewrite it as they have eaten 1 whole pizza and 5 slices out of 8 slices on the second pizza or to simplify in fraction form we have $1\frac{5}{8}$

Knowledge and understanding regarding this mathematical concept could be extended further by introducing further worded problems which can be solvable through the above strategy however it would require students to think deeper and find solutions through the paper folding methods or using similar whole shapes to figure out the problem. A step further would be introducing linear models as a manipulator tool once the area model is understood.

References

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