Hot Rocks & Cold Rocks

A Contextual Resource for Integer Operations

By Marcie Curcie with David Mattoon

Relating integers to temperature is a context that students can understand and are most likely already aware. Hot Rocks and Cold Rocks help students conceptually understand what is happening with integer operations through a contextual lens. Hot Rocks and Cold Rocks is based on zero pairs, and can lead into the use of color chips, pictures, and vertical number lines to understand integer operations.

The premise is that each Hot Rock increases the temperature of your drink by 1°. Each Cold Rock decreases the temperature of your drink by 1°. Hot Rocks make things hotter and Cold Rocks make things colder. This simple context can be introduced through a variety of different stories based on your students' interests. Cold Rocks and/or negative numbers will always be shown in parentheses. For example, 2 Cold Rocks would be shown as (-2).

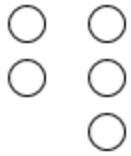
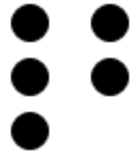
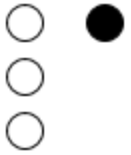
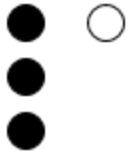
Note that the context questions provide the Hot Rock Cold Rock context to provide meaning to integer operations. For integer combination, the context provides meaning to integer subtraction; however, for understanding algebraic operations, seeing the "-" sign as "opposite" is often necessary. In fact, this difference in how one views the "-" sign is the primary source of confusion for students as the previous learning of subtraction interferes with new learning. One of the points of these lessons is to build on the students' understanding of subtraction through the Hot Rock Cold Rock context, and then connect it to viewing the "-" as the opposite or a negative; therefore, we recommend teaching both at the same time through manipulatives and multi-column/multi-representational notes.

Meaning is the focus of these lessons; not integer rules. Students should discover rules for themselves through this process. Rules might be reviewed as they are discovered by students or saved for the end. Rather than having to memorize multiple rules students can remember a single context, Hot Rocks Cold Rocks. Rather than having to memorize multiple rules students can learn that + means "and" & - means "opposite."

Adding Integers

Adding integers is addressed in this context by putting Hot and Cold Rocks into a drink. As you "put in" certain Rocks, you are "adding" them to your drink. In 6th grade, students learned that positive and negative numbers are opposites sides because they are in the same location on opposite sides of the number line (6.NS.5 and 6.NS.6). In this context, hot and cold are opposites and they have opposite reactions on the temperature of the drink.

Context Questions	**Visual**	**Symbolic**	**Verbal** *Read as:* *(+) means "and"* *(-) means "opposite"*	**Connecting Questions**
If I put in 2 Hot Rocks, what happens to the temperature of my drink? *It gets 2° hotter.*		+(2) = 2	And two	**What is the relationship between hot and cold?** *Opposite.* **What is the relationship between positive and negative numbers? How do you know?** *Positive and negative numbers are opposites because they are found on opposite sides of zero on the number line.* **What is the relationship between a positive number and a Hot Rock? A negative number and a Cold Rock?** *A Hot Rock increases the temperature by 1 which can be shown as +1 and a Cold Rock decreases the temperature by 1 which can be shown by -1.*
If I put in 2 Cold Rocks, what happens to the temperature of my drink? *It gets 2° colder.*		+(-2) = (-2)	And the opposite of two, which is negative two	
If I put in 1 Hot Rock and 1 Cold Rock, what happens to the temperature of my drink? *It remains the same.*		1 + (-1) = 0	One and the opposite of one, or one and negative one, which is zero (a zero pair)	**How does the lack of change in temperature when adding one Hot Rock and one Cold Rock relate to -1 and 1 being opposites?** *If you take one step forward (+1) and then do the opposite of taking one step backward (-1), then you are exactly where you started and there has been no change.*

				You may introduce Additive Inverse Property at this time.
If I put in 2 Hot Rocks and then 3 more Hot Rocks, what happens to the temperature of my drink? *It gets 5° hotter.*		2 + 3 = 5	Two and three, which is five	**Why does your drink get 5° hotter?** *You are putting in a total of 5 Hot Rocks which make the drink 5° hotter.*
If I put in 3 Cold Rocks and then 2 more Cold Rocks, what happens to the temperature of my drink? *It gets 5° colder.*		(-3) + (-2) = (-5)	The opposite of three and the opposite of two, or negative three and negative two, which is negative five	**Why does your drink get 5° colder?** *You are putting in a total of 5 Cold Rocks which make the drink 5° colder.*
If I put in 3 Hot Rocks and 1 Cold Rock, what happens to the temperature of my drink? *It gets 2° hotter.*		3 + (-1) = 2	Three and the opposite of one, or three and negative one, which is two	**Why does the temperature of the drink only get 2° hotter?** *There is a zero pair where one Hot Rock and one Cold Rock make no change to the temperature of the drink.* **How else can we describe the change in temperature?** *The temperature of the drink gets hotter by 3° and then cools down by 1° making an overall change of 2° hotter.* The answers can be given in any order.
If I put in 3 Cold Rocks and 1 Hot Rock, what happens to the temperature of my drink? *It gets 2° colder.*		(-3) + 1 = (-2)	The opposite of three and one, or negative three and one, which is negative two	**Why does the temperature of the drink only get 2° colder?** *There is a zero pair where one Hot Rock and one Cold Rock make no change to the temperature of the drink.* **How else can we describe the change**

				in temperature? *The temperature of the drink gets colder by 3° and then warms up by 1° making an overall change of 2° colder.* The answers can be given in any order.

Can we figure out if the overall change of the drink is going to be hotter or colder without "doing the math?" If so, how do you know? If not, why not? *Yes. If there are more Hot Rocks than Cold Rocks, the drink will get hotter and if there are more Cold Rocks than Hot Rocks then the drink will get colder.*

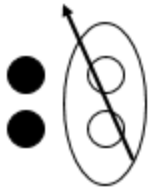
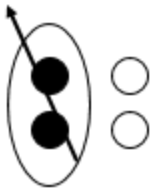
What connection does this have to adding positive and negative numbers? *If your addition problem has more negatives, then your answer will be negative. If your addition problem has more positives, then your answer will be negative.*

Instead of drawing a visual to help us determine the answer, is there a pattern that we could use that would work for every addition problem that includes integers? *Yes. If you have all the same type of number, either all positive or all negative, then you add them together and keep their same sign. If you have both positive and negative numbers, then you keep the sign of the number with the greatest absolute value (which sign you have more of) and then subtract to find how many more you have and that number gets the sign of the number with the greatest absolute value.*

Subtracting Integers

Subtracting integers is addressed in this context by taking Hot and Cold Rocks out of your drink. As you "take out" certain Rocks, you are subtracting them. When beginning with the context, students should become familiar with what happens when you "take out something that is cold" or "take out something that is hot." When connecting to the visual stage, a rich conversation regarding adding (or assuming) zero pairs must be had. Additionally, it allows for the conversation about how subtracting an integer is the same thing as adding its opposite by relating it each change in temperature at each stage of the problem.

The Hot Rock Cold Rock context leads into the mathematical concept of zero pairs as Hot Rocks and Cold Rocks are opposite and if you add one (or many) of each to drink then the temperature remains the same. If students struggle with the understanding that there are an infinite number of zero pairs already in the drink, the use of physically putting in zero pairs from which to draw upon prior to the start of each question can strengthen this concept.

Context Questions	**Visual**	**Symbolic**	**Verbal** *Read as:* *(+) means "and"* *(-) means "opposite"*	**Connecting Questions**
If I take out 2 Hot Rocks, what happens to the temperature of my drink? *It gets 2° colder.*		-(+2) = (-2)	Opposite of positive two is negative two	**What is the relationship between taking out 2 Hot Rocks and putting in 2 Cold Rocks?** *They are opposites and produce the same change. Both taking out 2 Hot Rocks and putting in 2 Cold Rocks make the drink get 2° colder.*
If I take out 2 Cold Rocks, what happens to the temperature of my drink? *It gets 2° hotter.*		-(-2) = +2	Opposite of negative two is positive two	**What is the relationship between taking out 2 Cold Rocks and putting in 2 Hot Rocks**? *They are opposites and produce the same change. Both taking out 2 Cold Rocks and putting in 2 Hot Rocks make the drink get 2° hotter.*

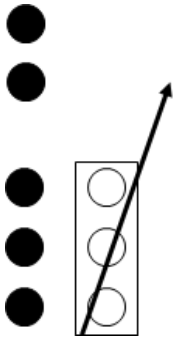
Can we use taking out a Hot Rock and putting in Cold Rock interchangeably? How do you know? *Yes, we can use them interchangeably because both instances produce the same change to the temperature of the drink; they make the drink get 1° colder.*

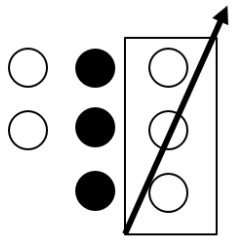
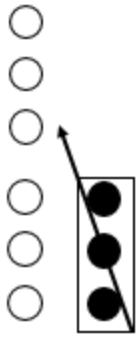
Can we also use putting in a Hot Rock and taking out a Cold Rock interchangeably? *Yes. Both instances produce the same change to the temperature of the drink; they make the drink get 1° hotter.*

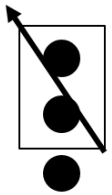
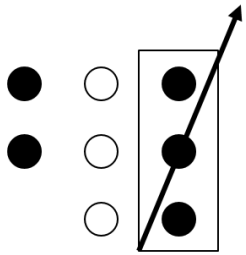
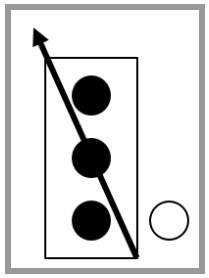
What operation is represented by the "taking out" of Rocks? *Subtraction.*

How could you use the flipping of a two color counter* to illustrate the same thing in these first two examples? *In the first example, flip over the two positives to make it the opposite. In the second example, start with the two positives and flip it over twice to illustrate doing the opposite twice ending up back where you started. And/or, in the second example, flip over the negative two to make it a positive two.*

*If you are using two single colored counters, you can show the opposite by switching out with the opposite color counter.

If I put in 5 Hot Rocks and then take out 3 Hot Rocks, what happens to the temperature of my drink? *It gets 2° hotter.* (pos - pos = pos)		5 - 3 = 2	Five, opposite of three; or five and negative three, which is two	**Is there another way to make the drink 2° hotter while still putting 5 Hot Rocks in first? How do you know?** *Instead of taking out 3 Hot Rocks, you can put in 3 Cold Rocks. When you take out 3 Hot Rocks the drink gets 3°colder and the same change happens when you put in 3 Cold Rocks. They are opposites that produce the same change.*
If I put in 2 Cold Rocks and then take out 3 Hot Rocks, what happens to the temperature of my drink? *It gets 5° colder.* (neg - pos = neg)		-2 - 3 = (-5) [(-2) + (-3) = (-5)]	Opposite of two, opposite of three; or negative two and negative three, which is negative five	**Is there another way to make the drink 5° colder?** *You can put in 5 Cold Rocks.* **What is the connection between this problem and putting in 5 Cold Rocks**? *Both make the drink get 5° colder. Both begin with putting in 2 Cold Rocks making it 2° colder. When you take out the 3 Hot Rocks, it gets 3° colder and 5° colder overall. The opposite of taking out 3 Hot Rocks is putting in 3 Cold Rocks which maintains the 3° colder change and the 5° colder change overall.*

If I put in 2 Hot Rocks and then take out 3 Hot Rocks, what happens to the temperature of my drinks? *It gets 1° colder.* (pos - pos = neg)		2 - 3 = (-1) [2 + (-3) = (-1)]	Two, opposite of three; or two and negative three, which is negative 1	**How does putting in 2 Cold Rocks and then taking out 3 Hot Rocks make the drink only 1° colder?** *When putting in the 2 Hot Rocks the drink gets 2° hotter. Then, when you take out the first 2 Hot Rocks you are back to the original temperature of the drink, but when you take out the third Hot Rock, you make the drink 1° colder.* **Is there another way that you can make the drink 1° colder if you still start by putting in 2 Hot Rocks?** *Yes, you can add 3 Cold Rocks to balance the change of the first 2 Hot Rocks you put it, and then the third Cold Rocks makes the drink 1° colder overall.*
What connection can you make between taking out Hot Rocks and adding Cold Rocks? Justify your answer. *They are opposites that produce the same change in the temperature of the drink. If I take out 3 Hot Rocks it is the same as adding 3 Cold Rocks because the drink gets 3° colder.*				
*For the visual images of subtracting negative numbers, we have included all zeros pairs that are related to the negative number being subtracted as doing so allows for students to see why -(-3) is equivalent to +3 and how doing the latter is more efficient.				
If I put in 3 Hot Rocks and then remove 3 Cold Rocks, what happens to the temperature of my drink? *It gets 6° hotter.* (pos - neg = pos)		3 - (-3) = 6 [3 + 3] = 6	Three, opposite of negative three; or three and three, which is six	**How does putting in 3 Hot Rocks and then taking out 3 Cold Rocks make the drink get 6° hotter?** *Putting in 3 Hot Rocks makes the drink 3° hotter and taking out 3 Cold Rocks makes the drink another 3° hotter, so in total, the drink is 6° hotter.* **What is the relationship between putting in 3 Hot Rocks and taking out 3 Cold Rocks?** *They are opposites. Both instances make the drink 3° hotter.*

If I put in 3 Cold Rocks and then take out 2 Cold Rocks, what happens to the temperature of my drink? *It gets 1° colder.* (neg - neg = neg)		-3 - (-2) = -1 -3 + 2 = -1 -3 - (-2) = -1	Opposite of three, opposite of negative two; or negative three and two, which is negative one	**How does putting in 3 Cold Rocks and then taking out 2 Cold Rocks make the drink 1° colder?** *Putting in 3 Cold Rocks makes the drink 3° colder and then when you remove 2 of the Cold Rocks there is only 1 Cold Rock left in the drink making it only 1° colder overall.* **Is there another way that you can make the drink 1° colder if you still start by putting in 3 Cold Rocks?** *Yes, you can add the 3 Cold Rocks and then add 2 Hot Rocks which will balance out 2 of the Cold Rocks leaving the drink only 1° colder.*
If I put in 2 Cold Rocks and then take out 3 Cold Rocks, what happens to the temperature of my drink? *It gets 1° hotter.* (neg - neg = pos)	  *The top image is prefered over this one (see note with asterisk above)	-2 - (-3) = 1 [(-2) + 3 = 1]	Opposite of two, opposite of negative three; or negative two and three, which is one	**How does putting in 2 Cold Rocks and then taking out 3 Cold Rocks make the drink 1° hotter?** *1) After you put in 2 Cold Rocks and then take out 3 Cold Rocks, the first 2 Cold Rocks you take out bring the drink back to its original temperature. Then, when you take out the third Cold Rock, its removal makes the drink get 1° hotter.* *2) When you put in 2 Cold Rocks the drink gets 2° colder. When you take out 3 Cold Rocks the drink gets 3° hotter. Looking at the changes together, the drink gets 1° hotter overall.* Students may provide these answers in any order.

What connection can you make between taking out Cold Rocks and adding Hot Rocks? *They are opposites that produce the same change in the temperature of the drink. If I take out 3 Cold Rocks it is the same as adding 3 Hot Rocks because the drink gets 3° hotter.*

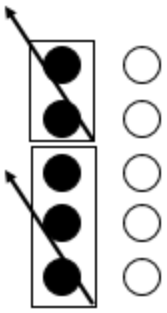
Does taking out Rocks (subtraction) and putting in the opposite Rocks (adding the opposite) instead always work? *Yes. The change is the same.*

You may solidify the concept of "adding the opposite" at this time.

How does the pattern you described using opposite Rocks apply to subtraction with positive and negative numbers? *If taking out Rocks is subtraction then its opposite is putting in Rocks or addition. Taking out Cold Rocks is like subtracting a negative which can also be viewed as the opposite which is adding a positive. Taking out Hot Rocks is like subtracting a positive which can also be viewed as the opposite which is adding a negative.*

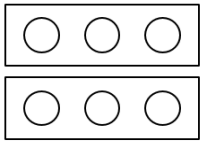
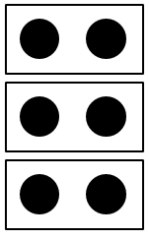
How could this idea of "adding the opposite" instead of subtracting be beneficial? *Adding the opposite makes integer operations more efficient in most cases, just like learning multiplication made repeated addition more efficient.*

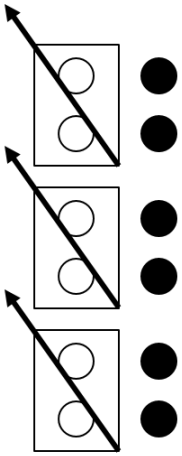
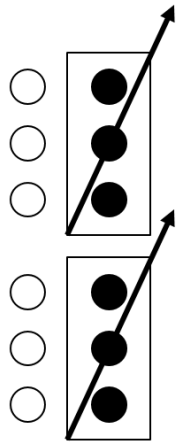
The example below is an additional example that addresses taking the opposite of the minuend first and then subtracting a negative subtrahend. It can be helpful to work through it for when students encounter problems when working in different contexts and to really solidify the understanding of a negative symbol as "taking the opposite" and subtraction of a negative as equivalent to adding a positive.

If I take out 2 Cold Rocks and then take out 3 more Cold Rocks, what happens to the temperature of my drink? *It gets 5° hotter.*		-(-2) - (-3) = 5 [2 + 3 = 5]	Opposite of negative two, opposite of negative three; or two and three, which is five	**Is there another way to make the drink 5° hotter?** *You can put in 5 Hot Rocks.* **What is the connection between taking out 2 Cold Rocks and then another 3 Cold Rocks and putting in 5 Hot Rocks?** *Both make the drink get 5° hotter. Taking out 2 Cold Rocks and then 3 more Cold Rocks is the same thing as taking out a total of 5 Cold Rocks which makes the drink 5° hotter. This same change would be seen if put in 5 Hot Rocks because taking out Cold Rocks and putting in Hot Rocks are opposites that produce the same change.*

Multiplying Integers

Multiplying integers is addressed in this context by putting in or taking out groups of Hot Rocks or Cold Rocks. The first factor denotes the number of groups being put in or taken out and the second factor denotes the amount and type of Rocks in each group. Students should be familiar with multiplication as the repeated addition of groups, but if not the context allows for those conversations to be addressed as well.

Context Questions	**Visual**	**Symbolic**	**Verbal** *Read as:* *(+) means "and"* *(-) means "opposite"*	**Connecting Questions**
If I put in 2 groups of 3 Hot Rocks, what happens to the temperature of my drink? *It gets 6° hotter.*		2(3) = 6 [3 + 3 = 6]	Two groups of three, which is 6	**What is the relationship between putting in 2 groups of 3 Hot Rocks and the drink getting hotter by 6°?** *The first group of 3 Hot Rocks makes the drink get 3° hotter and then adding a second group of 3 Hot Rocks makes it get another 3° hotter which is a total of 6° hotter.*
If I put in 3 groups of 2 Cold Rocks, what happens to the temperature of my drink? *It gets 6° colder.*		3(-2) = -6 [-2 + -2 + -2 = -6]	Three groups of negative two, which is negative six	**Why does the drink get colder by 6° when you put in 3 groups of 2 Cold Rocks?** *The first group makes the drink colder by 2°, then second group makes it 2° colder (4° colder overall), and then the third group makes it another 2° colder, making the drink 6° colder overall.*

If I take out 3 groups of 2 Hot Rocks, what happens to the temperature of my drink? *It gets 6° colder.*		-3(2) = -6 [-2 + -2 + -2 = -6] (-3) + (-3) = -6	The opposite of three groups of two, which is negative six (Commutative Property) Two groups of negative three, which is negative six	**How does taking out 3 groups of Hot Rocks make the drink colder by 6°?** *The first group of 2 Hot Rocks that you take out makes the drink colder by 2°. The second group makes it a total of 4° colder. And, the third group makes it a total of 6°.* **Is there another way we can think about this problem while keeping the numbers the same?** *Yes. We can think of it as "putting in 2 groups of 3 Cold Rocks" because it would yield us the same result of making the drink colder by 6°.*
		START HERE with only this representation (2)-3 = -1		**Is this multiplication?** *No* **How can you tell the difference between multiplication and combination (addition/subtraction)?** *When there is a sign between the digit and parenthesis, it is combination. When there is NO sign between the digit and parenthesis, it is multiplication.*
If I take out 2 groups of 3 Cold Rocks, what happens to the temperature of my drink? *It gets 6° hotter.*		-2(-3) = 6 [-(-3) -(-3) = 6] [3 + 3 = 6]	The opposite of two groups of negative three, or the opposite of the opposite of two groups of three, which is six	**How does taking out 2 groups of 3 Cold Rocks make the drink hotter by 6°?** *Taking out the first group of 3 Cold Rocks makes the drink hotter by 3° and then taking out the second group of 3 Cold Rocks makes the drink hotter by 3° more for a total of 6°.*

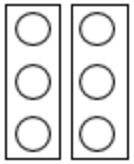
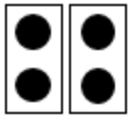
What pattern do you notice when putting in groups of Hot Rocks and taking out groups of Cold Rocks? *Both scenarios make the drink hotter.*

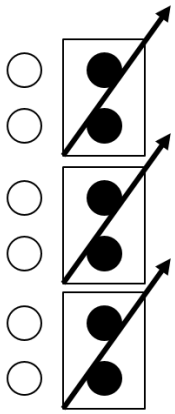
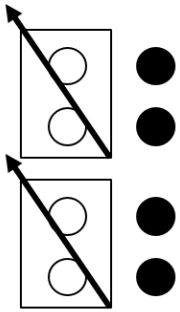
What pattern do you notice when putting in groups of Cold Rocks and taking out groups of Hot Rocks? *Both scenarios make the drink colder.*

What connection can be made between the patterns observed when putting in or taking out groups of Rocks and multiplying with positive and negative numbers? *If the factors have the same sign, either two positive numbers or two negative numbers, the answer is positive. If the factors have different signs, one positive and one negative number, the answer is negative.*

Dividing Integers

Dividing integers is addressed in this context by describing an overall change in the temperature of the drink by putting in or taking out an equal amount of groups. The quotient describes the amount and type of Rock in each of the equal groups.

Context Questions	**Visual**	**Symbolic**	**Verbal** *Read as:* *(+) means "and"* *(-) means "opposite"*	**Connecting Questions**
If the temperature of the drink got 6° hotter by putting in 2 equal groups, how many and what type of Rock were in each group? *There were 3 Hot Rocks in each group.*		6 / 2 = 3	Six divided by two is three	**How did you determine that each group would have 3 Hot Rocks?** *Since the temperature of the drink got hotter by 6° and we put in 2 equal groups, we know there has to be 3 Rocks in each group because six divided by 2 is 3. The Rocks added are Hot Rocks because we put them in and the drink got hotter.*
If the temperature of the drink got 4°colder by putting in 2 equal groups, how many Rocks were in each group? *There were 2 Cold Rocks in each group.*		-4 / 2 = -2	The opposite of four divided by two, or negative four divided by two, which is negative two	**How did you determine that each group would have 2 Cold Rocks?** *Since the temperature of the drink got colder by 4° and we put in 2 equal groups, we know there has to be 2 Rocks in each group because four divided by two is two. The Rocks added are Cold Rocks because we put them in and the drink got colder.*

If the temperature of the drink got 6° hotter by taking out 3 equal groups, how many Rocks were in each group? *There were 2 Cold Rocks in each group.*		6 / -3 = -2	Six divided by the opposite of three, or six divided by negative three, which is negative two	**How did you determine that each group would have 2 Cold Rocks?** *Since the temperature of the drink got 6° hotter by taking out 3 equal groups, we know there has to be 2 Rocks in each group because six divided by three is two. The Rocks in each group have to be Cold Rocks because we are taking them out and the drink is getting hotter.*
If the temperature of the drink got 4° colder by taking out 2 equal groups, how many Rocks were in each group? *There were 2 Hot Rocks in each group.*		-4 / -2 = 2	The opposite of four divided by the opposite of two, or four divided two, which is two	**How did you determine that each group would have 2 Hot Rocks?** *Since the temperature of the drink got 4° colder by taking out 2 equal groups, we know there has to be 2 Rocks in each group because four divided by two is two. The Rocks in each group have to be Hot Rocks because we are taking them out and the drink is getting colder.*

What pattern do you notice between the temperature change and equal groups being put in? *If the temperature increased, the equal groups being put in were made up of Hot Rocks. If the temperature decreased, the equal groups being put in were made up of Cold Rocks.*

What pattern do you notice between the temperature change and equal groups being taken out? *If the temperature increased, the equal groups being taken out were made up of Cold Rocks. If the temperature decreased, the equal groups being taken out were made up of Hot Rocks.*

How do these patterns relate to dividing positive negative numbers? *If the dividend and the divisor have the same sign, either two positive numbers or two negative numbers, the answer is positive. If the dividend and divisor have different signs, one positive and one negative number, the answer is negative.*

How do these patterns relate to the patterns observed when multiplying positive and negative numbers? *The patterns are the same. If you multiply/divide numbers with the same signs, the answer is positive; if you multiply/divide numbers with different signs, the answer is negative.*