AP Calculus BC

STRAND	UNIT TITLE/REPORT CARD LANGUAGE
POWER OBJECTIVE #1	**Strategies for Integration and Differential Equations**

SUPPORTING INDICATORS	Enduring Understanding 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change.
	• *Learning Objective 2.3E: Verify solutions to differential equations.*
	    • Essential Knowledge 2.3E1: Solutions to differential equations are functions or families of functions.
	    • Essential Knowledge 2.3E2: Derivatives can be used to verify that a function is a solution to a given differential equation.
	• *Learning Objective 2.3F: Estimate solutions to differential equations*
	    • Essential Knowledge 2.3F2: For differential equations, Euler's method provides a procedure for approximating a solution or a point on a solution curve.
	
	Enduring Understanding 3.3: The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.
	• *Learning Objective 3.3B(a): Calculate antiderivatives.*
	• *Learning Objective 3.3B(b): Evaluate definite integrals.*
	    • Essential Knowledge 3.3B5: Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, integration by parts, and nonrepeating linear partial fractions.

	Enduring Understanding 3.5: Antidifferentiation is an underlying concept involved in solving separable differentiable equations. Solving separable differential equations involves determining a function or relation given its rate of change.
	• *Learning Objective 3.5A: Analyze differential equations to obtain general and specific solutions.*
	  ◦ Essential Knowledge 3.5A1: Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line, exponential growth and decay, and logistic growth.
	  ◦ Essential Knowledge 3.5A2: Some differential equations can be solved by separation of variables.
	  ◦ Essential Knowledge 3.5A3: Solutions to differential equations may be subject to domain restrictions.
	  ◦ Essential Knowledge 3.5A4: The function F defined by $F(x) = c + \int_a^x f(t)dt$ is a general solution to the differential equation $\frac{dy}{dx} = f(x)$, and $F(x) = y_0 + \int_a^x f(t)dt$ is a particular solution to the differential equation $\frac{dy}{dx} = f(x)$ satisfying $F(a) = y_0$.
	• *Learning Objective 3.5B: Interpret, create, and solve differential equations from problems in context.*
	  ◦ Essential Knowledge 3.5B1: The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt} = ky$.

	• The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity" is $\frac{dy}{dt} = ky(a-y)$.
POWER OBJECTIVE #2	**Applications of Integration**

SUPPORTING INDICATORS	Enduring Understanding 3.2: The definite integral of a function over an interval is the limit of a Riemann sum over the interval and can be calculated using a variety of strategies.
	• *Learning Objective 3.2D: Evaluate an improper integral or show that an improper integral diverges.*
	  • Essential Knowledge 3.2D1: An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration.
	  • Essential Knowledge 3.2D2: Improper integrals can be determined using limits of definite integrals.
	Enduring Understanding 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.
	• *Learning Objective 3.4D: Apply definite integrals to problems involving area and volume.*
	  • Essential Knowledge 3.4D3: The length of a planar curve defined by a function or by a parametrically defined curve can be calculated using a definite integral.

POWER OBJECTIVE #3	Curves in Parametric, Vector, and Polar Form

SUPPORTING INDICATORS	Enduring Understanding 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.
	• *Learning Objective 2.1C: Calculate derivatives.*
	    • Essential Knowledge 2.1C7: Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates.
	
	Enduring Understanding 2.2: A function's derivative, which is itself a function, can be used to understand the behavior of a function.
	• *Learning Objective 2.2A: Use the derivatives to analyze properties of a function.*
	    • Essential Knowledge 2.2A4: For a curve given by a polar equation $r = f(\theta)$, derivatives of $r, x,$ and y with respect to θ and first and second derivatives of y with respect to x can provide information about the curve.
	
	Enduring Understanding 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change.
	• *Learning Objective 2.3C: Solve problems involving related rates, optimization, rectilinear motion, and planar motion.*
	    • Essential Knowledge 2.3C4: Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along curves given by parametric or vector-valued functions.
	
	Enduring Understanding 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.

	• *Learning Objective 3.4C: Apply definite integrals to problem involving motion.* • Essential Knowledge 3.4C2: The definite integral can be used to determine displacement, distance, and position of a particle moving along a curve given by parametric or vector-valued functions.
POWER OBJECTIVE #4	**Infinite Sequences and Series**

SUPPORTING INDICATORS	• Essential Knowledge 4.1A3: Common series of numbers include geometric series, the harmonic series, and *p*-series.
	• Essential Knowledge 4.1A4: A series may be absolutely convergent, conditionally convergent, or divergent.
	• Essential Knowledge 4.1A5: If a series converges absolutely, then it converges.
	• Essential Knowledge 4.1A6: In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are *n*th term test, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test.
	• *Learning Objective 4.1B: Determine or estimate the sum of a series.*
	  • Essential Knowledge 4.1B1: If *a* is a real number and *r* is a real number such that $\lvert r \rvert < 1$, then the geometric series $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$.
	  • Essential Knowledge 4.1B2: If an alternating series converges by the alternating series test, then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series.
	  • Essential Knowledge 4.1B3: If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value.

	Enduring Understanding 4.2: A function can be represented by an associated power series over the interval of convergence for the power series.
	
	• *Learning Objective 4.2C: Determine the radius and interval of convergence of a power series.*
	  • Essential Knowledge 4.2C1: If a power series converges, it either converges at a single point or has an interval of convergence.
	  • Essential Knowledge 4.2C2: The ratio test can be used to determine the radius of convergence of a power series.

POWER OBJECTIVE #5	**Taylor and Maclaurin Series**

SUPPORTING INDICATORS	Enduring Understanding 4.2: A function can be represented by an associated power series over the interval of convergence for the power series.
	
	• *Learning Objective 4.2A: Construct and use Taylor polynomials.*
	  • Essential Knowledge 4.2A1: The coefficient of the nth-degree term in a Taylor polynomial centered at $x = a$ for the function f is $\frac{f^{(n)}(a)}{n!}$.
	  • Essential Knowledge 4.2A2: Taylor polynomials for a function f centered at $x = a$ can be used to approximate function values of f near $x = a$.
	  • Essential Knowledge 4.2A3: In many cases, as the degree of a Taylor polynomial increases, the nth-degree polynomial will converge to the original function over some interval.
	  • Essential Knowledge 4.2A4: The Lagrange error bound can be used to bound the error of a Taylor polynomial approximation to a function.
	  • Essential Knowledge 4.2A5: In some situations where the signs of a Taylor polynomial are alternating, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the function.

	• *Learning Objective 4.2B: Write a power series representing a given function.* • Essential Knowledge 4.2B1: A power series is a series of the form $\sum_{n=0}^{\infty} a_n(x-r)^n$ where n is a non-negative integer, $\{a_n\}$ is a sequence of real numbers, and r is a real number. • Essential Knowledge 4.2B2: The Maclaurin series for $\sin(x), \cos(x),$ and e^x provide the foundation for constructing the Maclaurin series for other functions. • Essential Knowledge 4.2B3: The Maclaurin series for $\frac{1}{1-x}$ is a geometric series. • Essential Knowledge 4.2B4: A Taylor polynomial for $f(x)$ is a partial sum of the Taylor series for $f(x)$. • Essential Knowledge 4.2B5: A power series for a given function can be derived by various methods (e.g., algebraic processes, substitutions, using properties of geometric series, and operations on known series such as term-by-term integration or term-by-term differentiation. • *Learning Objective 4.2C: Determine the radius and interval of convergence of a power series.* • Essential Knowledge 4.2C3: If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval. • Essential Knowledge 4.2C4: The radius of convergence of a power series obtained by term-by-term differentiation or term-by-term integration is the same as the radius of convergence of the original power series.
POWER OBJECTIVE #6	**AP Calculus AB Review**
SUPPORTING INDICATORS	The AP Calculus BC exam consists of both AP Calculus AB and BC content. Therefore, a significant amount of time is spent reviewing these concepts. Any AB review will be attached to this Power Objective.