Assignment no 1

MTH647

SOLVED BY :- 😡🕊️ ᵛïℝ τ ʊ̄ą̣ĺîåñ ꞵíɾɖ𓆏🕊️😡

Question 1:

The function $f(x)=\frac{x}{|x|}$ is not continuous at (x = 0)since (|x|) changes sign at (x = 0).

Interval (x < 0):

For(x < 0), (|x| = -x), so (f(x) = $\frac{x}{-x}$ = -1).

Therefore, (f(x) = -1) for (x < 0).

Interval (x > 0):

For (x > 0),(|x| = x), so (f(x) = $\frac{x}{|x|}$ = 1).

Therefore, (f(x) = 1) for (x > 0).

SOLVED BY :- 😡🕊️ ᵛïℝ τ ʊ̄ą̣ĺîåñ ꞵíɾɖ𓆏🕊️😡

Conclusion:

(f(x)) has a piecewise continuity:

(f(x) = -1) for (x < 0)

(f(x) = 1) for (x > 0)

Question 2:

Given the steady-state solutions ($U_m(x)$) and ($U_n(x)$) of the system:

$$\frac{\partial^2 U}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}, 0 < x < a, t > 0.$$

SOLVED BY :- 😡🕊️ ѵïℝ τ ʊäl̲ĩåñ ℬίɾʠ🕊️😡

with boundary conditions:

$$U(o,t) = t_0, \frac{\partial}{\partial x}U(a,t) = 0$$

We need to evaluate:

$$\int_0^a U_m(x)U_n(x)dx$$

Consider

$$\frac{\partial^2 U}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$$

$$\frac{\partial^2 U}{\partial x^2}k = \frac{\partial u}{\partial t}$$

For steady state time is independent.

So $\frac{\partial u}{\partial t}$ =0

$$\frac{\partial^2 U}{\partial x^2}k = 0$$

$$\frac{\partial^2 U}{\partial x^2} = 0$$

Intg on both sides

SOLVED BY :- 😡🕊️ ѵïℝ τ ʊäl̲ĩåñ ℬίɾʠ🕊️😡

$$\int \frac{\partial^2 U}{\partial x^2}.dx = 0$$

$$\frac{\partial U}{\partial x} = A$$

$$Again$$

$$U(x) = Ax + b \rightarrow (i)$$

By using B.C in eq (i)

U(o)=0+b

U(0)=b

Diff eq(i)

$$\frac{\partial U(x)}{\partial x} = A \Rightarrow \frac{\partial U(a)}{\partial x} = A$$

Now, By comparing to B.C:

$B = t_0$ so ($U_m(x)$)=($U_n(x)$) = t

Putting the values in integrals:

$$\int_0^a U_m(x)U_n(x)dx = \int_0^a t_0.t_0 dx$$

$$= \int_0^a {t_0}^2 dx$$

$$= t_0^2 \int_0^a dx = t_0^2 \left|x\right|_0^a = t_0^2(a)$$

$$= at_0^2$$

$$\int_0^a U_m(x)U_n(x)dx = at_0^2$$