Towards an Equitable Futures Discounting Framework

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Executive Summary

This project addresses the critical issue of short-termism in economic decision-making through the exploratory development of an analytical framework for comparing and applying various discounting approaches. The inadequacy of traditional economic discounting methods in accurately capturing long-term impacts, particularly in intergenerational decision-making, has resulted in suboptimal policy and investment decisions. This whitepaper considers the process of using 20 distinct discounting methods in a model, employing comparative analysis and sensitivity testing.

The primary deliverable is a proposed interactive computational model featuring visualisation capabilities and parameter adjustment functionalities – if implemented in institutional decision-making processes, this tool has the potential to influence trillions of dollars in investments and policy decisions over the many decades, and has scope even outside sectors such as climate change mitigation and infrastructure development. The results of this project, upon completion of the backend model and probabilistic simulations, include the successful development of an economic discounting tool. The model demonstrates robust functionality across various discounting methodologies and is now primed for initial user testing and stakeholder feedback.

Introduction

The discipline of economics has long grappled with the challenge of evaluating future costs and benefits in present-day decision-making. This challenge is particularly acute in the context of long-term projects and policies, where the impacts extend far beyond the immediate future. The method used to compare present and future values, known as discounting, plays a crucial role in shaping these decisions. However, the application of traditional discounting methods has often led to an undervaluation of future outcomes (both good and bad), resulting in short-sighted policies and investments that pose detrimental long-term ramifications.

The choice of discounting method can dramatically affect the perceived value of long-term projects, yet there is a lack of consensus on which approach is most appropriate for different scenarios. This lack of agreement often leads to inconsistent decision-making and potential misallocation of resources. Furthermore, the complexity of various discounting methods makes it challenging for decision-makers to fully understand and appropriately apply them. In short, the principles of economic discounting – and their application – lack futures literacy.

Efforts towards reforming discounting methods by lowering social discount rates (SDRs) faces slow progress and adoption. While such direct intervention would be ideal in theory, perhaps more nuanced solutions are necessitated; if we cannot modify conventional methods, we can instead employ some of the more future-friendly existing discounting methodologies by making them more usable and understandable for decision-makers. This report explores such a solution.

Theory of change

Current challenges in economic discounting

The infamous tragedy of the time horizon sums up the detrimental effects of short-term thinking on decision-making, particularly in the context of planning and policy. Drawing a parallel to the well-known "tragedy of the commons," which describes how individuals acting in their self-interest can deplete shared resources which leads to negative consequences for the collective, this serves as a cautionary reminder of the importance of long-term thinking in decision-making processes.

When making decisions that affect the trillions of future citizens yet to be born, such calculations aren't as straightforward, and face a wide array of problems, particularly the following:

- **Inadequacy of traditional methods**: Conventional discounting approaches often fail to capture the complexities of long-term impacts, or simply undermine them.
- **Lack of consensus**: There is no agreement among experts on which discounting method is most appropriate for different scenarios. This discord leads to inconsistent decision-making across sectors and jurisdictions. For transnational issues, this problem is even more pertinent; since they require coordinated action across multiple countries, the dangers of inconsistent discounting can cause misaligned incentives, delayed action, and equity issues for burden-sharing across nations.
- **Complexity and accessibility**: The intricacy of various discounting methods makes it challenging for decision-makers to fully understand and appropriately apply them. This complexity barrier often results in the misapplication or oversimplification of discounting techniques.
- **Sensitivity to parameter changes**: Even seemingly miniscule alterations in discounting parameters can dramatically affect the perceived value of long-term projects. However, many decision-makers lack the tools to conduct comprehensive sensitivity analyses.
- **Political and social myopia**: Decision-makers, influenced by immediate concerns, fears, or pressures, prioritise short-term solutions over long-term strategies. As a result, this leads to excessive use of reactive policies (instead of proactive ones) to address symptoms over root causes. Fear-driven decisions – fear of losing public support, fear of electoral consequences, or fear of backlash from interest groups – lead to hasty decisions that prioritise short-term popularity over sustainable solutions, causing a persistent and cyclical neglect of systemic issues.

Proposed theory of change

The first and most concrete step is **tool development**, aimed at addressing the **complexity and inaccessibility** of current methods. By creating an intuitive software system that evaluates various discounting approaches, from traditional to hyperbolic models, we tackle the **inadequacy of traditional methods** by offering more appropriate alternatives. This tool will empower decision-makers, not just economists, to engage with complex economic models, thus directly addressing the **lack of consensus** by providing a unified platform that encourages consistent decision-making across sectors.

A key feature of this tool is **sensitivity analysis**. By incorporating advanced sensitivity analysis capabilities, we address the issue of **sensitivity to parameter changes**. Users will better understand how small input changes can affect long-term valuations, leading to more informed and stable decision-making.

Finally, the most crucial part of the theory of change is **adoption**. Ensuring the tool reaches policymakers, business leaders, and NGOs is vital for combating **political and social myopia**. By enabling these stakeholders to engage more deeply with long-term economic analysis, we aim to shift the focus from reactive, short-term policies to sustainable, long-term strategies.

Expected outcomes and impacts

The integration of sensitivity analysis capabilities will allow for more robust evaluation of long-term projects, **accounting for uncertainties and parameter variations**. The greatest benefit from this lies in potential for policy improvements. Enhanced long-term economic analysis is expected to result in policy changes that better account for future costs and benefits. Since it enables stakeholders to scrutinise the assumptions behind long-term economic decisions more easily, it could lead to more robust and **defensible policy choices**, allowing long-termist policymakers to gain traction.

Another supplementary outcome would be **standardisation**. As (and if) the tool gains adoption, it may contribute to the standardisation of discounting practices across different fields and jurisdictions, leading to more **consistent long-term decision-making**.

Policymakers, business leaders, and investors are likely to adopt this tool because it directly addresses their current challenges. For policymakers, the ability to evaluate the long-term impacts of decisions with greater precision will help them create policies that are both forward-thinking and politically defensible. The tool's transparency and advanced analysis capabilities offer a clear advantage: it empowers them to justify their decisions with robust data, making it easier to gain support from stakeholders who might otherwise focus on short-term gains.

For the private sector, **the promise of more accurate long-term valuations can lead to better investment outcomes**. Companies and investors, constantly in search of a competitive edge, will find value in a tool that allows them to identify projects with substantial long-term potential. As the tool helps to align business strategies with long-term market trends, it can foster a shift towards more sustainable and profitable investments.

Additionally, **the educational impact cannot be overstated**. As this tool becomes part of academic curricula, it will prepare future leaders to think more critically about long-term economic challenges. This foundational shift in how upcoming professionals approach economic decision-making could have a lasting impact on global policy and investment landscapes.

Literature Review

Traditional economic discounting models

It is undeniable that exponential discounting is the most widely used model in economics and finance. This model assumes a constant discount rate over time, resulting in an exponential decline in the present value of future cash flows. it is expressed as:

$$PV = FV / (1 + r)^t$$

where PV is present value, FV is future value, r is the discount rate, and t is time.

Its popularity stems from its simplicity, mathematical tractability, and time consistency (the relative preference between two future outcomes doesn't change as time passes). It is extensively used in finance for valuing investments, in cost-benefit analysis for public projects, and in economic modelling. However, it is criticised for potentially undervaluing long-term impacts, especially in environmental economics and policy making for issues like climate change, suggesting why it needs to be replaced by or used alongside other methods (which is precisely the aim of the model).

In a different context, Weighted Average Cost of Capital (WACC) discounting is a fundamental concept in corporate finance and is widely used for capital budgeting decisions. WACC represents the average cost of financing for a firm, considering both equity and debt. The formula is:

$$WACC = \Sigma (w_i * r_i)$$

where w_i is the weight of each financing source and r_i is its cost.

It is used as a discount rate for evaluating investment projects: if a project's return exceeds the WACC, it is considered value-creating. WACC is popular because it accounts for a company's capital structure and provides a clear hurdle rate for investments. It is widely taught in business schools and used in practice for making capital allocation decisions. However, it may not be appropriate for projects with different risk profiles than the company as a whole, and it doesn't explicitly consider long-term or societal impacts.

While not as ubiquitous as the previous two, Ramsey Discounting is a key model in welfare economics and has gained prominence in climate economics. Developed by Frank Ramsey in 1928, this model links the discount rate to economic growth and social preferences. The humble Ramsey equation is:

$$r = \delta + \eta g$$

where δ is the pure rate of time preference, η is the elasticity of marginal utility, and g is the growth rate of consumption.

It is grounded in economic theory and considers both impatience (δ) and the decreasing marginal utility of consumption as societies become wealthier (ηg). Ramsey Discounting has become particularly important in climate economics, featuring prominently in the Stern Review on the Economics of Climate Change. Although valued for its theoretical rigour and explicit consideration of intergenerational welfare, it can be sensitive to parameter choices and requires long-term economic projections.

The Social Discount Rate (SDR), delineated in Partha Dasgupta's work, has grown to become a critical parameter in policy analysis, quantifying the time preference of society for present versus future consumption. It is employed in cost-benefit analyses to calculate the net present value of long-term projects. The SDR's magnitude significantly impacts policy decisions, particularly in domains with extended time horizons such as climate change mitigation: a lower SDR ascribes greater weight to future benefits, potentially justifying more aggressive climate action, while conversely, a higher SDR prioritises near-term outcomes. Hence, policymakers must carefully calibrate the SDR to balance immediate societal needs against long-term sustainability goals. The SDR's application extends beyond environmental policy, influencing decisions in infrastructure development, healthcare, and education, where benefits may accrue over decades or centuries.

Alternative approaches to discounting

No single discounting approach has even been universally agreed upon, with much debate on the subject. For instance, in the Weitzman-Gollier puzzle on the appropriate way to discount future cash flows, Weitzman (1998) argues for a low discount rate for the distant future, suggesting that future generations should not be penalised heavily for the uncertainty of future benefits. He emphasises that the value of future benefits should be considered more carefully, especially in the context of irreversible investments and the potential for catastrophic outcomes. Gollier (2004) challenges this view by suggesting that the discount rate should reflect the uncertainty of future consumption growth. He argues that if future consumption is uncertain, the appropriate discount rate should be higher, as it reflects the risk associated with future returns. Some economists, in pursuit of identifying a discount rate that is consistent with our basic moral intuitions but also conforms with economists' traditional notion of economic efficiency, have proposed an SDR of zero.

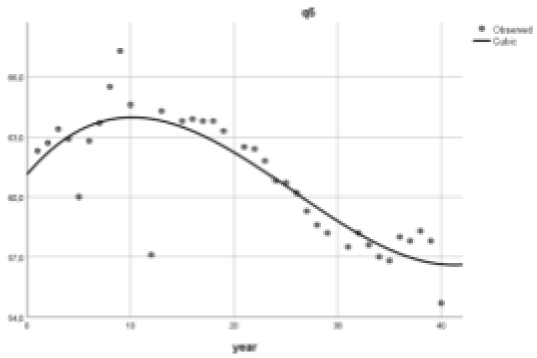
Similarly, in Nordhaus vs. Stern, discourse centres on the appropriate discount rate for long-term climate change policies. Nordhaus advocates for a higher discount rate based on observed market returns, emphasising intergenerational equity and opportunity costs. He uses a descriptive approach, deriving parameters from empirical data. Contrarily, Stern argues for a lower discount rate, employing a prescriptive approach based on ethical considerations. Stern's approach incorporates a near-zero pure rate of time preference and lower elasticity of marginal utility of consumption. Despite vast differences in time preference (δ) and elasticity of marginal utility (η), both of them use the same per capita growth rate (g) of ~1.3%; yet, these parameters result in significantly different policy recommendations. A summary of this is detailed in Appendix A.

Sector-specific discounting practices have also evolved to address unique challenges. In healthcare economics, for instance, the use of health-adjusted life years (HALYs) and quality-adjusted life years (QALYs) has led to discussions about whether health outcomes should be discounted at the same rate as monetary costs. Some countries, like the UK, have adopted differential discounting approaches for health effects and costs in their health technology assessments.

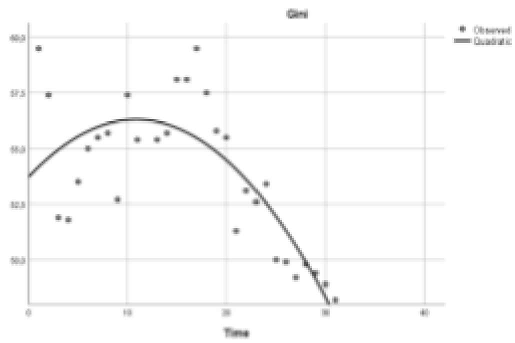
Long-term equity considerations

A [Rawlsian perspective] on long-term discounting would emphasise the welfare of the worst-off future generations. The “maximin” principle posits that we should maximise the minimum level of well-being across all generations, including potentially catastrophic future scenarios. For instance, given uncertainty about climate impacts, a Rawlsian approach would err on the side of caution by aligning more with Stern's precautionary stance. Hence, Rawls would likely support Stern's lower discount rate, as it gives more weight to future generations' welfare.

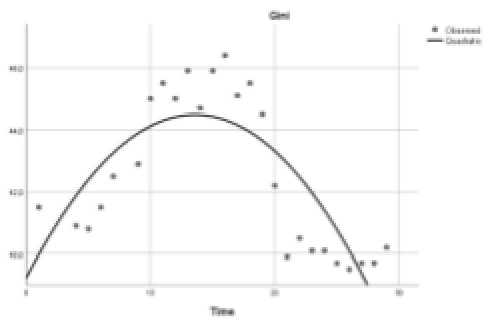
What about the aforementioned ~1.3% per capita growth rate? It makes certain underlying assumptions: technological progress will continue, resource constraints will be overcomed, and no major global catastrophes will significantly impede growth. These assumptions lead to a high growth rate, which justifies less aggressive preventive policy in the medium-term. Not to mention, there are also assumptions made about the environmental Kuznets curve, which history has proved to be difficult to forecast and varies dramatically across nations even during similar cyclical periods, see charts below:



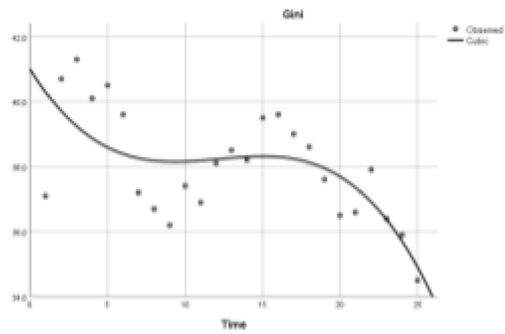
Brazil, 1981-2020



Honduras, 1989-2019



Uruguay, 1992-2020



Georgia, 1996-2020

Source: Sandqvist, Rickard. *Does Post-Industrialized Countries Face a Second Kuznets Curve with the IT Revolution?*, *Stockholms Universitet*, 2022, https://www.diva-portal.org/smash/get/diva2:1664687/FULLTEXT01.pdf. Accessed 2024.

Proposed Discounting Model

Model overview and key components

The core of the model is a discounting engine implementing 20 methods, enabling robust comparative analysis *without a weighted index*. While advocating for future-friendly methods is a secondary aim, this has its own challenges, discussed in the limitations section. The intuitive user interface allows easy input of project parameters and instant visualisation of results across different discounting models. A sensitivity analysis module shows how changes in inputs affect long-term valuations.

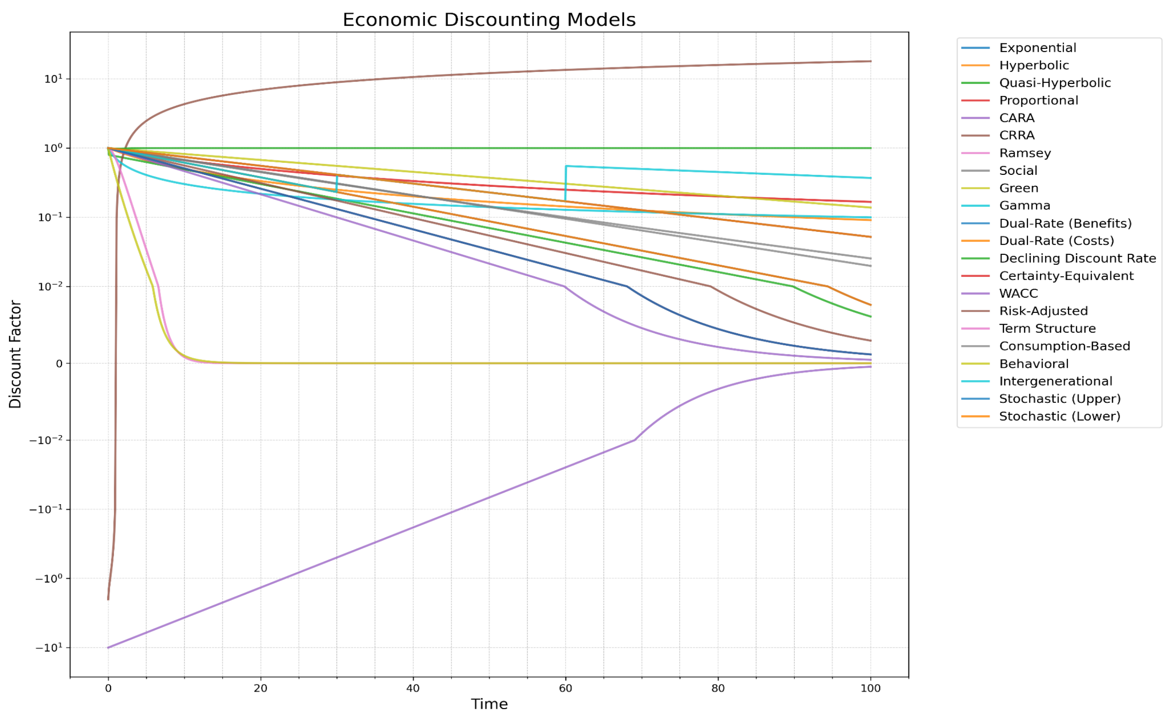
Under the hood, a Python-based backend handles long-term calculations, with potential for expansion. The model includes a structured database for efficient data storage and retrieval, dynamic visualisations for customizable charts, and graphs tailored to various stakeholders. The tool will be published with documentation and guides.

The current model is executed by a fast-performing code. In each iteration of modelling, there are two key outputs, which can be further manipulated by the user:

- A table array of discounting calculations derived from the inputs for all 20 discounting methods
- A logarithmic graph visualising the discount rates up to 100 years

Some anomalies on the graph were observed after running simulations. Gamma discounting had a split graph (see below) due to the incorporation of the discount factor (DF) in its calculation. This element was retained to avoid oversimplification of the graph.

An example of the graph, derived from Monte Carlo simulations, is shown below:



Incorporating diverse discounting methods

A key aspect of this model is its ability to facilitate a side-by-side comparison of various discounting methods **without** relying on a weighted index. This approach allows users to directly observe the implications of applying different discounting techniques to their project, rather than obscuring the results through a single composite score. The model handles the parameters and assumptions of each method **independently**, for each of the discounting methods applied, as listed below (formulae are explained in Appendix A)

Discounting Method	**Formula**
Intergenerational Discounting	$r = \rho + (1 - \alpha)\eta g$
Green Discounting	$r\_green = r - \alpha$
Social Discounting	$(1 + g)\text{^}(-\eta) / (1 + \rho)$
Declining Discount Rate (DDR) Model	$r\_t = r\_0\ e\text{^}(-\alpha t)$
Ramsey Discounting	$r = \delta + \eta g$
Gamma Discounting	$E[DF] = \int e\text{^}(-rt)\ f(r)\ dr$
Proportional Discounting	$PV = FV / (1 + rt)$
Dual-Rate Discounting	$PV = \Sigma\ (B\_i / (1 + r\_i)\text{^}t)$
Consumption-Based Discounting	$r = \rho + \gamma g\_c$
Stochastic Discounting	$E[PV] = E[\Sigma\ (CF\_t / \Pi(1 + r\_i))]$
Hyperbolic Discounting	$PV = FV / (1 + kt)$
Behavioural Discounting Models	*Various*
Quasi-Hyperbolic Discounting	$PV = \beta FV / (1 + r)\text{^}t$
CRRA Discounting	$U(c) = (c\text{^}(1\text{-}\eta) - 1) / (1 - \eta)$
Certainty-Equivalent Discounting	$CE = E[X] - \lambda\ Var(X)$
Term Structure Discounting	$PV = \Sigma\ (CF\_t / (1 + r\_t)\text{^}t)$
CARA Discounting	$U(c) = -e\text{^}(-ac) / a$
Exponential Discounting	$PV = FV / (1 + r)\text{^}t$
Risk-Adjusted Discounting	$r\_adj = r\_f + \beta(r\_m - r\_f)$
WACC Discounting	$\Sigma\ (w\_i * r\_i)$

By preserving the integrity of each discounting method's underlying assumptions, the tool empowers users to understand the specific factors driving the results and make informed decisions accordingly.

Mathematical framework

This subsection briefly explains some discounting formulae used, as well as general assumptions in model operation:

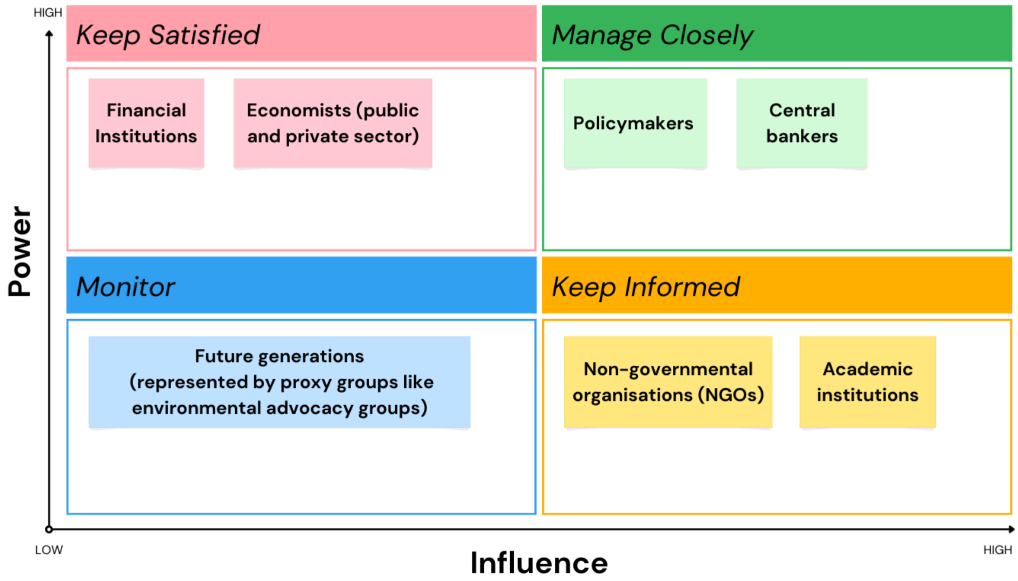
1. Generalised discounting
    a. Define a general function D(t) representing the discount factor at time t
    b. $PV = \Sigma[FV(t) * D(t)]$ for $t = 0$ to T where FV(t) is the future value at time t, and T is the total time horizon
2. Comparison metrics
    a. Net Present Value (NPV): $NPV = \Sigma[FV(t) * D(t)] - Initial\ Investment$
    b. Internal Rate of Return (IRR): Solve for r where: $0 = \Sigma[FV(t) / (1+r)\text{^}t] - Initial\ Investment$
    c. Benefit-Cost Ratio (BCR): $BCR = PV\ of\ Benefits / PV\ of\ Costs$
3. Sensitivity analysis
    a. Partial derivatives of PV with respect to key parameters
    b. Monte Carlo simulations done for probabilistic sensitivity analysis
4. Time horizon effects
    a. Asymptotic behaviour of different discounting methods as $t \to \infty$
    b. Analysis of crossover points where different methods intersect
    c. Attempt to ensure non-extrapolatory behaviour with some viable models (~7) as $t > 100$
5. Parameterization
    a. Defined a set of parameters $\theta = \{\theta\_1, \theta\_2, ..., \theta\_n\}$ that fully describe each discounting method
    b. Expressed $D(t)$ as a function of these parameters: $D(t, \theta)$

More details of mathematical justifications, proofs of properties, and some pseudocode are provided in Appendix C.

Stakeholder Analysis and Application

Identification of key stakeholders

The stakeholder matrix below classifies key stakeholders for this project:



Primary stakeholders in this context include policymakers at various levels of government, central bankers, economists in both public and private sectors, and financial institutions — these groups are at the forefront of implementing and utilising discounting methods in their decision-making processes.

Policymakers, particularly those involved in fiscal policy and long-term planning, are crucial stakeholders. For instance, the U.S. Office of Management and Budget (OMB) plays a pivotal role in setting discount rates for cost-benefit analyses of government projects. Their decisions, informed by tools like the proposed discounting evaluation model, can have far-reaching consequences on public investment and intergenerational resource allocation.

Secondary stakeholders, while not directly involved in the decision-making process, have significant interests in the outcomes. These include non-governmental organisations (NGOs) focused on sustainable development, academic institutions conducting research on economic theory and policy, and, crucially, future generations whose welfare is directly impacted by present-day economic decisions. The Intergovernmental Panel on Climate Change (IPCC), for example, represents a conglomerate of stakeholders whose work on climate change mitigation relies heavily on accurate long-term economic forecasting.

A unique challenge in this stakeholder analysis is the representation of future generations. While they cannot actively participate in current decision-making processes, their interests must be considered to ensure intergenerational equity. This necessitates the inclusion of proxy representatives, such as environmental advocacy groups or specialised government agencies like the UK's Committee on Climate Change, which explicitly considers long-term impacts in its policy recommendations.

Potential use cases and applications

This model presents a versatile tool with wide-ranging applications across various sectors with distinct decision-making challenges. In public policy and governance, the model can significantly enhance the quality of cost-benefit analyses for large-scale infrastructure projects. In environmental and climate change policy, the model's ability to incorporate and compare various discounting methods is particularly valuable. The perennial debate surrounding the appropriate social discount rate for climate change mitigation efforts, as highlighted in the aforementioned Stern Review and subsequent discussions, underscores the need for a tool that can transparently evaluate different approaches. By allowing policymakers to visualise the long-term implications of various discount rates on the perceived costs and benefits of climate action, the model can contribute to more informed and potentially more aggressive climate policies.

Corporate decision-making and financial markets stand to benefit significantly from the model's application. In the context of Environmental, Social, and Governance (ESG) investing, the model can provide a more nuanced approach to valuing long-term sustainability initiatives.

In the field of international development, the model can play a crucial role in optimising aid allocation and project selection. International financial institutions like the World Bank and regional development banks can employ the tool to evaluate the long-term impacts of their investments in developing countries. By allowing for the comparison of different discounting approaches, the model can help these institutions balance immediate needs with long-term development goals, potentially leading to more equitable development strategies.

The academic community can leverage this model to advance economic theory and empirical research on intertemporal choice and social preferences. Researchers can use the tool to conduct sensitivity analyses on existing economic models, potentially uncovering new insights into the relationship between discount rates and long-term economic outcomes. This could lead to refinements in economic theory and contribute to the ongoing discourse on how best to value future outcomes in economic decision-making.

Implementation strategies for different sectors

Implementing the economic discounting evaluation model across various sectors requires tailored strategies that address the unique challenges and operational contexts of each domain.

In the government and public sector, a phased implementation approach could prove most effective. This strategy might begin with pilot programs in select agencies, such as the **Environmental Protection Agency (EPA)** or the **Department of Transportation**, where long-term project evaluation is critical. These pilot programs would serve as proof-of-concept and allow for refinement of the model based on real-world application.

In the corporate sector, implementation strategies should focus on seamless integration with existing financial planning and risk assessment tools. Developing industry-specific modules or templates could facilitate adoption across various business sectors.

Limitations and Mitigation Strategies

Identified limitations of the model

A primary challenge lies in the **quantification of certain long-term impacts**, particularly those related to environmental and social outcomes. While the model provides a framework for incorporating these factors, the inherent difficulty in assigning monetary values to intangible or complex long-term effects (such as biodiversity loss or social cohesion) remains a significant limitation.

The potential for **misuse or misinterpretation of results** presents another critical limitation. The model's comprehensive nature and ability to generate multiple scenarios could lead to **cherry-picking of results** that align with predetermined positions. This risk is particularly acute in politically sensitive contexts, where different stakeholders might selectively emphasise scenarios that support their agendas.

Computational complexity and processing time pose practical limitations, especially for users without access to high-performance computing resources. The model's ability to run multiple discounting scenarios and conduct extensive sensitivity analyses, while valuable, may result in significant processing times for complex projects. This could potentially limit its applicability in **time-sensitive decision-making** processes.

The model's focus on economic valuation may **not fully capture non-economic values** that are crucial in many decision-making contexts. For instance, cultural significance, ethical considerations, or intrinsic environmental values may be inadequately represented in purely economic terms. This limitation could lead to an incomplete picture in scenarios where such non-economic factors play a pivotal role.

Not to mention, the model's effectiveness is heavily dependent on the **quality and availability of input data**. In many long-term planning scenarios, particularly those involving emerging technologies or unprecedented environmental changes, reliable data may be scarce. This limitation could affect the accuracy and reliability of the model's projections, especially for very long-term horizons – this is the unfortunate reason why I've limited the current model's projection to 100 years, hence the asymptotic graph.

Lastly, the inherent difficulty in **validating long-term projections** presents a fundamental limitation. While the model can provide a range of scenarios based on different assumptions, **verifying the accuracy** of these projections over extended time frames (e.g., 50-100 years) remains challenging. This limitation underscores the need for cautious interpretation of results and regular updating of the model as new data becomes available.

Proposed mitigation strategies

To address the identified limitations, several mitigation strategies are proposed.

Firstly, to tackle the challenge of **quantifying long-term impacts**, the development of **supplementary qualitative assessment tools** is recommended. These tools could include structured frameworks for evaluating non-monetary impacts, such as the use of multi-criteria decision analysis (MCDA) techniques. This approach would allow for a more holistic assessment that combines quantitative economic analysis with qualitative evaluations of social and environmental factors.

To mitigate the **risk of misuse or misinterpretation**, implementing **comprehensive user training and certification programs** is crucial. These programs would ensure that users understand the model's underlying assumptions, appropriate application contexts, and limitations. Additionally, incorporating **built-in safeguards and warning systems** within the model interface could alert users to potential misapplications or inconsistencies in their analyses.

Addressing the limitation of **capturing non-economic values** requires the integration of **alternative valuation methodologies**. This could involve incorporating techniques from ecological economics, such as the concept of Total Economic Value (TEV), which includes **use and non-use values**. Furthermore, the model could be expanded to include modules for **rights-based approaches or multi-stakeholder deliberative processes** to capture values that resist monetization.

To tackle **computational complexity and processing time issues**, optimization of computational algorithms is essential. This could involve leveraging advanced techniques in **parallel processing and machine learning** to enhance efficiency. Additionally, developing a c**loud-based version** of the model could democratise access to high-performance computing resources, enabling more users to conduct complex analyses.

Establishing **rigorous data quality standards and validation protocols** is crucial to address the dependency on input data quality. This could involve something like creating a **curated database of verified long-term projections and historical data**, regularly updated through collaborations with academic institutions and international organisations. Implementing **automated data quality checks** and **uncertainty quantification methods** within the model would also enhance the reliability of inputs and outputs.

To address the challenge of **validating long-term projections**, a system for **regular model updates and long-term performance tracking** should be established. This could involve creating a **feedback loop** where model projections are compared against real-world outcomes as they unfold over time. Implementing a **version control system** and maintaining **transparency in model evolutio**n would allow users to understand how projections change with updated data and methodologies.

On a lighter note, I believe the first and most dangerous potential limitation has already been subverted: considering Chesterton's fence argument by recognising the need for discounting at all. Although the proposal for a zero-percent discount rate was noted in the literature review, we avoided the trap of completely dismissing discounting, and instead taking a more balanced and realistic approach with this model.

Areas for further research and development

As the field of economic discounting continues to evolve, these areas warrant further research and development to enhance the proposed model's capabilities and relevance.

Integrating insights from behavioural economics presents a promising avenue for research. While the model currently incorporates some behavioural aspects through hyperbolic discounting options, there's potential for deeper integration. Research could focus on incorporating more nuanced models of time preference, such as those accounting for magnitude effects, sign effects, and sequence effects in discounting. This could lead to more accurate representations of real-world decision-making processes, particularly in contexts involving individual choices or public preferences.

Exploring adaptive discounting methodologies represents another critical area for development. As global conditions change rapidly, static discounting approaches may become less relevant. Research into dynamic discounting models that adjust based on evolving economic, environmental, and social conditions could significantly enhance the model's long-term applicability. This might involve developing algorithms that update discount rates based on real-time data or incorporating scenario-based approaches that allow for flexible discount rate adjustments.

Although this would be an ambitious project, the development of sector-specific modules could greatly expand the model's utility. While the current model offers a generalised approach, tailored modules for sectors such as healthcare, energy, or education could provide more targeted insights. This research direction would involve collaborating with sector experts to identify unique discounting considerations and developing specialised interfaces and calculation methods for each sector.

Research on intergenerational equity frameworks is crucial for addressing one of the fundamental challenges in long-term discounting. This could involve exploring alternative ethical frameworks for valuing future outcomes, such as sufficientarianism or prioritarianism, and developing mathematical representations of these approaches within the discounting model. Such research could lead to more ethically robust methods for evaluating projects with multi-generational impacts.

Investigating the applications of artificial intelligence and machine learning in discounting models offers exciting possibilities. Research could focus on developing AI algorithms to optimise parameter selection, predict long-term trends, or identify patterns in historical discounting decisions. Machine learning techniques could also be employed to enhance the model's sensitivity analysis capabilities, potentially uncovering complex relationships between input variables and long-term outcomes.

As previously discussed, although there is presently no weighted index of discounting methods used in the model due to potential for controversy, an optional, calculated comparison of the discounting methods that favour future outcomes could be conducted to provide an additional feature in the model, or perhaps even be posed as an alternative model.

Lastly, studying discounting in the context of global catastrophic risks represents an important frontier. As the world grapples with existential threats such as climate change or engineered pandemics, traditional discounting approaches may prove inadequate. Research in this area could explore novel discounting methodologies that appropriately weight low-probability, high-impact black swan events in long-term decision-making processes.

Next Steps and Recommendations

Short-term implementation plan

The short-term implementation plan for this discounting model focuses on rapid development, testing, and initial deployment to key stakeholders. The plan is structured over a 12-month period, divided into a few key phases.

- Months 1-3 will be dedicated to the **development and testing of the core model**. This phase involves finalising the mathematical frameworks for each discounting method, implementing the calculation engines, and conducting rigorous internal testing. **Collaboration** with academic experts in economic theory and computational economics will be crucial during this phase to ensure the model's theoretical soundness and computational efficiency, hence I shall contact subject-matter experts who can provide relevant guidance.
- Months 4-6 will focus on creating an improved **user interface and visualisation tools**. So far, I've not done any noteworthy work on the front end, even though the goal is to develop an intuitive, user-friendly interface that makes complex economic concepts accessible to a broad range of users. This phase will involve close collaboration between economists, UX designers, and data visualisation experts (or someone with a combination of all three). Emphasis will be placed on creating dynamic, interactive visualisations that clearly illustrate the implications of different discounting approaches.
- Months 7-8 will be devoted to **beta testing with select institutions**. This will involve partnering with government agencies, academic institutions, and private sector organisations to test the model in real-world scenarios. Beta testers will be chosen to represent a **diverse range of potential use cases**, from environmental policy planning to corporate investment strategies. Feedback from this phase will be crucial for identifying and addressing any usability issues or computational bugs.
- Concurrent with beta testing, months 7-9 will also focus on the development of **comprehensive user documentation and training materials**. This will include detailed **user manuals, video tutorials, and interactive online training modules**. The aim is to create a robust support system that enables users to fully leverage the model's capabilities while understanding its limitations and appropriate use contexts.
- Months 10-11 will see the **initial rollout of the model to key government agencies**. This phase will prioritise agencies involved in long-term policy planning, such as environmental protection agencies, infrastructure development departments, and economic planning units.

Long-term research agenda

The long-term research agenda for the economic discounting evaluation model is designed to ensure its continued relevance and effectiveness in addressing complex, long-term economic challenges. This agenda spans a 5-10 year horizon and focuses on areas of development and exploration.

A primary focus will be the ongoing refinement of discounting methodologies. This will involve continuous monitoring of advances in economic theory and empirical research related to intertemporal choice and social preferences. Specific areas of investigation will include the development of more sophisticated hyperbolic discounting models, exploration of state-dependent discounting approaches, and research into the implications of uncertainty and irreversibility for discount rate selection. Collaboration with behavioural economists will be crucial in incorporating new insights into time preferences and decision-making under uncertainty.

Integrating emerging economic theories represents another critical aspect of the long-term agenda. This will involve staying abreast of developments in fields such as ecological economics, complexity economics, and post-growth economics. The goal is to ensure that the model remains flexible enough to incorporate alternative economic paradigms as they gain traction in academic and policy circles – this might involve developing new modules or alternative calculation frameworks within the existing model structure.

Developing AI-enhanced predictive capabilities is a key objective for enhancing the model's long-term utility. This research stream will explore the application of machine learning algorithms for improving long-term forecasting accuracy, optimising parameter selection, and identifying complex patterns in historical economic data. Collaboration with data scientists and AI researchers will be essential in this endeavour, potentially leading to the creation of a semi-autonomous system capable of adapting discounting approaches based on real-time data and emerging trends.

Expanding the model to address global, interconnected challenges is crucial given the increasing complexity of long-term policy issues. This will involve developing new frameworks for modelling the interactions between economic, environmental, and social systems on a global scale. Research will focus on incorporating systems thinking approaches and developing methods for addressing cross-border externalities and global public goods in discounting calculations.

Collaboration with interdisciplinary research teams will be a cornerstone of the long-term agenda. This will involve establishing partnerships with experts in fields such as climate science, public health, and technology foresight to ensure that the model can effectively address multifaceted, long-term challenges. These collaborations will aim to bridge the gap between economic theory and other disciplines, potentially leading to improved performance.

Policy implications and recommendations

The implementation of the comprehensive economic discounting evaluation model has far-reaching policy implications across various sectors and levels of governance. These implications necessitate a set of targeted recommendations to maximise the model's positive impact on long-term decision-making, some of which are explored here.

- In the realm of environmental policy, the model's ability to compare multiple discounting approaches could significantly influence climate change mitigation strategies. Policymakers should consider mandating the use of this model in cost-benefit analyses of major climate policies, ensuring that a range of discounting scenarios are evaluated. This could lead to more robust justifications for ambitious climate action, as the model may reveal the long-term benefits of early intervention across various discounting assumptions.
    - **Recommendation**: Integrate the model into national and international climate policy assessment frameworks, such as those used by the Intergovernmental Panel on Climate Change (IPCC) and national environmental protection agencies.
- For infrastructure development, the model's capacity to evaluate long-term projects under different discounting scenarios could reshape investment priorities. Policymakers should consider requiring the use of this model for all major public infrastructure projects, particularly those with lifespans exceeding 50 years.
    - **Recommendation**: Update infrastructure planning guidelines to incorporate multi-scenario discounting analyses using the new model, ensuring that long-term benefits are adequately captured in project evaluations.
- In healthcare policy, the model's flexibility could inform more nuanced approaches to valuing future health outcomes. Policymakers should explore using the model to refine health technology assessment methodologies, potentially leading to more balanced investments in preventive care and long-term health interventions.
    - **Recommendation**: Commission a comprehensive review of healthcare discounting practices, using the new model to assess the impact of different approaches on long-term health outcomes and healthcare system sustainability.
- To enhance transparency in long-term economic analysis, policymakers should consider mandating the disclosure of full discounting assumptions and sensitivity analyses in all major policy proposals, which can be facilitated by usage of this model. This would foster more informed public debate and scrutiny of long-term policy decisions.
    - **Recommendation**: Develop standardised reporting templates for discounting analyses, requiring policymakers to present results under multiple discounting scenarios and clearly communicate the implications of different approaches.

Conclusion

The development and implementation of the comprehensive economic discounting evaluation model represents a significant advancement in the field of long-term economic decision-making. By providing a flexible, user-friendly platform for comparing and applying various discounting approaches, this model (partially) addresses critical gaps in existing tools and methodologies.

Throughout this report, we have explored the model's potential applications across diverse sectors. The model's ability to simultaneously evaluate multiple discounting scenarios offers decision-makers insights into the long-term implications of their choices, potentially reshaping how we approach intergenerational resource allocation and sustainability.

As we move forward, it is crucial to recognize that this model is not just a technical tool; with further improvements, it can be a catalyst for a broader shift in how we conceptualise and value the future in our economic decisions.

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Appendices

Appendix A: Breakdown of discounting methods

The *equity rating* (column 3) reflects how well each discounting method considers the interests of future generations. **Methods that give more weight to long-term outcomes or explicitly consider future welfare** scores higher, with 1 being low and 5 being a high future consideration. While **not** currently included in my model, this rating could serve as a weighting factor when aggregating results from different methods in the future, especially for projects with **multi-generational effects.** Incorporating this rating would allow the model to adjust its emphasis on long-term considerations based on the specific needs of each policy decision. Yet, more thought is needed before these ratings are incorporated.

The table below includes all discounting methods used in the model:

Discounting Method	**Formula**	**Equity rating (1-5)**	**Main areas of application**
Intergenerational Discounting	$r = \rho + (1 - \alpha)\eta g$	4.8	Climate policy, sustainable development
Green Discounting	$r\_green = r - \alpha$	4.5	Environmental policy, conservation projects
Social Discounting	$(1 + g)\text{^}(-\eta) / (1 + \rho)$	4.2	Public policy, social welfare analysis
Declining Discount Rate (DDR) Model	$r\_t = r\_0\ e\text{^}(-\alpha t)$	4.0	Environmental economics, long-term policy
Ramsey Discounting	$r = \delta + \eta g$	3.8	Climate economics, long-term policy analysis
Gamma Discounting	$E[DF] = \int e\text{^}(-rt)\ f(r)\ dr$	3.7	Long-term project evaluation, climate policy
Proportional Discounting	$PV = FV / (1 + rt)$	3.5	Environmental economics, long-term projects
Dual-Rate Discounting	$PV = \Sigma\ (B\_i / (1 + r\_i)\text{^}t)$	3.5	Policy analysis, multi-faceted projects
Consumption-Based Discounting	$r = \rho + \gamma g\_c$	3.5	Macroeconomic policy, growth models
Stochastic Discounting	$E[PV] = E[\Sigma\ (CF\_t / \Pi(1 + r\_i))]$	3.3	Financial modelling, risk analysis, long-term planning
Hyperbolic Discounting	$PV = FV / (1 + kt)$	3.2	Behavioural economics, psychology, marketing
Behavioural Discounting	*Various*	3.0	Behavioural economics,

Models			marketing, public policy
Quasi-Hyperbolic Discounting	$PV = \beta FV / (1 + r)^t$	2.8	Behavioural economics, policy analysis
CRRA Discounting	$U(c) = (c^{(1-\eta)} - 1) / (1 - \eta)$	2.5	Macroeconomics, growth models, finance
Certainty-Equivalent Discounting	$CE = E[X] - \lambda\ Var(X)$	2.5	Financial analysis, risk management
Term Structure Discounting	$PV = \Sigma (CF_t / (1 + r_t)^t)$	2.0	Financial economics, bond valuation
CARA Discounting	$U(c) = -e^{(-ac)} / a$	2.0	Financial economics, risk analysis
Exponential Discounting	$PV = FV / (1 + r)^t$	1.7	Finance, economics, policy analysis
Risk-Adjusted Discounting	$r_{adj} = r_f + \beta(r_m - r_f)$	1.6	Investment analysis, project evaluation
WACC Discounting	$\Sigma (w_i * r_i)$	1.3	Corporate finance, capital budgeting

References for what each symbol means in the above formulae:

- PV: Present Value
- FV: Future Value
- r: Discount rate
- t: Time period
- k: Hyperbolic discount factor
- β: Present bias factor
- a: Coefficient of absolute risk aversion
- c: Consumption
- η: Coefficient of relative risk aversion (it's also used as elasticity of marginal utility)
- δ: Pure rate of time preference
- g: Growth rate of consumption or economy
- ρ: Social discount rate
- SDF: Stochastic Discount Factor
- α: Adjustment factor (e.g., for green discounting or declining discount rates)
- DF: Discount Factor
- f(r): Probability density function of discount rates
- B_i: Benefits of type i
- r_i: Discount rate for type i
- r_0: Initial discount rate
- CE: Certainty Equivalent
- E[X]: Expected value of X
- λ: Risk aversion parameter
- Var(X): Variance of X
- w_i: Weight of funding source i
- r_f: Risk-free rate
- β: Beta (systematic risk)
- r_m: Market return
- CF_t: Cash flow at time t
- γ: Elasticity of marginal utility of consumption
- g_c: Growth rate of consumption
- s: Psychological time

Appendix B: Comparative analysis and precedents

Review of similar approaches

Several models have been developed over the years to apply different discounting approaches, each with its own strengths and limitations. This review focuses on key models that share similarities with the proposed tool.

The Dynamic Integrated Climate-Economy (DICE) model, developed by William Nordhaus in the 1990s, is one of the most influential models for climate economics. While not exclusively focused on discounting, DICE incorporates a discounting framework that has been widely used and debated. The model uses a constant exponential discount rate, around 3-4% per year. As previously discussed, this poses severe threats:

- Limited flexibility in discounting approaches, primarily relying on constant exponential discounting, leading to poor intergenerational consideration.
- Inadequate representation of uncertainty, particularly for catastrophic outcomes.
- Oversimplification of complex climate-economy interactions.
- Lack of transparency in some underlying assumptions and calculations

The PAGE (Policy Analysis of the Greenhouse Effect) model, developed by Chris Hope in the 1990s, is another integrated assessment model that incorporates discounting. PAGE was notably used in the Stern Review and allows for Monte Carlo simulations to account for uncertainty. While more flexible than DICE, it still offers limited options for different discounting approaches. The model's complexity can make it challenging for non-experts to use and interpret results. Like DICE, it struggles to fully capture non-market impacts and extreme climate scenarios.

The Social Cost of Carbon (SCC) Model Intercomparison Project, led by the U.S. Environmental Protection Agency, compared results from DICE, PAGE, and FUND (Framework for Uncertainty, Negotiation and Distribution) models. While this project provided valuable insights into the impact of different discount rates on climate policy, it was limited in scope and did not offer a unified platform for comprehensive discounting analysis. Its focus was primarily on climate-related applications, limiting broader economic analysis. Moreover, there was limited exploration of alternative discounting methodologies beyond those used in the three main models.

The UK government's Green Book supplementary guidance on discounting is an example of a more flexible approach, introducing declining discount rates for long-term projects. However, this is more of a guideline than a comprehensive model, and it lacks the analytical capabilities of a full evaluation tool.

In summary, while these models and approaches have made significant contributions to the field of economic discounting, they all share common deficiencies: limited flexibility in comparing multiple discounting approaches, inadequate user-friendly interfaces for non-expert users, insufficient integration of uncertainty and risk factors, and limited scope in addressing diverse economic, environmental, and social outcomes.

Case studies and lessons learned

The application of economic discounting models in real-world scenarios provides valuable insights into the strengths and limitations of various approaches.

Large-scale infrastructure projects provide a rich source of case studies in discounting practices. For instance, the Three Gorges Dam project in China, one of the largest hydroelectric power stations in the world, offers an instructive example. The project's economic evaluation, conducted in the 1990s, used a discount rate of 12% for financial analysis and 8% for economic analysis. These high rates reflected the high opportunity cost of capital in China at the time. However, critics argued that such rates undervalued long-term environmental and social impacts. This case highlights the challenges of balancing immediate economic considerations with long-term sustainability in infrastructure planning.

In the realm of healthcare economics, the evaluation of global vaccination programs provides valuable lessons in discounting practices. The Global Alliance for Vaccines and Immunization (GAVI) has grappled with the challenge of comparing costs and benefits occurring over different time horizons. In some analyses, GAVI has used differential discounting, applying a lower rate to health effects than to costs. This approach reflects the ethical consideration that future health gains should not be excessively devalued. The debates surrounding these practices have contributed to ongoing discussions about the appropriate discounting methodology for health interventions with long-term impacts.

The UK government's adoption of a declining discount rate schedule for long-term projects represents a significant development in public sector discounting practices. This approach, implemented in 2003 and updated in 2018, uses a 3.5% rate for the first 30 years, declining to 1% for periods beyond 300 years. This method has been applied to various long-term policy evaluations, including flood defence strategies and nuclear decommissioning plans. The UK's experience offers valuable lessons in implementing more nuanced discounting approaches in public policy and has influenced practices in other countries.

Comparative advantages of the proposed model

The proposed model offers some advantages over other existing approaches. Firstly, its comprehensive comparison of 20 distinct discounting methods, ranging from traditional exponential models to more nuanced ecological and hyperbolic approaches, provides unparalleled breadth in economic decision-making tools. This feature allows users to simultaneously evaluate the implications of various discounting philosophies, fostering a more holistic understanding of long-term project valuations.

The model's user-friendly interface stands out as a significant improvement over many existing economic tools. By translating complex economic concepts into intuitive visualisations and interactive elements, it democratises access to sophisticated discounting techniques. This accessibility is crucial for bridging the gap between theoretical economics and practical policy implementation, potentially leading to more informed decision-making across various sectors.

Advanced sensitivity analysis capabilities represent another pivotal advantage. The model allows users to instantly visualise how small changes in input parameters affect long-term valuations across different discounting methods. This feature is particularly valuable in contexts of high uncertainty, such as climate change mitigation or long-term infrastructure planning. By providing a clear picture of how various assumptions impact outcomes, the model enables more robust and defensible economic analyses - particularly for long-termist proposals, as suggested earlier.

The model's flexibility in incorporating new discounting theories as they emerge ensures its long-term relevance. This adaptability is crucial in a field where economic thinking is constantly evolving. For instance, the model could easily integrate new approaches to social discount rates or novel methods for valuing ecosystem services as they gain academic consensus.

Moreover, transparency in assumptions and calculations is a core strength of the proposed model. Unlike many "black box" economic tools, this model provides clear visibility into the underlying methodologies and data sources. For example, all the default value settings are made public to the user in the user interface input dialogue itself. This transparency not only builds trust in the model's outputs but also facilitates critical examination and refinement of economic assumptions.

Lastly, the potential for customisation across different sectors sets this model apart. While maintaining a consistent core framework, the model can be tailored to address sector-specific considerations, whether in healthcare economics, environmental policy, or corporate finance, since there are two outputs from the model with one of them being a table of discounting values, which can be further used for more industry-specific and complex analysis. This versatility makes it a valuable tool across a wide range of applications, from government policy-making to corporate strategic planning.

Appendix C: Mathematical foundations

Proofs of properties

Mathematical underpinnings of the model. Some proofs of properties – which explain the discounting methods' behaviour in the model – are shown below:

1. **Monotonicity of exponential discounting function**

Theorem: The exponential discounting function $D(t) = 1 / (1 + r)^t$ is monotonically decreasing for $r > 0$ and $t \geq 0$.

Proof: Let $D(t) = 1 / (1 + r)^t$, where $r > 0$ and $t \geq 0$. To prove monotonicity, we need to show that $D'(t) < 0$ for all $t \geq 0$.

$D'(t) = -ln(1 + r) * (1 + r)^{(-t)}$

Since $r > 0, ln(1 + r) > 0$ *and* $(1 + r)^{(-t)} > 0$ for all t. Therefore, $D'(t) < 0$ for all $t \geq 0$.

Hence, D(t) is monotonically decreasing.

This property ensures that future values always decrease over time, reflecting the time value of money. It guarantees consistent valuation of future cash flows, preventing illogical scenarios where later cash flows are valued more highly than earlier ones.

2. **Convexity of hyperbolic discounting function**

Theorem: The hyperbolic discounting function $D(t) = 1 / (1 + kt)$ is convex for $k > 0$ and $t \geq 0$.

Proof: Let $D(t) = 1 / (1 + kt)$, where $k > 0$ and $t \geq 0$. To prove convexity, we need to show that $D''(t) > 0$ for all $t \geq 0$.

$D'(t) = -k / (1 + kt)^2$ $D''(t) = 2k^2 / (1 + kt)^3$

Since $k > 0$ and $t \geq 0$, $(1 + kt)^3 > 0$ for all t. Therefore, $D''(t) > 0$ for al*l* $t \geq 0$.

Hence, D(t) is convex.

Convexity in hyperbolic discounting captures the human tendency to heavily discount the near future but less so the distant future. This property allows the model to reflect more realistic human behaviour in long-term decision-making.

3. **Monotonicity of quasi-hyperbolic discounting function**

Theorem: The quasi-hyperbolic discounting function $D(t) = \beta / (1 + r)^t$ for $t > 0$ and $D(0) = 1$ is monotonically decreasing for $0 < \beta < 1, r > 0,$ and $t \geq 0$.

Proof: For $t > 0, D(t) = \beta / (1 + r)^t$, where $0 < \beta < 1$ and $r > 0$. We need to show that $D(t+1) < D(t)$ for all $t \geq 0$.

For $t = 0$: $D(1) = \beta / (1 + r) < 1 = D(0)$, since $\beta < 1$ and $r > 0$.

For t > 0: $D(t+1) / D(t) = [\beta / (1 + r)^(t+1)] / [\beta / (1 + r)^t] = 1 / (1 + r) < 1$

Therefore, $D(t+1) < D(t)$ for all $t \geq 0$.

Hence, D(t) is monotonically decreasing.

This property combines immediate gratification bias with consistent long-term discounting. It allows the model to capture both present bias and rational long-term planning, providing a more nuanced approach to intertemporal choice.

4. Limit behaviour of ecological discounting function

Theorem: The limit of the ecological discounting function $D(t) = exp(-r * t * (1 - exp(-\varphi * t)))$ as t approaches infinity is 0 for $r > 0$ and $\varphi > 0$.

Proof: Let $D(t) = exp(-r * t * (1 - exp(-\varphi * t)))$, where $r > 0$ and $\varphi > 0$. We need to show that $lim(t\to\infty)\ D(t) = 0$.

$lim(t\to\infty)\ D(t) = lim(t\to\infty)\ exp(-r * t * (1 - exp(-\varphi * t))) = exp(lim(t\to\infty)\ [-r * t * (1 - exp(-\varphi * t))])$

Inside the exponential: $lim(t\to\infty)\ [-r * t * (1 - exp(-\varphi * t))] = -r * lim(t\to\infty)\ [t * (1 - exp(-\varphi * t))]$ $= -r * lim(t\to\infty)\ [t - t * exp(-\varphi * t)] = -r * [lim(t\to\infty)\ t - lim(t\to\infty)\ (t * exp(-\varphi * t))]$

The first limit diverges to infinity.

For the second limit: $lim(t\to\infty)\ (t * exp(-\varphi * t)) = lim(t\to\infty)\ (t / exp(\varphi * t))$

Using L'Hôpital's rule: = $lim(t\to\infty)\ (1 / (\varphi * exp(\varphi * t))) = 0$

Therefore, $lim(t\to\infty)\ [-r * t * (1 - exp(-\varphi * t))] = -\infty$

Hence, $lim(t\to\infty)\ D(t) = exp(-\infty) = 0$

This property ensures that very long-term environmental impacts approach zero value, but at a slower rate than standard discounting. It allows the model to balance long-term environmental concerns with economic considerations in a mathematically consistent manner.

Pseudocode

The current code and its capabilities are run on limited computing capacity. However, the following pseudocode can be used to generate more capable code in the foreseeable future:

```
IMPORT required libraries

// Define discounting functions

FUNCTION exponential_discounting(x, r)

FUNCTION hyperbolic_discounting(x, k)

FUNCTION quasi_hyperbolic_discounting(x, beta, r)

... [other discounting functions]


FUNCTION calculate_discounting_rates(x, params)

    Initialise empty dictionary 'rates'

    FOR EACH discounting method

        Calculate rate using corresponding function

        Add result to 'rates' dictionary

    RETURN rates


FUNCTION plot_rates(rates, x)

    Set up plot figure and axes

    FOR EACH model, rate in rates

        Plot rate vs x

    Add labels, title, legend, and grid

    Save plot as image file

    Display plot


FUNCTION get_user_input(prompt, default)

    Display prompt with default value

    IF user enters value
```

```
        RETURN user's value
    ELSE
        RETURN default value
FUNCTION main()
    Print instructions for parameter input
    Initialise 'params' dictionary
    FOR EACH parameter
        Get user input or use default value
        Store in 'params' dictionary
    Generate x values (time range)
    Calculate rates using calculate_discounting_rates()

    Create DataFrame from rates
    Display first 10 rows of DataFrame
 Save full DataFrame to CSV file
    Call plot_rates() to create and display graph
IF this is the main program
    Call main()
END PROGRAM
```