AP Physics Midterm Review Sheet

By Christopher M. Donohue

Test 1: Review/Background Material

Need to Know Calculus:

Derivatives:

$y = cx^{n} \rightarrow \frac{dy}{dx} = cnx^{n-1}$

$y = e^{ax} \rightarrow \frac{dy}{dx} = ae^{ax}$

$y = ln(ax) \rightarrow \frac{dy}{dx} = \frac{1}{x}$

$y = sin(ax) \rightarrow \frac{dy}{dx} = acos(ax)$

$y = cos(ax) \rightarrow \frac{dy}{dx} = - asin(ax)$

$\frac{df}{dt} = \frac{df}{dx}\frac{dx}{dt}$(Chain rule)

Integrals:

$\int x^{n}dx = \frac{1}{n+1}x^{n+1} + C, x \neq - 1$

$\int e^{ax}dx = \frac{1}{a}e^{ax} + C$

$\int \frac{dx}{x+a} = ln|x + a| + C$

$\int cos(ax)dx = \frac{1}{a}sin(ax) + C$

$\int sin(ax)dx = \frac{-1}{a}cos(ax) + C$

U-substitution: If $\int f(g(x))dx$, then $u(x) = g(x) \rightarrow \frac{du}{dx} = g'(x) \rightarrow dx = \frac{du}{g'(x)} \rightarrow \int \frac{1}{g'(x)} f(u)du$

Note: Most of these equations are on the reference table, but knowing them by heart saves time

Vectors:

Scalars-any quantity that can be completely described by a numerical value (and an appropriate unit)

Vectors-any quantity whose complete description requires a magnitude (with units) and a direction

Dot Product:

$\vec{A} \bullet \vec{B} = AB\cos\theta$ $\qquad$ $A_x B_x + A_y B_y + A_z B_z = \vec{A} \cdot \vec{B}$

Cross Product: $\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$

The Direction of a cross product can be determined using the right hand rule:

1. Draw the vectors, A and B, tail to tail
2. Curl your fingers of your right hand from A to B through the smallest angle between them
3. The direction your thumb points is the direction of the of AxB

Alternative way to find the direction:

1. Point your pointer finger in the direction of the first vector
2. Point your middle finger in the direction of the second vector
3. The direction your thumb points is the direction of the cross product

Unit Vectors:

The unit vector for the x-direction is i hat: î

The unit vector for the y-direction is j hat: ĵ

The unit vector for the z-direction is k hat

The unit vector notation isn't actually important to know for the test

Unit Analysis:

- ➢ the use of base units, mass, time, and length, to analyze more complex situations
- ➢ Probably no question specifically asking to do this, but it is helpful for checking answers, especially if those answers are in variables

Test 2: Kinematics

Definitions in terms of derivatives:

$v(t) = \frac{ds}{dt}$

$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Formulas:

$x = x_0 + vt + \frac{1}{2}at^2$

$v = v_0 + at$

$v^2 = v_0^2 + 2ax$

★$s_y = s_{0y} + tan(\theta)s_x - (\frac{g}{2v_0^2cos^2(\theta)})s_x^2$★

Notes: 1. The first three are on the reference table, but the fourth it's so it should be memorized or know how it is derived, hence the stars

2. These are only good for constant acceleration

Projectiles:

x/horizontal	y/vertical
$a_x = 0$	$a_y = -g$(negative depends on which direction is set as positive)
$v_x = constant$	$v_y = v_{o_y} - gt$

$s_x = v_x t$	$s_y = s_{o_y} + v_{o_y} t - \frac{1}{2} g t^2$ $v_y^2 = v_{o_y}^2 - 2gy$

General Procedure:

1. Calculate components
2. Calculate "time of fight"
3. Calculate various answers

Remember: "range" is the horizontal distance traveled when a projectile returns to original vertical displacement

Uniform Circular Motion:

$a_c = \frac{v^2}{r}$

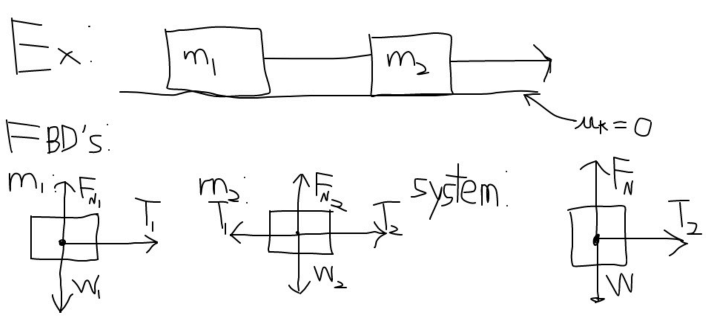
Test 3: Forces & Newton's Laws

Newton's Laws of Motion:

1. Law of Inertia-a body at rest stays at rest and a body in motion remains in motion unless acted upon by an outside net force
    a. Applies in all inertial reference frames (reference frames that are not accelerating)
2. $F_{Net} = \Sigma F = ma$
3. Every action has an equal and opposite reaction.

Free Body Diagrams and Internal Forces

- FBD's simplifies diagram showing all of the forces acting on a particular object
- Don't draw in the components, do that over to the side
- Label your positive direction and if on an inclined plane, the parallel and perpendicular axes
- Internal forces are a consequence of Newton's third law of motion
- If dealing with more than one object, a FDB of the system must not include internal forces
- Examples:



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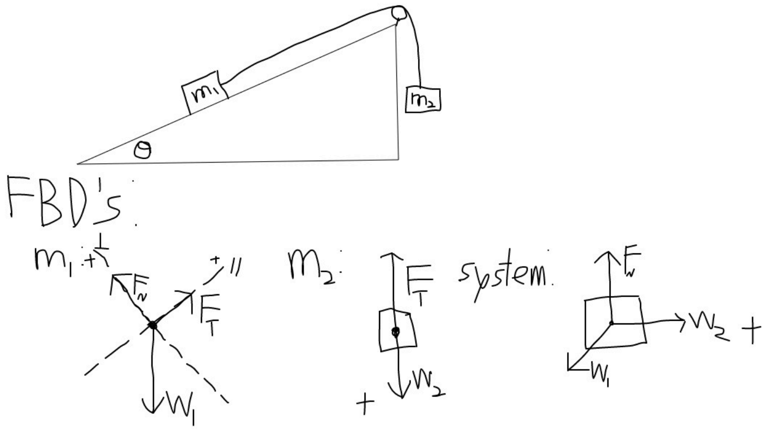


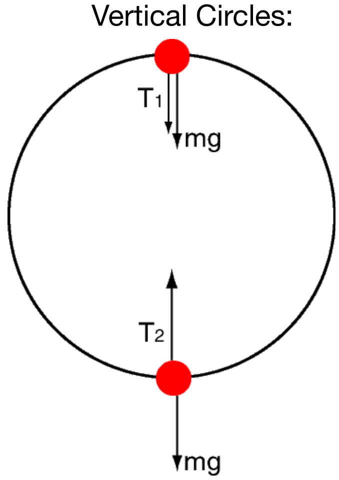
➢

Friction Forces:

- ➢ Forces that tend to oppose the relative motion between surfaces sliding or "trying" to slide, against each other
- ➢ The resistance to relative motion can cause an object to accelerate
- ➢ Static friction (F_{f_s})
    - ○ Occurs when surfaces have no relative motion
    - ○ The magnitude of static friction force is variable up to some maximum value (dependent on specific surfaces involved)
    - ○ $F_{f_s} \leq \mu_s F_N$
- ➢ Kinetic friction (F_{f_k})
    - ○ Occurs when surfaces have any relative motion
    - ○ The magnitude of kinetic friction force is constant for the specific surfaces involved (independent of surface area and relative velocity)
    - ○ $F_{f_k} = \mu_k F_N$

Vertical Circles and Pendulum

Vertical Circles:



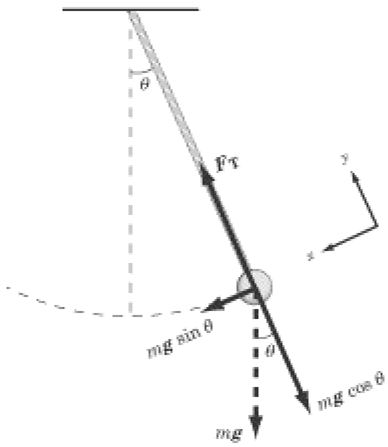
Top (12 o'clock): $F_T + mg = \frac{mv^2}{r}$

Bottom (6 o'clock): $F_T - mg = \frac{mv^2}{r}$

If the speed is constant, then F_T changes: $F_{T\ Bottom} > F_{T\ Top}$ (on an involved roller coaster or Ferris wheel, F_N would change)

The minimum velocity to maintain the circular pathway is $v_{min} = \sqrt{rg}$.

Regular Pendulum:

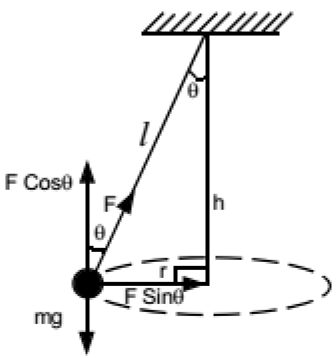


Radial: $F_T - mgcos\theta = \frac{mv^2}{r}$

Tangential: $gsin\theta = a_T$

At zero degrees, a_T=0 and at ninety degrees a_T=g

Conical Pendulum:



$\cos\ \theta = \frac{h}{l}$

$\therefore h = l\ \cos\theta$

$v = \sqrt{rgtan\theta}$

General Procedure:

1. Draw the Free body diagram
2. Find the components of any forces
3. Set up and write the equation of motion in both directions for objects and/or the system as a whole
4. Look for common variables between the equations and see if the equations can be combined somehow to isolate one of the variables
5. Plug in and solve for the variables
6. If needed, go back to original equations of motion to find other unknowns

Equations of Motion-Constant & Non-constant

- ΣF→a→v(t)→x(t)

- ➢ With non-constant forces, there is non-constant acceleration
- ➢ “Standard” kinematics equations no longer hold😟
- ➢ Integral calculus is now necessary😯

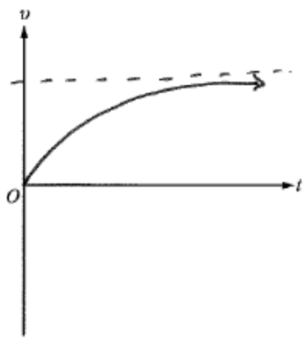
1. Time-dependent forces
    1.1. Non-constant braking of automobile
    1.2. Waves traveling through medium (force & acceleration vary sinusoidally)
2. Velocity-dependant forces
    2.1. Drag (or retarding) forces experienced by a body moving through a fluid medium
    2.2. These forces increase with increasing velocity
    2.3. Ex: moving through air or water (projectiles, free-falling objects, etc.)
3. Position-dependent forces
    3.1. A spring compressed/stretched from equilibrium
    3.2. Usually more easily analyzed using techniques of work and energy
    3.3. Ex: springs, pendulums

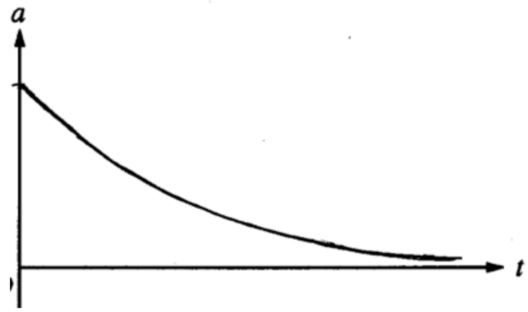
Time Dependent:

$$a(t) = \frac{dv}{dt} \rightarrow dv = a(t)dt \rightarrow \int_{v_0}^{v} dv = \int_{0}^{t} a(t)dt \rightarrow v(t) = v_o + \int_{0}^{t} a(t)dt$$

Drag Forces & Projectiles:

- ➢ Drag forces are velocity dependent; therefore, the acceleration is a function of velocity
- ➢ $a(v) = \frac{dv}{dt} \rightarrow \frac{dv}{a(v)} = dt \rightarrow \int \frac{dv}{a(v)} = \int dt = t$
- ➢ Yields time as a function of velocity [t(v)]→needs to be rearranged to yield velocity as a function of time [v(t)]
- ➢ If F_{Drag}=bv→$v_{terminal} = \frac{mg}{b}$ and $v(t) = \frac{mg}{b}(1 - e^{\frac{-bt}{m}})$





(sort of what the displacement looks like, really it could be flipped if you set the starting point as the zero point)

Test 4: Work & Energy

Definitions:

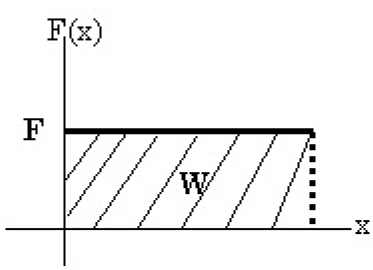
Constant: $W = \vec{F} \cdot \vec{S} = |\vec{F}||\vec{S}|\cos\theta$

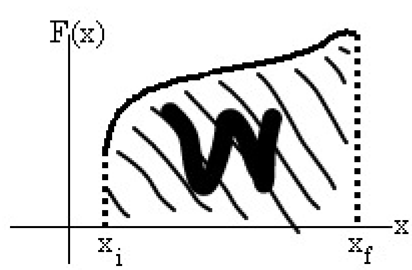
Non-constant:

$$W = \int \vec{F} \cdot d\vec{s}$$

Additionally: $W = \Delta E$

Being an integral, work can be expressed graphically as well:





Types of Energy:

Potential:

Spring Potential: $U_s = \frac{1}{2}kx^2$

Gravitational Potential (near Earth): $U_g = mgh$

Kinetic: $K = \frac{1}{2}mv^2$ (Note: this formula works even if the velocity is non-constant)

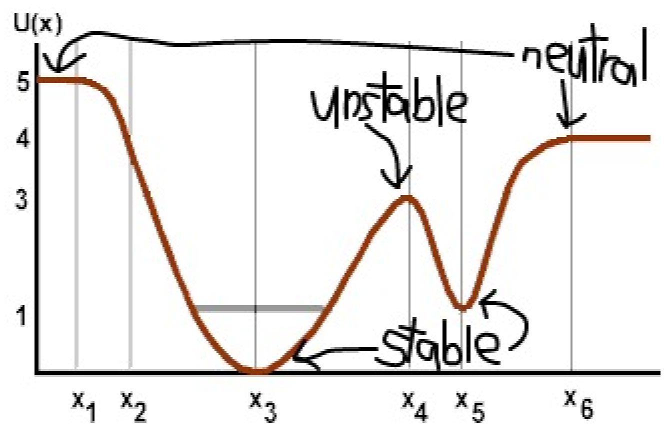
Conservative Forces:

- If a body moves under the action of a force that does no total work during any round trip, that force is "conservative"
- If work done by a force in moving a body from some initial location to some final location is independent of the path taken, the force is "conservative"
- Examples of conservative forces:
  - Gravity, springs, electrostatics, magnetism
- Examples of non-conservative forces:
  - Friction, air resistance

Conservation of Energy:

- If the only forces acting on a system are conservative forces, then energy is conserved in the system
- If Work is done by a conservative force in changing the relative configuration of the system, there is a corresponding change in potential energy: $\Delta U = -W_{conservative}$
- Following from this property: $\Delta K = -\Delta U \rightarrow$ $(K+U)_1=(K+U)_2$
- If non-conservative forces are in play, that equation can be written as: $(K+U)_1=(K+U)_2-\Delta E$
- It can also be written as $\Delta E=\Delta U+\Delta K$

Relationship between Force & Potential Energy

- As a consequence of the formulas in the last section, it follows: $U = -\int F dx$ and $\frac{-dU}{dx} = F$
- It also follows that $v = \pm\sqrt{\frac{2}{m}(E - U(x))}$
- Motion is restricted to positions where $E \geq U(x)$
- 

- Stable equilibrium: particle at rest at this location will remain at rest. Any deviation from that location will result in the presence of a restoring force (force pushing object back towards equilibrium position). Graphical represented as a relative minimum.

- Unstable equilibrium: particle at rest at this location will remain at rest. Any deviation from that location will result in the presence of a force (force pushing object away from equilibrium position.) Graphically represented as a relative maximum.
- Neutral equilibrium: particle at rest at this location will remain at rest. Any deviation from that location will not result in any change to the particle's speed. Graphically represented as a horizontal line.

Power

- Although Slater didn't teach it in class, it is still part of the curriculum
- $P = \frac{W}{\Delta t} = \frac{dW}{dt} = Fv$

Test 5: System of Particles, Momentum, and Impulse

Center of Mass:

- For systems that are really complex and hard to evaluate by analyzing individual parts, determining properties about the center of mass can be extremely helpful
- Center of mass is a location which behaves as if all of the mass of the system were located (concentrated) at that point
  - Essentially, a "weight average" (similar to atomic mass)
- Formula: $x_{com} = \frac{1}{M_{Total}}(x_1 m_1 + x_2 m_2 + ... + x_n m_n)$
- Since velocity and acceleration are just different order derivatives of displacement, the velocity and the acceleration of the center of mass can be determined using the same formula just with all the x's replaced with v's or a's
- If there are no external forces acting on the system, a_{com}=0 m/s
- If there are external forces, the system can be analysed by applying Newton's second law to the center of mass: $F_{external}=Ma_{com}$
- To determine the of mass of a system not consisting of point masses, the centers of mass of the objects can be combined to together to determine the overall center of mass (such as is a wire frame)

Linear Momentum & Impulse:

- $p = mv$
- $F = \frac{dp}{dt}$
- $J = \Delta p = \int F(t)dt$
- Force is the slope of a p vs. t graph
- The area under a F vs. t graph is the impulse or change in linear momentum
- Remember that linear momentum has a direction, so if an object changes direction, that change must be included in the change in linear momentum
  - For example, if a ball bounces off a wall
- Momentum is conserved as long as no external forces are acting on the system
- If there are external forces, usually it's a normal force

Properties of Elastic & Inelastic Collisions

Elastic	Inelastic
Momentum is conserved	Momentum is conserved
$ME_{Before}=ME_{After}$ $(K_{Before}=K_{After})$	$ME_{Before}\neq ME_{After}$ $(K_{Before}\neq K_{After})$(some ME is converted to other forms;usually heat or light)
Examples: 1. Carts with springs 2. Collisions with hard spheres (ball bearings/billiard balls) 3. Atomic particles	Examples: 1. Bullets or related objects 2. Objects that stick together 3. Obvious changes in shape

In both cases, momentum AND TOTAL ENERGY are conserved

Conversation of Momentum in 2-Dimensions

General Procedure:

1. Break up the components of the momentums
2. Set up equations for the conservation of momentum in the vertical and horizontal directions
3. Possibly set up an equation for the conservation of Kinetic Energy if the collision is elastic (remember that Kinetic energy is not a vector, so you can't have a vertical and horizontal Kinetic Energy.)
4. See if you have enough equations for the amount of unknowns you have and do some algebraic manipulation to isolate one of them in terms of known quantities (warning: although the physics is basically done by this point, this may be the hardest step of all)
5. Use the unknown that has been found to solve for the other unknowns

Test 6: Rotation Everyone's favorite

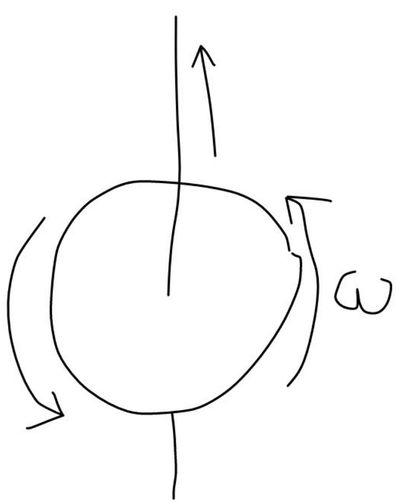
Pure Rotation of a Rigid Body:

- ➢ Every point on the body moves in a circular path and the centers of these circles all lie on a common straight line (axis of rotation)
- ➢ A reference line (perpendicular to the axis of rotation) moves through the same angle in a given time interval as only other reference line (also perpendicular to the axis)
- ➢ If the body rotating isn't a rigid body, the rotational equations don't work (this probably wouldn't be too much of a problem)

Rotational Kinematics:

- ➢ Linear vs. angular displacement
    - $s = r\theta$
    - s=arc length (linear displacement)
    - θ=angular displacement
    - r=radius
    - "Radians" is a nae and not necessarily a unit and may be inserted or dropped out for convenience

- Linear vs. angular velocity
  - Avg angular velocity: $\overline{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$
  - Instantaneous angular velocity: $\omega = \frac{d\theta}{dt}$
  - If we differentiate $s = r\theta$with respect to time:
    - $v = r\omega$
- Linear vs. angular velocity
  - Avg angular acceleration: $\overline{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i}$
  - Instantaneous angular acceleration: $\alpha = \frac{d\omega}{dt}$
  - Following the logic from before: $a = r\alpha$
  - Radial acceleration: $a = r\omega^2$
- Formulas:
  - $\omega = \omega_0 + \alpha t$
  - $\theta = \theta_0 + \omega_0 + \frac{1}{2}\alpha t^2$
  - $\omega^2 = \omega_0^{\ 2} + 2\alpha\theta$
- Direction:
  - Angular velocity (ω) and the angular acceleration (α) are all vectors and therefore have a direction associated with them
  - Very small angular displacements (θ) can also be described as vectors, but is they are larger, they aren't
  - The direction of these vectors is oriented along the axis of rotation
  - If your fingers of your right hand curl around the axis in the direction of rotation or the direction the rotation is speeding up or slowing down in, your thumb extends in the direction associated with these vectors



Additional information:

If there are two wheels or gears linked by a spring and they don't slip, their linear velocities are the same, so $r_1\omega_1 = r_2\omega_2$

Torque (τ):

- "Rotational force" that produces a rotational acceleration ("twisting effect") as opposed to a “linear” force that produces a linear, or “translational” acceleration

- Formula: $\tau = I\alpha$ and $\tau = F x r = rF\sin\theta = rF_{\perp}$
- The direction of the torque is given by the right hand rule (see first section)
- They are added as vectors

Rotational Inertia/Moment of Inertia:

- The measure of resistance of an object to a rotational acceleration (analogous to a resistance to a linear acceleration, mass)
- Formulas: $I = \sum m_i r_i^{\ 2} = \int r^2 dm$
  - In class, Slater showed how these formulas were derived, but it really isn't that important to know
- Also remember: $K = \frac{1}{2} I\omega^2$
- Rotational Inertia depends not only on the mass, but its distribution
- Also, depends on what the axis of rotation is
- It is not an intrinsic property

Parallel Axis theorem

- The rotational inertia of any object around any arbitrary axis is equal to the rotational inertia around a parallel axis through the center of mass plus the mass of the object multiplied by the square of the distance between the two axes
- $I = I_{COM} + Mh^2$
- Since $Mh^2 \geq 0$, I_{COM} is always smallest possible rotational inertia for an object

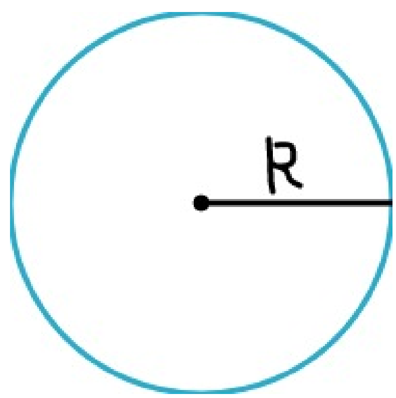
Need to Know Derivations of Rotational Inertia

Hoop:

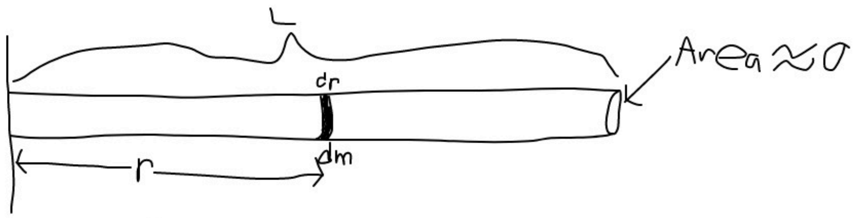
$I = \Sigma m_i r_i^{\ 2}$

$I = R^2 \Sigma m_i$ and $M = \Sigma m_i$

$I = MR^2$



Thin Rod:



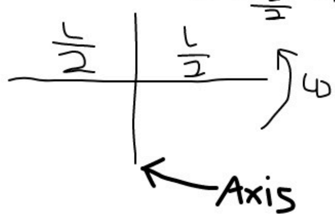
$$\lambda = \frac{M}{L} = \frac{dm}{dr} \longrightarrow dm = \frac{M}{L}\,dr$$

Around End

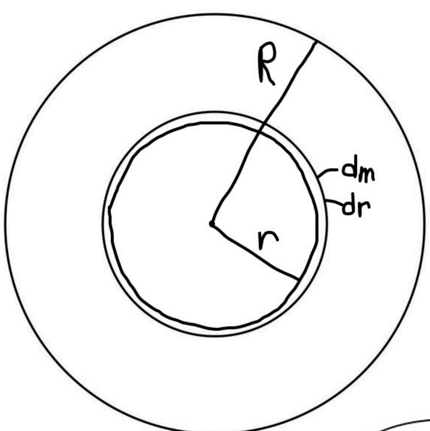
$$I = \int r^2\,dm = \frac{M}{L}\int_0^L r^2\,dr = \frac{M}{L}\left(\frac{r^3}{3}\Big|_0^L\right) = \frac{M}{L}\left(\frac{L^3}{3} - 0\right) = \frac{1}{3}ML^2$$

Around Center

$$I = \int r^2\,dm = \frac{M}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} r^2\,dr = \frac{M}{L}\left(\frac{r^3}{3}\Big|_{-\frac{L}{2}}^{\frac{L}{2}}\right) = \frac{M}{L}\left(\frac{L^3}{3(8)} - \frac{-L^3}{3(8)}\right) = \frac{1}{12}ML^2$$



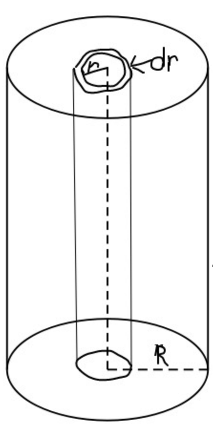
Disk:



$$\sigma = \frac{M}{A} = \frac{dm}{dA} = \frac{dm}{2\pi r\,dr} \longrightarrow dm = \frac{M}{A}2\pi r\,dr = \frac{M}{\pi R^2}2\pi r\,dr = \frac{2M}{R^2}r\,dr$$

$$I = \int r^2\,dm = \frac{2M}{R^2}\int_0^R r^3\,dr = \frac{2M}{R^2}\left(\frac{r^4}{4}\Big|_0^R\right) = \frac{M}{2R^2}\left(R^4 - 0\right) = \frac{1}{2}MR^2$$

Cylinder:



$$\rho = \frac{M}{V} = \frac{dm}{dV} \longrightarrow \frac{M}{\pi R^2 L} = \frac{dm}{2\pi L r\,dr} \longrightarrow dm = \frac{2M}{R^2}r\,dr$$

Then do the same integration as you would for a disk.

Summary of need to know moments of Inertia:

Hoop: $I = MR^2$

Rod (around the end): $I = \frac{1}{3}ML^2$

Rod (around center): $I = \frac{1}{12}ML^2$

Disk: $I = \frac{1}{2}MR^2$

Cylinder: $I = \frac{1}{2}MR^2$

Other object's Rotational Inertias will be given to you on the test.

Work & Power

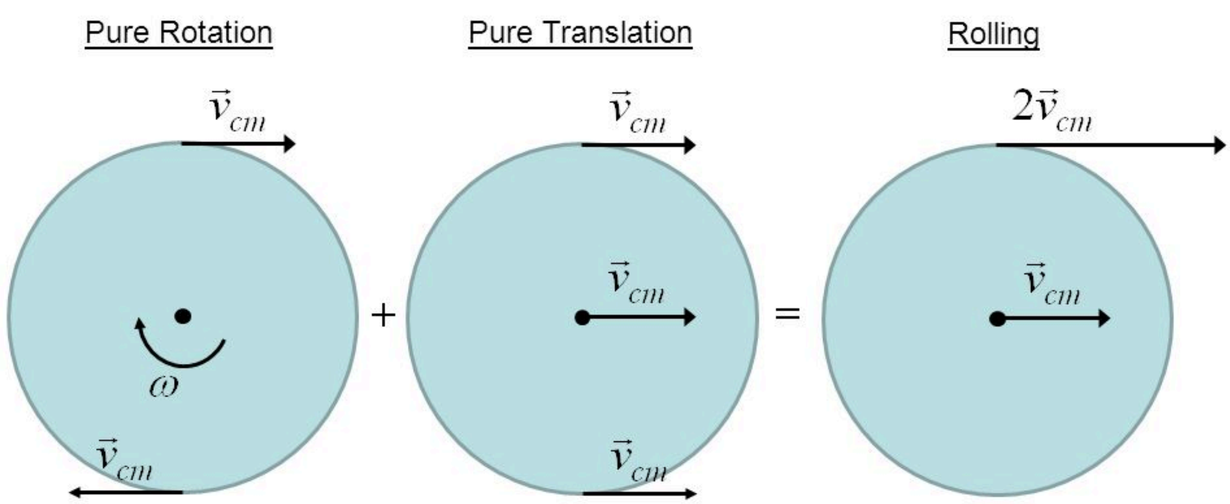
- Not that important in this unit
- $W = \tau\theta$ or $W = \int \tau d\theta$
- $P = \frac{dw}{dt} = \tau\omega$

Combined Rotational & Translational Kinetic Energy

- K_T=K_R+K_T=$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
- Suppose that the total kinetic energy is constant:
    - If I is small, the rotational kinetic energy is small, and therefore, the translational kinetic energy is greater
    - If I is big, the rotational kinetic energy is big, and therefore, the translational kinetic energy is smaller
- Therefore, sphere beats cylinder, but cylinder beats disk

Rolling Without Slipping:

- An object rolls across a surface such that there is no relative motion between the surfaces at the point of contact
- There is static friction between the surfaces so no work is done by the frictional force and no energy is dissipated
- Can be viewed as the superposition of rotational and translational motion

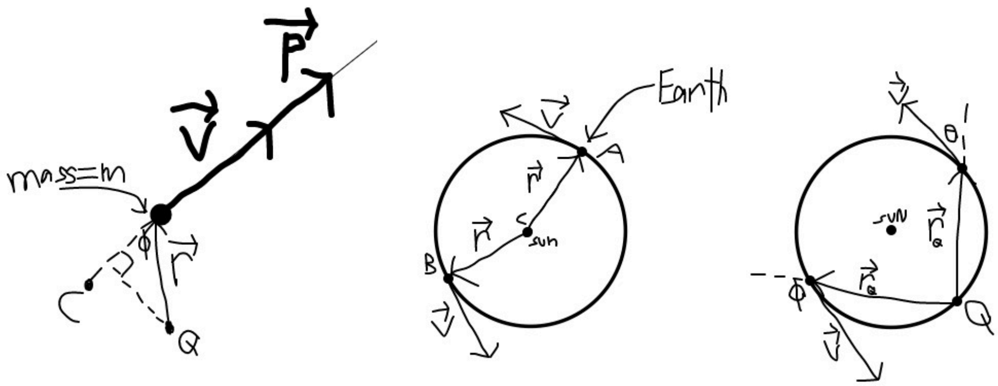


Only rolling without slipping is the following true: s=rθ, v=rω, and a=rα.

Angular Momentum:

- l-single particle
- L-system of particles
- $l_Q = rxp = (rxv)m = rmvsin\Phi = mvr_\perp$, direction is given by the right hand rule
- If we choose a different reference point, like point C: l_C=0 because r and v are parallel (sinϕ=0)

- ➢ Therefore: angular momentum is not an intrinsic property (unlike linear momentum, p) and depends on the choice of the reference point
- ➢ (All of this can be visualized with the image on the next page)



For the middle imagine:
- ➢ l_C (at point A)=$r_c x p_c = m(rxv) = mrv$(since sin(90°)=1)
- ➢ l_C (at point B)=$r_c x p_c = m(rxv) = mrv$(since sin(90°)=1)
- ➢ Angular momentum (relative to point C0 is conserved (constant)
- ➢ This is a special case

For the right imagine:
- ➢ l_Q (at point A)=mrvsinθ
- ➢ l_Q (at point B)=mrvsinϕ
- ➢ l_Q (at point Q)=0
- ➢ Angular momentum is clearly not conserved

What to get out of this example:
Angular momentum is conserved for a body orbiting another larger body with the reference point being the larger body
Conservation of Angular Momentum:
- ➢ If $l_Q = rxp$, then: $\frac{dl_Q}{dt} = \frac{dr_Q}{dt}xp + r_Q\frac{dp}{dt} = 0 + r_QxF = \tau_Q$
- ➢ If a torque acts on an object, then the angular momentum isn't constant
- ➢ In situations where $\tau_{Net} = 0$, then angular momentum is constant
- ➢ It is also true that $L = I\omega$and therefore, $\frac{dL}{dt} = I\frac{d\omega}{dt} = I\alpha = \tau_{Net}$
- ➢ When the net torque acting on a system is zero, the total angular momentum of the system is constant
- ➢ Examples:
  - ○ Spinning ice skater-arms of closer to body (I decreases and ω increases)\
  - ○ Drivers and gymnastics-”tuck” position lower I and increases ω)
  - ○ Collapsing stars
  - ○ Spinning objects (rotating bicycle wheels, gyroscopes, football “spiral”, artillery shells and bullets), spinning increases stability

Test 7 (Part 1): Oscillation & Simple Harmonic Motion

Definitions:
- ➢ Periodic motion-regular repetition
- ➢ Restoring force-force acting so to return the system to equilibrium position (pendulum,

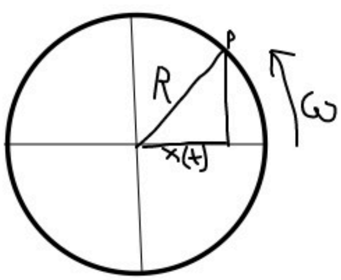
springs, etc.)

- Simple harmonic motion (SHM)-restoring force is proportional to displacement
- Amplitude-maximum displacement
- Period-amount of time for 1 complete cycle/oscillation/wave
- Frequency- number of cycles/second
- Damping- reducing the amplitude of the vibrations (caused by air resistance, friction, daspot, etc.)
- Potential energy is at a minimum at equilibrium
- "Turning points"- direction changes (ex: pendulum at highest point)
  - Motion (U is at maximum and K is at minimum)

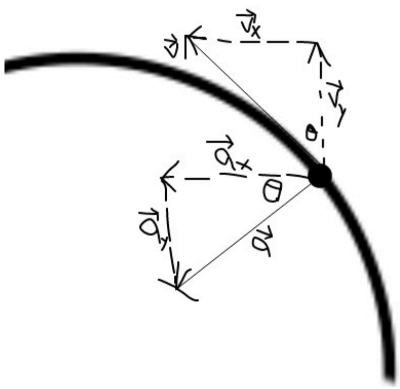
Simple Harmonic Oscillator

- An object that moves with simple harmonic motion (no fucking duh)
  - I.e. $F_{restoring}$αdisplacement from equilibrium
- For example, a mass on a spring
- Since $F =- kx$and $F = ma \rightarrow - kx = ma = m\frac{d^2x}{dt^2} \rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$
- Solution form: $x(t) = Acos(\omega t + \Phi)$
- A is the maximum displacement from equilibrium
- ϕ is the phase constant which depends on the starting position
- Only works if $\omega = \sqrt{\frac{k}{m}}$
- Maximum velocity: Aω
- Maximum acceleration: $A\omega^2$
- Velocity: $v(t) =- A\omega sin(\omega t + \Phi)$
- Acceleration: $a(t) =- A\omega^2 cos(\omega t + \Phi)$

Circular Motion vs. Simple Harmonic Motion:



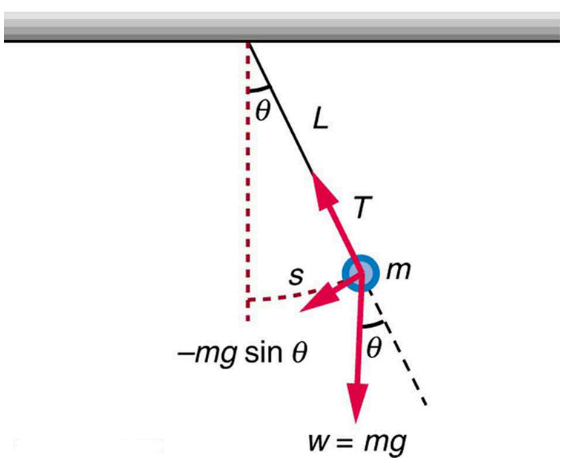
- Object moves with uniform circular motion with an angular speed of ω, ω=θt and radius R
- Project the motion onto the x-axis
- $x_x(t) = Rcos(\omega t)$and $x_y(t) = Rsin(\omega t)$, so $x = \sqrt{x_x^{\ 2} + x_y^{\ 2}} = \sqrt{R^2cos^2(\omega t) + R^2sin^2(\omega t)} = R$(since the cosine and sine squares combine to be one)
- Just take the derivative and do the same square root equation to determine similar information
- Velocity: $v_x =- R\omega sin(\omega t)$and $v_y = R\omega cos(\omega t)$: $v = R\omega$
- Acceleration: $a_x =- R\omega^2 cos(\omega t)$ and $a_y =- R\omega^2 cos(\omega t)$: $a = R\omega^2$(notice how it's the same equation from the rotation unit)



Period and Other Information:

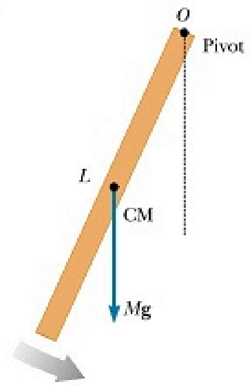
- ➢ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = \frac{1}{f}$
- ➢ Remember: gravity does not affect the restoring force or vertical oscillation, but it does however affect the energy of the system
- ➢ Importance of S.H.M.: Although we've concentrated on springs (which are the most obvious and easily available examples of simple harmonic motion), many other situations with small amplitude vibrations, reduce to simple harmonic motion
  - Ex: acoustics, optics, mechanics, circuits, and atomic physics

Pendulum:



- ➢ The radial component of weight are F_T contribute to the centripetal acceleration (changes in direction
- ➢ Only the tangential component of the weight changes the speed of the object
- ➢ $F =- mgsin\theta = ma_{tan}$
- ➢ since the sine function can be approximated by θ due to Taylor series, the equation can be written as $F =- mg\theta = ma_{tan}$
- ➢ s=rθ (for a small θ, s=x and r=L),so x=Lθ→θ=x/L
- ➢ So $F =- mg\frac{x}{L} =- (\frac{mg}{L})x$since there's a constant, there is S.H.M.
- ➢ For a pendulum, $k = \frac{mg}{L}$so therefore: $T = 2\pi\sqrt{\frac{L}{g}}$

Physical Pendulum:

- In a physical pendulum, the mass is not entirely concentrated at the "end" of the pendulum
- 

- $\tau = rxF = - dMg sin\theta = I_p \alpha \rightarrow - dMg sin\theta = I_p \frac{d^2\theta}{dt^2} \rightarrow \frac{d^2\theta}{dt^2} + \frac{dMg}{I_p}\theta = 0$
- $\omega = \sqrt{\frac{dMg}{I_p}} \rightarrow T = 2\pi\sqrt{\frac{I_p}{dMg}}$

Energy within Simple Harmonic Motion

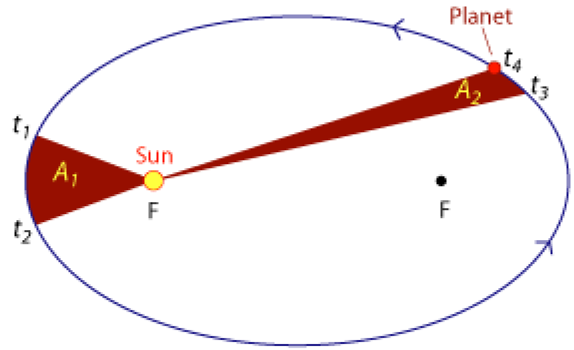
(assume oscillator moves horizontally with no damping effects from friction, air resistance, etc.)

- $U_s = \frac{1}{2}kx^2$ where $x(t) = Acos(\omega t)$
- Maximum potential energy: $U_s = \frac{1}{2}kA^2$
- $K = \frac{1}{2}mv^2$ where $v(t) = - A\omega sin(\omega t)$
- Through some math: $K = \frac{1}{2}kA^2 sin^2(\omega t)$
- Maximum kinetic energy: $K = \frac{1}{2}kA^2$
- $E_{total} = K + U_s = \frac{1}{2}kA^2$

Test 7 (Part 2): Gravitation

Kepler's Laws

1. Law of Ellipses
   a. All planets love in elliptical orbits having the sun at a focus
   b. Aphelion: largest distance from sun
   c. Perihelion: shortest distance from the sun
   d. "Tools" the analyze orbital motion
      i. $T = \frac{2\pi}{\omega} \rightarrow \omega = \frac{2\pi}{t} \rightarrow v = r\omega = r\frac{2\pi}{T} \rightarrow T = \frac{2\pi r}{v}$
      ii. $F_c = F_g \rightarrow \frac{mv^2}{r} = \frac{GMm}{r^2} \rightarrow v = \sqrt{\frac{GM}{r}}$(M is the mass of the central body)
2. Law of Equal Area
   a. A line connecting the sun to any planet sweeps out equal areas in equal times



Kepler's Second Law:
Law of Equal Areas
(greatly exaggerated)

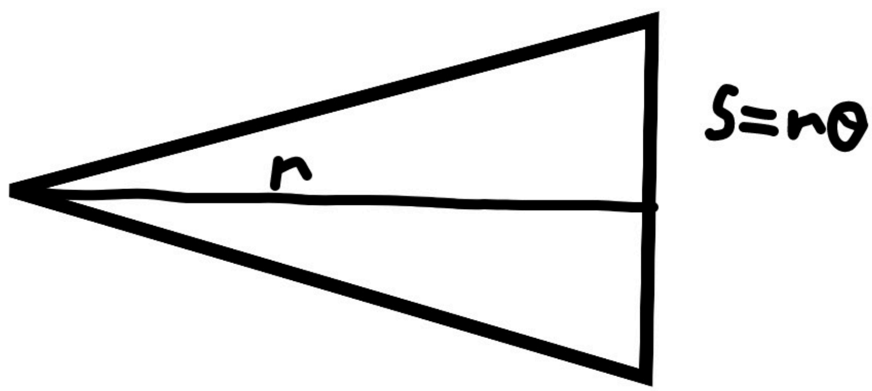
b.

c. Proof:

i. $\frac{dA}{dt} = \lim_{\Delta t \to o} \frac{\Delta A}{\Delta t} = \lim_{\Delta t \to 0} \frac{1}{2} r^2 \frac{\Delta \theta}{\Delta t} = \frac{1}{2} r^2 \omega$

ii. $L = I\omega = mr^2\omega \rightarrow r^2\omega = \frac{L}{m} \rightarrow \frac{dA}{dt} = \frac{L}{m}$

iii. L is constant (in isolated system) therefore dA/dt is constant



iv. Note: knowing this proof is completely useless for doing well on the midterm

d. Conservation of Angular Momentum

i. With same orbit, L is observed

ii. $L = rxp = rmv \rightarrow$ if r increases, then v decreases proportionally

iii. However, with different orbits (with different r values):

1. $L = rxp = rmv = rm\sqrt{\frac{GM}{r}} \rightarrow L\alpha\sqrt{r}$

e. Other Information:

i. Velocity of orbiting body varies with distance, r, from central body

ii. $K\alpha\frac{1}{r}$

iii. $K\alpha v^2$

iv. $v^2\alpha\frac{1}{r}$

v. $K = \frac{1}{2}m\frac{GM}{r}$

3. Law of Periods

a. The square of the period of any planet about the sun is proportional to the cube of the planet's mean distance from the sun

b. $\frac{GMm}{r} = m\omega^2 r \rightarrow \omega = \sqrt{\frac{GM}{r}}$

c. $T = \frac{2\pi}{\omega} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = \frac{2\pi}{\sqrt{GM}} r^{\frac{3}{2}} \rightarrow T^2 = \frac{4\pi^2}{GM} r^3 \rightarrow \frac{T^2}{r^3} = \frac{4\pi^2}{GM} = k$

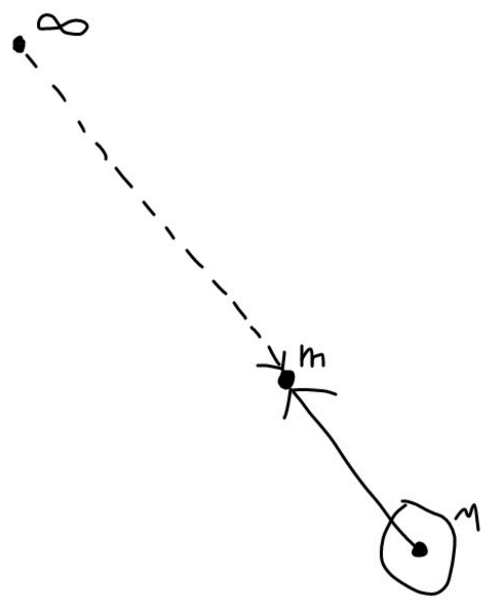
d. $T^2 \alpha r^3$ and $k\alpha\frac{1}{M}$

e. Note: the proof to show these equations and proportion is not that important to know

Calculation of Gravitational Acceleration at any Location

$w = F_g \rightarrow mg = \frac{GMm}{r^2} \rightarrow g = \frac{GM}{r^2}$

Gravitational Potential Energy

- Near surface of Earth: U_g=mgh
- Distance farther from Earth (U_g in general):
- Recall U_g=work done against gravity in lifting a mass
- The change in U_g is also defined as the negative of the work done on mass (by gravity) as its position changes from a to b
- $\Delta U_g = -W_{ab}$
- Consider 2 particles to be separated by a distance a and then brought closer together= separation by a and b distance
- If we use infinity as our (original)reference position and r as out final distance
- 

- $\Delta U = -W_{Gravity} = -\int_{\infty}^{r} F \bullet ds = \int_{\infty}^{r} \frac{GMm}{r^2} dr = GMm \int_{\infty}^{r} r^{-2} dr = GMm(\frac{-1}{r}|^{r}_{\infty}) = \frac{-GMm}{r}$

  where U(∞)=0

Hole through the Earth:

$M' = \rho V' = \rho \frac{4}{3}\pi r^3$

$F = \frac{-GM'm}{r^2} = \frac{-G\rho\frac{4}{3}\pi m r^3}{r^2} = -G\rho\frac{4}{3}\pi m r$

$k = \frac{4\pi}{3} G\rho m$

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\frac{4G\rho\pi m}{3}}} = \sqrt{\frac{3\pi}{G\rho}}$

Escape Velocity

- ➢ Minimum velocity needed to leave gravitational influence of central body (and, therefore, avoid orbiting or falling back)
- ➢ Upon reaching infinity, the object will have K=0 (note: we are looking for minimum velocity) and U_g=0
- ➢ Therefore, its total energy is equal to zero
- ➢ $E_{total} = 0 = \frac{1}{2}mv^2 - \frac{GMm}{r} \rightarrow \frac{GMm}{r} = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2GM}{r}}$
- ➢ Note: Knowing how to derive this formula is probably actually important for the test, since some points might be granted for it
- ➢ If E_T<0→object is gravitationally bound to central body
- ➢ If E=0→minimum escape velocity
- ➢ If E_T>0→will escape (and have non-zero velocity (and K) at infinity (WHAT!?))