Summary:
Professional communication specifications

This page gives a checklist for what makes a "professionally communicated proof." Examples: **LaTeX "how to" guide**; a **sample proof**; full **portfolio draft and writing revision**.

To earn "revision complete" or "excellent" a proof must do *all* of these:

1. Include a true and correctly worded theorem statement just before the proof.
2. At the very beginning of the proof, state all assumptions and explain what will be proved (even if this repeats what was in the theorem statement).
3. State the proof method being used. If using proof by contrapositive, contradiction, cases, or induction, clearly state the new assumptions or statement being used.
4. Give clear, correct, and organized justification for each statement. Clearly show which results justify each statement.
5. Have no important errors or omissions in mathematical reasoning or justification.
6. Define all variables.
7. Cite definitions clearly and correctly each time one is used.
8. Follow all instructions on the portfolio sheet, including additions or changes to these specifications (most often, this includes adding examples and an Overleaf link).

In addition, to earn "revision complete", a proof must do *most* of these. This allows for occasional errors in up to 3 items, or consistent errors in 1 item.
To earn "excellent", a proof must do *all* of these thoroughly with no noticeable omissions.

A. Be typed in a new Overleaf document (template, LaTeX guide).
B. Use correct spelling, grammar, punctuation, sentence structure, and start a new paragraph when a new idea begins.
C. Use "we" instead of "I" or "you".
D. Never begin a sentence with a mathematical symbol.
E. Include every equation or formula in a sentence, including punctuation where appropriate.
F. Display (center) important equations and formulas and align "=" signs.
G. Use appropriate symbols in formulas and equations; avoid excessive symbols (or writing out formulas in words) otherwise.
H. Use equation numbers sparingly, and always refer to them in the text.
I. Write exponents, fractions, multiplication signs, and all other mathematical symbols in professional "textbook" format.
J. Write all variables in italics.
K. Include a summary at the end of the proof and an "end of proof" symbol.

Detailed explanation and examples are on the following pages

Details and examples:
Professional communication specifications

This page gives explanations and examples for the professional communication specifications.
Here are two complete examples of excellent professional communication:
A short **sample proof**, and a full **portfolio problem solution** with much more detail.

Items that must all be done in order to earn "revision complete":

1. **Include a true and correctly worded theorem statement just before the proof.**
2. **At the very beginning of the proof, state all assumptions and explain what will be proved (even if this repeats what was in the theorem statement).**

   Details: The theorem statement should be a simple declarative **statement**, like we've studied in class. Here is a typical textbook problem:

   > Prove that if n is an integer and n^2 is odd, then n is odd.

   To write a solution to this problem, begin with something like the following:

   > **Theorem:** If n is an integer and n^2 is an odd integer, then n is an odd integer.

   Then skip a line, write "*Proof*" and begin the proof on the same line. Right away, clearly state *all* assumptions and say what you will prove. This may feel like you're repeating things you just wrote in the theorem statement. That's OK! A proof should be fully self-contained.

   > *Proof.* We assume that n is an integer and n^2 is an odd integer. We will prove that n is an odd integer.

   You should always check some examples to see if a theorem statement is true or not. If it is false, give a single clear counterexample (with explanation) and then follow any additional instructions on the portfolio sheet.

3. **State the proof method being used. If using proof by contrapositive, contradiction, cases, or induction, clearly state the new assumptions or statement being used.**

   Details: Always state the proof method being used just after the assumptions, such as "We give a direct proof."

   The other proof methods listed begin in Chapter 3. When using any of these proof methods, first tell the reader that you are using them and clearly state the assumptions or new theorem statement before you continue the proof:

> **Theorem:** If n^2 is odd, then *n* is odd.
>
> *Proof:* We will prove the contrapositive of this statement, which is "If *n* is even, then n^2 is even." So, we assume that *n* is an even integer. We will prove that n^2 is an even integer as well.
>
> **Theorem:** If $x^2 = 2$ then *x* is an irrational number.
>
> *Proof*: We will prove this theorem by contradiction. So, we assume that the negation of the statement is true. That is, we assume that $x^2 = 2$ and *x* is a rational number.

4. **Give clear, correct, and organized justification for each statement. Clearly show which results justify each statement.**
5. **Have no important errors or omissions in mathematical reasoning or justification.**

Details: A proof is made of statements that build up on each other. Once you've justified one statement, it becomes a building block that can be used to justify future statements. So, the sentences in your proofs must be put in a logical order so that each statement follows from the work before it.

Each statement's justification must be clear and unambiguous in addition to being mathematically correct. Don't worry about being *too* direct or repetitive; always say exactly how you know that a statement is true. Leave nothing to the reader's imagination! The words "because" and "by" are helpful to indicate justifications.

Example:

> **Theorem:** If *n* is an even integer, then n^2 is an odd integer.
>
> *Proof.* We assume that *n* is an even integer. We will prove that n^2 is an odd integer.
>
> Because *n* is even, by definition we can write *n = 2k* for some integer *k*. Using algebra, we know that $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$. Because *k* is an integer, by the closure properties of integers, $2k^2$ is also an integer. …

Each sentence contains a statement (in yellow). The justifications are highlighted in green. It's OK for the justifications to come before the statements, although they can also go afterwards (for example, "We can write $n^2 = (2k)^2 = 4k^2$ using algebra.") This is just a matter of finding a way to make a sentence sound natural, rather than an issue of correctness.

You can also justify statements using outside facts that are prerequisites for this class, such as facts from calculus. You can (and should!) leave out details below the level of this class (including excessive algebra or arithmetic).

6. **Define all variables.**

   Details: Before using any variable, you must explain what it is. *Never* use a variable without defining it first. You can do so in this style: “Let b be the base length of the triangle.” or “Let $x \in R$.” In particular, notice that each example tells you the meaning of the variable or the set it comes from (which is like a meaning). It is also OK to define the variable *immediately* after using it, as in “Because n is even, we know that $n = 2k$ for some integer k” or “Therefore, $n = 2k$ for some $k \in Z$.” Using an undefined variable is a critical logical mistake!

7. **Cite definitions clearly and correctly each time one is used.**

   Details: Every definition has two parts: A word (like “even”), and a meaning for that word. Using a definition involves connecting one of these two parts to the other, so that you can use the other part.

   For example, if you know a number n is even, the definition lets you include the other half of the definition as a statement in your proof: There exists an integer k so that $n = 2k$. Definitions work both ways, so if you know that $n = 2k$ where k is an integer, then the definition lets you state that n is even.

   To cite a definition, you must explain why you can use it by including a “warrant” or reason for why you can use it in this situation. Then, state the conclusion that the definition lets you reach. Use the definition’s word where it best fits and use the phrase “by definition” to connect these items.

   It can sometimes feel repetitive to cite a definition each time you use it. That’s OK! It is better to be clear and thorough, than to leave some justification out.

   Examples:

   - ❏ “Because we are given that n is even, by definition there exists an integer k such that $n = 2k$.” Here the warrant is that we are *given* n is even. Then the conclusion comes after “by definition.”
   - ❏ “Because we have written $n = 2k$ where k is an integer, by definition n is an even integer.” This time the warrant is that we’ve already written $n = 2k$ for an integer k. The conclusion is that we can say n is called “even.”

Items that must be “mostly” done:

Reminder: “Mostly” means occasional errors in up to 3 items, or consistent errors in 1 item.

A. **Type each problem in a new Overleaf document.**

   Use this Overleaf **template** and refer to this **LaTeX reference** document for help with symbols and formatting. You can also find these links on Blackboard.

B. **Use correct spelling, grammar, punctuation, and sentence structure and start a new paragraph when a new idea begins.**

This is just like in MTH 150 (or MTH 120 & 130). Proofs often have only 1 or 2 sentences per paragraph, and it's OK to start a new paragraph as soon as there's a new idea, even if it makes for a very short paragraph.

C. **Use "we" instead of "I" or "you".**
The idea is to stress that you and the reader are doing the mathematics together. This really is standard in math, so it's good practice with it now!

D. **Never begin a sentence with a mathematical symbol.**
This avoids problems with whether you should capitalize a variable or not. Symbols also make it harder to tell where a sentence begins.

Good example: "Because $n = 2k$, by definition we know that n is even."
Bad example: "$n = 2k$, so by definition we know that n is even."

E. **Include every equation or formula in a sentence, including punctuation where appropriate.**
Every equation can be read in words, and so it must fit properly within a sentence. This includes putting a punctuation mark at the end of a sentence, even if the sentence ends with a formula. For example, notice the comma, colon, and period in this complete sentence:

Using algebra, we have:

$$x^2 + 2x + 1 = 0.$$

Read out loud, this forms a complete sentence including a verb: "Using algebra, we have: x squared plus $2x+1$ is equal to zero."

F. **Display (center) important equations and formulas and align "=" signs.**
This means putting important equations, centered, on their own line. Only center *important* equations that are key to the proof or its conclusion. Minor or "helper" equations can be left inline. If the left side of the equation does not change, don't repeat it. For example:

Using algebra, we obtain:

$$\begin{aligned} x \cdot y &= (2m+1)(2n+1) \\ &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1. \end{aligned}$$

Since m and n are integers, we conclude that…

You can find examples of how to do this in the sample Overleaf file.

G. **Use appropriate symbols in formulas and equations; avoid excessive symbols (or writing out formulas in words) otherwise.**
Do *not* try to write out formulas in words. But also, never use a symbol or abbreviation in place of a word, such as: ∀, ∃, =, ∈, s.t. etc.

Good example: "Because there exists an integer k so that $n = 2k$…"

Bad example: "Because $\exists k$ s.t. *n* equals 2 times *k*..."
Another bad example: "By adding the square of *x* to both sides, we find that…"

H. Use equation numbers sparingly, and always refer to them in the text.
If you don't need to refer to an equation by number, leave out the number. You can see examples of both in the sample Overleaf file.

I. Write exponents, fractions, multiplication signs, and all other mathematical symbols in professional "textbook" format.
Do *not* use shorthand or "text" format. See examples in the sample Overleaf file.

Yes!	**Noooooooo!**
x^2	x^2
$\frac{x+1}{5}$	(x+1)/5, or worse: x+1/5
$\sqrt{7}$	sqrt(7)
xy or $x \cdot y$	x*y

J. Write all variables in italics.
This helps them stick out and distinguishes *words* like "a" from *variables* like *a*.

K. Include a summary at the end of the proof and an "end of proof" symbol.
It's considered good form to sum up the proof once you've finished the main writing, and to clearly show the reader where the proof ends. Just state what you actually proved, and put a box at the end of that line. This can be summarized or simplified compared to the original theorem statement, as long as it is still true.

Example:

Thus we have shown that if *n* is an integer and n^2 is odd, then *n* is odd. ■

Special rules for writing solutions to conjectures

Some of these instructions will appear on any portfolios that are labeled “Conjecture”. Be sure to read them there, and follow them!

If a result is labeled a “Conjecture”, you’ll need to take care before attempting to prove it. Each of the following is a “required” specification that, if not completed carefully, means your proof is not professionally communicated.

1. Test whether the conjecture is true by creating a variety of examples. It may be false!
2. If a conjecture is true, state an appropriate theorem or proposition and prove it.
3. If a conjecture is false, provide a single carefully chosen counterexample to show that it is false.
4. If a biconditional statement is false, determine if one of the two conditional statements within it is true. If so, state an appropriate theorem for this conditional statement and prove it. Separately, provide a counterexample for the other false conditional statement.
5. If a conjecture asks you to prove that two sets are equal and is false, then determine if one of the sets is a subset of the other set. If so, state an appropriate theorem or proposition and prove it. Provide an example that shows that the other subset is false.
6. If a conjecture asks you to prove that a function is a bijection and is false, then determine if the function is an injection or is a surjection. State an appropriate theorem stating only the true items and prove it. Provide a counterexample to any parts that are false.

This list is adapted from Ted Sundstrom’s “Mathematical Reasoning: Writing and Proof” v3.0.

Summary:
Professional communication specifications

This page gives a checklist for what makes a "professionally communicated proof." *See the complete list on Blackboard, including examples and explanations of each specification.*

To earn "revision complete" or "excellent" a proof must do *all* of these:

1. Include a true and correctly worded theorem statement just before the proof.
2. At the very beginning of the proof, state all assumptions and explain what will be proved (even if this repeats what was in the theorem statement).
3. State the proof method being used. If using proof by contrapositive, contradiction, cases, or induction, clearly state the new assumptions or statement being used.
4. Give clear, correct, and organized justification for each statement. Clearly show which results justify each statement.
5. Have no important errors or omissions in mathematical reasoning or justification.
6. Define all variables.
7. Cite definitions clearly and correctly each time one is used.
8. Follow all instructions on the portfolio sheet, including additions or changes to these specifications (most often, this includes adding examples and an Overleaf link).

In addition, to earn "revision complete", a proof must do *most* of these. This allows for occasional errors in up to 3 items, or consistent errors in 1 item.
To earn "excellent", a proof must do *all* of these thoroughly with no noticeable omissions.

A. Be typed in a new Overleaf document.
B. Use correct spelling, grammar, punctuation, sentence structure, and start a new paragraph when a new idea begins.
C. Use "we" instead of "I" or "you".
D. Never begin a sentence with a mathematical symbol.
E. Include every equation or formula in a sentence, including punctuation where appropriate.
F. Display (center) important equations and formulas and align "=" signs.
G. Use appropriate symbols in formulas and equations; avoid excessive symbols (or writing out formulas in words) otherwise.
H. Use equation numbers sparingly, and always refer to them in the text.
I. Write exponents, fractions, multiplication signs, and all other mathematical symbols in professional "textbook" format.
J. Write all variables in italics.
K. Include a summary at the end of the proof and an "end of proof" symbol.