7.5 SCALAR & VECTOR PROJECTIONS

Recall that the amount of work done in moving a object from one point to another can be defined by $W = \vec{f} \cdot \vec{s}$ or $W = |\vec{f}||\vec{s}| cos\,\theta$. Also, work can simply be defined by $W = Fd$ where F is the magnitude of the force moving the object and d is the distance between the two points. However, this relation is only valid when the force acts in the direction the object moves. Thus, it would be helpful to determine the force vector and/or scalar in the direction of the object. We will use scalar and vector projection to help us determine this.

Scalar Projection of $\vec{a}$ onto $\vec{b}$ is ON, where $ON = |\vec{a}| cos\theta$

$0^o \leq \theta < 90^o$ $\quad$ $90^o < \theta \leq 180^o$ $\quad$ $\theta = 90^o$

Calculate the projection of $\vec{a}$ onto $\vec{b}$**:**

Calculate the projection of $\vec{b}$ onto $\vec{a}$

Calculating Scalar Projections:

The scalar projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$.

The scalar projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$.

In general $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \neq \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

Ex. 1 For the vectors $\vec{a} = (-3, 4, 5\sqrt{3})$ and $\vec{b} = (-2, 2, -1)$, calculate each of the following scalar projections:

a) $\vec{a}$ onto $\vec{b}$

b) $\vec{b}$ onto $\vec{a}$

Another observation to make about scalar projections is that the scalar projection of $\vec{a}$ on $\vec{b}$ is independent of the length of $\vec{b}$. E.g.

Note: we can modify the **scalar projection** to slightly to determine the **vector projection**. The vector projection of $\vec{a}$ on $\vec{b}$ is just the corresponding scalar projection of $\vec{a}$ on $\vec{b}$ multiplied by the unit vector $\vec{b}$ which is $\frac{\vec{b}}{|\vec{b}|}$ pointing in the direction **of** $\vec{b}$.

Vector Projection of $\vec{a}$ on $\vec{b}$
Vector Projection of $\vec{a}$ on $\vec{b}$=

Ex. 2 Find the scalar and vector projection of $\vec{OA} = (-3, -1)$ on $\vec{OB} = (4, -2)$. Include a diagram.

Scalar projections are sometimes used to calculate the angle that a position vector $\vec{OP}$ makes with each of the positive coordinate axes.

Ex. 3 Determine the angle that the vector $\vec{OP} = (2, 1, 4)$ makes with the x-axis. Include a diagram.

Ex. 4 For the vector $\vec{OP} = (2, 1, 4)$, determine the direction cosine and the corresponding angles that this vector makes with the positive y- and z-axes.

Direction Cosines for $\vec{OP} = (a, b, c)$

If α, β, and γ are the angles that $\vec{OP}$ makes with the positive x-axis, y-axis and z-axis, respectively, then

$$cos\ \alpha = \frac{(a,b,c)\cdot(1,0,0)}{|\vec{OP}|} = \frac{a}{\sqrt{a^2+b^2+}}$$

$$cos\ \beta = \frac{(a,b,c)\cdot(0,1,0)}{|\vec{OP}|} = \frac{b}{\sqrt{a^2+b^2+}}$$

$$cos\ \gamma = \frac{(a,b,c)\cdot(0,0,1)}{|\vec{OP}|} = \frac{c}{\sqrt{a^2+b^2+}}$$