Constrained Bayesian Methods for Testing Union-Intersection and Intersection-Union Hypotheses

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Abstract

The Union-Intersection (UI) and Intersection-Union (IU) hypotheses testing problems are considered for all possible combinations of united and intersected sub-sets of hypotheses. Constrained Bayesian Method (CBM) is developed for solving these problems. Optimal decision rules are derived for all stated combinations of hypotheses. Theorems on the optimality of the derived decision rules in the sense of the restrictions on Type-I and Type-II error rates to the desired levels are proved. The proposed theoretical methods are enhanced for practical examples. Extensive simulation results are presented to confirm the theoretical results and to illustrate the properties of the proposed procedures for a finite sample.

Key Words: Constrained Bayesian Method, Intersection-Union hypotheses, Statistical hypothesis, Type-I and Type-II error rates, Union-Intersection hypotheses.