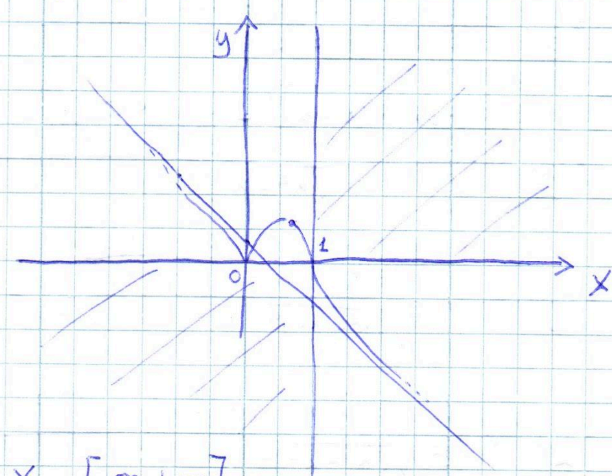
$y = \sqrt[3]{x^2(1-x)}$ $\qquad \mathcal{D}: \mathbb{R}$

segno $y>0 \quad x^2(1-x)>0 \quad$ $x^2>0 \ \forall x \neq 0$, $\ 1-x>0 \quad x<1$

$\cap$ asse x $\quad x=0 \quad x=1$

$\cap$ asse y $\quad y=0$



$$\lim_{x\to\pm\infty} \sqrt[3]{x^2(1-x)} = \mp\infty$$

$$\lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim \frac{\sqrt[3]{x^2(1-x)}}{x} = \lim \frac{\sqrt[3]{x^3\left(\frac{1}{x}-1\right)}}{x} =$$

$$= \lim_{x\to\infty} \frac{x\sqrt[3]{\frac{1}{x}-1}}{x} = -1 = m$$

$$q = \lim_{x\to\infty} f(x) - mx = \lim_{x\to\infty} \sqrt[3]{x^2-x^3} + x = [-\infty+\infty] =$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt[3]{x^2-x^3}+x\right)\left(\sqrt[3]{(x^2-x^3)^2} - x\sqrt[3]{x^2-x^3}+x^2\right)}{\left(\sqrt[3]{(x^2-x^3)^2} - x\sqrt[3]{x^2-x^3}+x^2\right)} = \lim_{x\to\infty} \frac{x^2 \cancel{-x^3} \cancel{+x^3}}{\sqrt[3]{x^4-2x^5+x^6} - x\sqrt[3]{x^3\left(\frac{1}{x}-1\right)}+x^2}$$

$$= \lim_{x\to\infty} \frac{x^2}{x^2\left(\sqrt[3]{\frac{1}{x^2}-\frac{2}{x}+1} - \sqrt[3]{\frac{1}{x}-1}+1\right)} = \frac{1}{3}$$

as. obliquo $y = -x + \frac{1}{3}$

Punti stazionari e crescenza

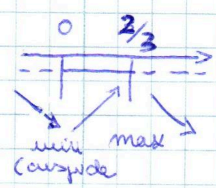
$$y' = \frac{1}{3}(x^2-x^3)^{-\frac{2}{3}}(2x-3x^2) = \frac{x(2-3x)}{3\sqrt[3]{(x^2-x^3)^2}}$$

C.E. y' $\quad x \neq 0 \quad x \neq 1$

$$\lim_{x\to 0} y' = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{x(2-3x)}{3\sqrt[3]{x^4-2x^5+x^6}} = \lim_{x\to 0^\pm} \frac{\cancel{x}(2-3x)}{3\cancel{x}\sqrt[3]{x-2x^2+x^3}} = \mp\infty \text{ cuspide}$$

$$\lim_{x\to 1} \frac{x(2-3x)}{3\sqrt[3]{(x^2-x^3)^2}} = \frac{-1}{0^+} = -\infty$$ punto a tg verticale

$$y'>0 \quad \frac{x(2-3x)}{3\sqrt[3]{(x^2-x^3)^2}} > 0 \quad x(2-3x)>0$$



$$f\left(\frac{2}{3}\right) = \sqrt[3]{\frac{4}{9}\left(1-\frac{2}{3}\right)} = \sqrt[3]{\frac{4}{27}} = \frac{\sqrt[3]{4}}{3}$$

$$y' = \frac{2x - 3x^2}{3(x^2 - x^3)^{2/3}}$$

$$y'' = \frac{1}{3} \cdot \frac{(2-6x)(x^2-x^3)^{2/3} - (2x-3x^2) \cdot \frac{2}{3}(x^2-x^3)^{-\frac{1}{3}}(2x-3x^2)}{(x^2-x^3)^{4/3}} =$$

$$= \frac{1}{2} \frac{(2-6x)\sqrt[3]{(x^2-x^3)^2} - \dfrac{2(2x-3x^2)^2}{3\sqrt[3]{x^2-x^3}}}{(x^2-x^3)^{4/3}} =$$

$$= \frac{1}{2} \frac{\dfrac{3(2-6x)(x^2-x^3) - 2(2x-3x^2)^2}{3\sqrt[3]{x^2-x^3}}}{\sqrt[3]{(x^2-x^3)^4}} =$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\left[3(1-3x) \cdot x^2(1-x) - (2-3x)^2 x^2\right]}{3\sqrt[3]{(x^2-x^3)^5}} =$$

$$= \frac{x^2\left[3(1-x-3x+3x^2) - (4-12x+9x^2)\right]}{3\sqrt[3]{(1-x)^5 x^{10}}} =$$

$$= \frac{1}{3\sqrt[3]{(1-x)^5 x^4}}\left(3 - \cancel{12x} + \cancel{9x^2} - 4 + \cancel{12x} - \cancel{9x^2}\right) = -\frac{1}{3(1-x)\sqrt[3]{(1-x)^2 x^4}}$$

$y'' > 0 \quad x > 1 \qquad x = 1$ punto di flesso (come osservato prima)