

Assignments # 2

Math 622
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Question#1

The work done by the force of a body of mass m with initial speed $v_1 = 2 \times 10^4 \text{ m/s}$ / To final speed $v_2 = 4 \times 10^4 \text{ m/s}$ is $w = 1.2 \times 10^7 \text{ joule}$. Find the mass of the body in grams.

Sol:

$$v_1 = 2 \times 10^4 \text{ m/s}$$

$$v_2 = 4 \times 10^4 \text{ m/s}$$

$$w = 1.2 \times 10^7 \text{ joule}$$

find mass = ?

$$w = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$w = \frac{1}{2}(mv_2^2 - mv_1^2)$$

$$1.2 \times 10^7 = \frac{1}{2}(m(4 \times 10^4)^2 - m(2 \times 10^4)^2)$$

$$1.2 \times 10^7 = \frac{1}{2}m(4 \times 10^4)^2 - (2 \times 10^4)^2$$

$$1.2 \times 10^7 = \frac{1}{2}m(16 \times 10^8) - (4 \times 10^8)$$

$$1.2 \times 10^7 = \frac{1}{2}m(16 \times 10^8 - 4 \times 10^8)$$

$$1.2 \times 10^7 = \frac{1}{2}m(12 \times 10^8 \text{ m/s})$$

$$1.2 \times 10^7 = m(6 \times 10^8 \text{ m/s})$$

$$\frac{1.2 \times 10^7}{6 \times 10^8 \text{ m/s}^{-1}} = m$$

$$0.2 \times 10^{7-8} = m$$

$$0.2 \times 10^{-1} = m$$

$$0.02 \text{ kg} = m$$

$$0.02 \times 10^3 \text{ g} = m$$

$$0.02 \times 1000 \text{ g} = m$$

$$20 \text{ g} = m$$

Hence proved

Question#2

Check whether the force field $F = 2xyi + (x^2 - z^2)j - 2yzk$ is conservative or not.

Sol:

$$F = 2xyi + (x^2 + y^2)j - 2yzk$$

$$\nabla \times F = 0$$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & x^2 - z^2 & -2yz \end{vmatrix}$$

$$\nabla \times F = \frac{\partial}{\partial y}(-2yz) - \frac{\partial}{\partial z}(x^2 - z^2)i - \frac{\partial}{\partial x}(-2yz) - \frac{\partial}{\partial z}(2xy)j + \frac{\partial}{\partial x}(x^2 - z^2) - \frac{\partial}{\partial y}(2xy)k$$

$$\nabla \times F = (-2z + 2z)i - 0j + (2x - 2x)k$$

$$= 0i - 0j + 0k$$

$$= 0$$

the force field Is conservative.