The Centipede Game: Analysis and Implications

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Motivating the Centipede Game

Game theory's purpose is to make positive statements regarding strategic interactions between players. Due to human behaviour however, theory and practice often differ. Its creation by mathematicians such as John von Neumann, and later famously expanded upon by John Nash in the 1960s, has had an enormous influence primarily in economics but also in all fields that engage in strategic interaction. There have been many ex-post theories in economics, although a primary interest and fascination has been to predict ex-ante. Game theory has provided such a possibility. Economics and its game theoretical contributions assume rationality among individuals, and have been central to the predictions made. A longstanding criticism of economics has been concerning the assumption of absolute rationality among individuals, and with reason. Can we truly believe every individual to be fully rational? There are certainly cases of irrationality in our own behaviors and fears that we have realized and perhaps not cared to admit. For a study defined as being the "...study of human behaviour...between ends and scarce means…", there has been a minute effort within the field at incorporating the aspects of human behavior that differ from rationality (Robbins, 1932). The study of behavioral economics has attempted to humanize economic concepts to compensate for this. The necessity of this newer field of study can be exemplified by a game which has been known as the Centipede Game. The game has long been contested in its theoretical prediction as it has been repeatedly shown that human behavior and the logical solution do not coincide.

The uncertainty in others' preferences, beliefs, and reputation are all large factors in individuals' decisions. Reputation plays an integral role behind the decisions of individuals as it is indicative of people's preferences. Experimental research on participants has shown discordance among the theoretical prediction by backwards induction and these experimental results. Justifications for such discordance, by numerous authors, include explanations such as: the subjectivization of certain payoffs, the framing of decisions, the level of altruism, beliefs about human behaviour, lack of information, and various other behavioural economic theories. The direct economic implications of the centipede game have been hindered by the theoretical model's requirement of complete information and rationality. We argue that the centipede game provides its usefulness in isolating such deviations from rationality. Specifically, the centipede game can provide a foundational basis for individuals' behaviors in interactive environments involving money such as business decisions, customer-based interactions, networking and job-seeking, and the cultivation of business reputations.

The Centipede Game

In game theory, backward induction is used to evaluate subgame perfect Nash equilibria in sequential games of perfect information. It is a solution method that starts from the end of a game tree to determine the optimal sequence of events. This process continues backwards through the sequence of events until the best decision at every event is determined. The centipede game is a 2 player extensive-form game where each player takes turns deciding to take the larger portion of a repeatedly increasing payoff sum or to pass and let the other player decide on the increasing payoff sum. Once an active player takes the payout offered, the game is terminated, with the active player receiving the larger of the two payoffs. If a player decides to pass, it strictly decreases their payoff. If the other player then too passes, they are faced again with an equal set of decisions as before, but with an increased payoff opportunity (Rabinowicz, 1998). The game has a finite number of turns in which all players are aware of all information; they are aware of the total number of rounds, the payoff at each node, and their choice matrices.

History and Economic Relevance

While backward induction has its roots in chess, its relevance to economics was proposed by Selten (1978) when he applied it to chain store pricing in the event of new competitors. From Colman (2003), the Chain-store game is described as such: “A chain-store has branches in 20 cities, in each of which there is a local competitor hoping to sell the same goods. These potential challengers decide one by one whether to enter the market in their home cities. Whenever one of them enters the market, the chain-store responds either with aggressive predatory pricing, causing both stores to lose money, or cooperatively, sharing the profits 50–50 with the challenger.” One may assume that the chain store has incentive to price predatorily to drive them out of the market and thus deter future entrants. Backward induction however leads to an opposite conclusion. Beginning in the final subgame, since the chain has no more entrants to deter it will cooperate. Thus the twentieth challenger will choose to enter. Since the final subgame is determined, the penultimate subgame is determined, as well as the subgame before that and before that, and the resemblance to the centipede game becomes clear. Knowing that the chain-store's best response is to cooperate, each challenger should respond by entering. By induction, we see that the chain-store will always cooperate and the challenger always enters. Selten also describes the results of a strategy of deterrence however and states that such a strategy would often lead to greater payoffs. Such a strategy's exactitudes are left up to the player's discretion on when in the game they believe deterrence to lose efficacy. While the logic of induction is valid, Selten himself states that he would still adopt the deterrence strategy and "met nobody who said that he would behave according to [backward] induction theory." (Selten, 1978). The logical proof of backward induction and the fruitful results of deterrence therefore induce a paradox. Selten provides a number of theories and explanatory strategies on the discrepancy between logical theory, and actual results in his preliminary paper on this game. The centipede game was formalized by Rosenthal (1981) soon after, and has endured as a subject of discussion in game theory and behavioural economics.

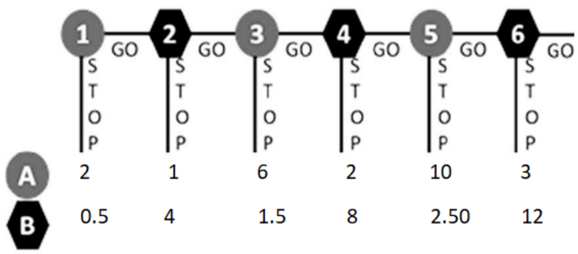
Experimental Research

In the 1992 study "An Experimental Study of The Centipede Game" conducted by McKelvey and Palfrey, three versions of the Centipede Game were experimented using 20 different subjects for each of the experiments. The researchers used different variations based off the version of the game brought forward by Aumann in 1988. In Aumann's version of the game, there are two pools of money where each player alternates between choosing to take either the larger of the two pools or to pass. When a player chooses to pass, both payoffs become increasingly larger with each pass (McKelvey, Palfrey, 1992). There are 6 nodes in total. The researchers of the 1992 study acknowledged that they do not follow the payoff structure presented by Aumann, primarily due to budgetary constraints. The three versions consisted of a four node game with payoffs of $0.40 and $0.10 to the first and second player respectively, where the payoffs increased by a multiple of two; a six node game which incorporated the same payoffs and simply extended these payoffs for another two nodes; and finally a higher-payoff version where each payoff at each node of the four nodes was multiplied by four. There were twenty different subjects for each version of the game, and they were split into two groups that distinguished between players who went first and second. Subjects knew they were never matched with each other more than once. The findings of the study were that most players did not choose to take in the primary nodes. In the games where players had advanced to the final node, 25 % of them passed in the 4-node game, 15 % in the six-node game, and 31 % in the high payoff game. This finding shows significant discordance with the theoretical Nash Equilibrium solved by backwards induction. Such findings seem to imply the existence of altruistic players who choose to pass at each node, and players who mimic an altruistic player in order to receive a higher pay off by choosing to take at later nodes (Nagel, Tang, 1998). This can be further corroborated by the finding that as the participants got closer to the final node, the probability of taking increased substantially. As repetitions of the games increased, players chose to take rather than pass much earlier and with greater frequency, implying a learned behavior to the equilibrium. This finding coupled with the fact that participants chose to take more often in the higher payoff game, has also been reported by Parco, Rapoport and Stein (2002) with a single-play, 3 player version of the centipede game. McKelvey and Palfrey reasoned that their studies' results are clearer if the games are considered as ones with incomplete information rather than complete information. The uncertainty exists at the beginning of the experiments where players do not know whether their opponent is an altruist or not: "If subjects believe there is some small likelihood that the other player is an altruist, players adopt mixed strategies in early rounds of this experiment with the probability of taking as the pile gets larger." (McKelvey, Palfrey, 1992). The researchers also noted that in general there did not seem to be a single pure-strategy employed by the players, as there tended to be variations in their choices in different rounds of the games played.

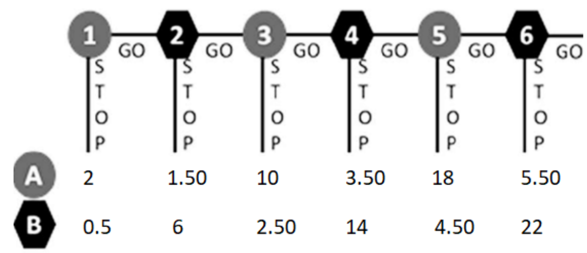
Lab Specifications and Data

In our lab, there were two separate treatments of the centipede game. Both treatments were a modified form of the the Take-it-or-Leave-it game in which only the player who takes the exit-node receives a payoff. The lab's modification to this game is that the player who has not taken the exit-node is still compensated with a payoff that is smaller than that of the "exiter". Each player is randomly assigned as Player A or Player B, and alternate being the active and waiting players, A being the initial active player. The game involves a terminal node in the 6th round. The first treatment of the game incorporated a linear growth of the payoffs as players advanced. The second treatment was identical except the linear growth rate of the payoffs was increased. That is, for all nodes past the first, payoffs were increased in the second treatment, and the difference of payoffs in each node between the first and second treatment of the game increased as the game continued. Diagrams for each treatment are shown below.

First Treatment



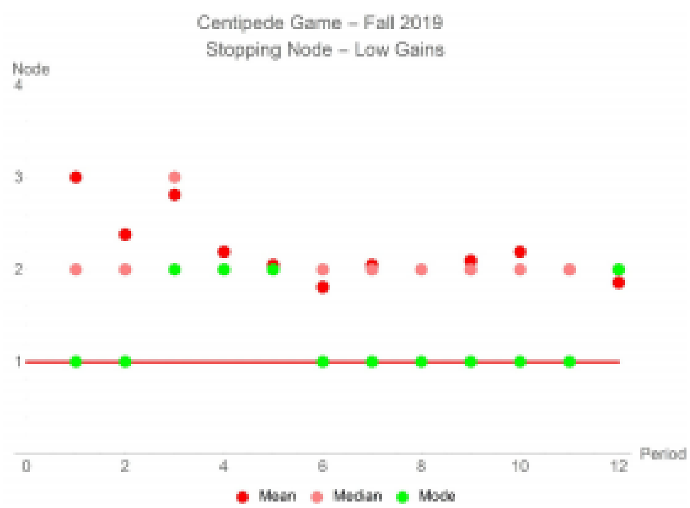
Second Treatment

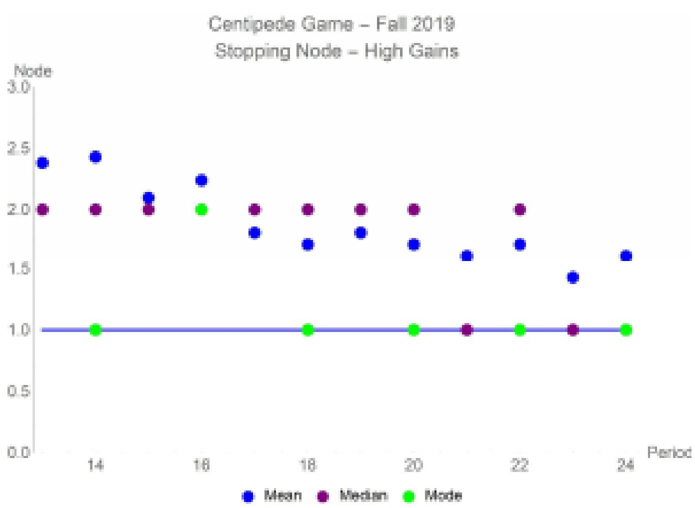


The first treatment has player A deciding to either stop at the first node of (A, B) = (2, 0.50) or to continue the game and allow player B to either continue or stop at node two of (A, B) = (1, 4). This alternating sequence continues at an increase of 2 for the active player and 0.50 for the waiting player for a total of 6 rounds terminating in a node of (A, B) = (3, 12). The second treatment has player A deciding to either stop at the first node of (A, B) = (2, 0.50) or to continue the game and allow player B to either continue or stop at node two of (A, B) = (1.50, 6). This alternating sequence continues at an increase of 4 for the active player and 1 for the waiting player for a total of 6 rounds terminating in a node of (A, B) = (5.50, 22).

Lab Discussion

In experimental data outside of our lab, neutral, cooperative and competitive renditions of the centipede game all saw an average exit-node that did not converge to the theoretical Nash equilibrium (Pulford, Krockov, Colman, Lawrence, 2016).





This is a common finding in many experiments which evaluate how backwards induction compares with the public's behaviour (Pulford et al, 2016). In the first treatment of the game, our data resembled past experiments in its substantial deviation from the backward induction equilibrium.

In our second treatment however, we converged to an exit-node much closer to this equilibrium. The difference between the two games is the doubling of the payoff growth rate. The result of the second treatment in our lab was an earlier exit node with less variance over repeated trials.

Due to the increased rate of payoff growth, cooperation now yields greater payoffs but the stakes are also greater. The effects of this change delve into psychological and social theory as well as behavioural economics. Gossen's first law (1854) states that marginal utility is diminishing. Applying this to our lab, we may assume that players value later stage payoffs as less than their dollar equivalence and are thus satisfied with an earlier exit-node. Another explanation for our results is described by Richard Thaler (1985) as “Mental Accounting”. In his paper titled “Mental Accounting and Consumer Choice”, he states that humans value money (or payoffs), in a relative rather than absolute manner. Specifically, the categorization of money has an effect on people’s treatment of such money. In the second treatment of the game, as payoff growth has increased, we may expect cooperation to be equal or greater in response so that players reap the increased payoffs. It is possible that players in our game are subject to mental accounting and valuing payoffs based on their categorization of the results of the game. For example, in the first treatment the payoff of a 4th node exit for player A is 2 and for player B is 8, and in the second treatment is 3.5 and 14 respectively. Player A should have more motivation to continue to the fourth node of the game in the second treatment than the first as the payoffs are greater, yet this is not the case. Mental accounting reasons that this may be due to a different categorization of the payoffs. In the first treatment, when player A earns a payoff of 2 compared to player B’s 8, they see this as a “loss” of the game. The value of their payoff is therefore diminished due to their interpretation of it being money earned from a loss as opposed to money earned from a win. Conversely, player A may value their payoff of 8 as greater than 8 due to its positive connotations. In the second treatment, when player A earns a payoff of 3.5 compared to player B’s 14, player A may see this as an even greater loss and the value of 3.5 may be diminished further, possibly even below a subjective value of 2. The perception of losing by a greater margin may lead player A to avoid loss further than in the first treatment. This could be considered an issue of framing which is a cognitive bias based on positive or negative connotations associated with outcomes. In our experiment, the perceived severity of the wins and losses may have affected the players’ behaviour. Results indicating increased payoffs leading to a convergence towards the theoretical equilibrium were corroborated by other studies such as “Effects of Financial Incentives on the Breakdown of Mutual Trust” (Parco, Rapaport, Stein, 2002). Prospect theory, proposed by Kahneman and Tversky (1979), describes people’s aversion to risk. They assert that people overvalue options that are certain and undervalue those involving risk. Since all games of strategic interaction involve uncertainties, players will be faced with risks. In the centipede game, players may take a set payoff with certainty. Prospect theory suggests that certainty is most attractive when avoiding a loss, such as taking the exit-node and certain payoff. Their model also assumes that the greater the loss, the greater an agent’s aversion to it. As payoffs increase, the perceived loss of each node foregone increases as well. While the certain payoff plays a role, prospect theory would argue that the certain aversion to loss plays an even greater role in individuals’ behaviour. The increased payoffs in our experiment seemingly led to the logically paradoxical behavior of exiting earlier despite greater payoffs for cooperation. When accounting for various behavioural economic theories such as mental accounting, framing, and prospect theory, we see the dissonance between human reasoning and economic rationality; the latter often failing in actuality.

Economic Implications: Reputation, Networking and Business Strategy

The results seen in the literature of the centipede game have shown that there exists altruism in game play, as well as individuals who do not follow conventional rationality by choosing to not stop at the initial node. This has differed from the less optimistic theoretical equilibrium, where it is predicted that the initial player will end the game by stopping immediately. This implies that there can be mutual gains realized among individuals in certain competitive environments, even when the motives or beliefs of the players are not fully known. The literature also shows however, that with increased monetary pressure, people show an increased tendency to preserve ones-self rather than maintain the previously held co-operative demeanor. The real-world implications resulting from this observed behavior concern interactions between individuals that involve money, or trade. The findings of these studies demonstrate the necessity of reputation in high-stakes business environments for the occurrence of beneficial interactions. This has been known long before the observation of subject's behaviors during the centipede game. For instance, in China, there has been a term for such a reputation: "Guanxi". Guanxi is a term used to define "the system of networks and connections which facilitate business" (Oxford Dictionary, 2019). An individual with an abundance of this unit of measure of one's trusts and connections, will be able to prosper in interactions conducted in Chinese business. This individual's counterpart who lacks in Guanxi, may find it much more difficult to flourish in such instances. The behavior of subjects in the studies of the game shows us that it is necessary to know whether the other player is an altruist or not. The term "Shadow Banking" describes the creation of credit by non-bank financial intermediaries which are thus not subject to any regulatory oversight (Luo, 2018). This industry is one where there exists interaction among individuals with money, and where there is a lack of complete information. In the recent decline in China's shadow lending sector, many investors have been unable to retrieve their investments. These losses have been partially accredited to business owners running off with investors' money, which is an action comparable to players choosing to take with higher frequency when larger payoffs are on the table when paired with different players (Tech Crunch, 2019).

In the Aumann model of the game, it has been repeated that mutual gains can be realized when not choosing to play the theoretical equilibrium as one's strategy. The literature examined places emphasis on altruistic players, and implies that in a sense, they are the losers of the interaction. This of course, can be argued as being contextual. In the context of the centipede game which has very narrowly defined rules, conventions and explicit payoffs, being altruistic may be a less than optimal strategy if one wants to maximize their payoffs in the game. However, in the context of business, and specifically customer-oriented business, employing the altruist strategy may be the simplest way to optimize these interactions for a business. In an interaction with a player who would be the customer, it will prove to be fruitful for the business to be the altruist so long as the game proceeds longer than the first node. With increasing payoffs as the nodes go on, it should not matter whether employing the altruist strategy garners a win for the business, since they already have larger payoffs than when starting. The data on human behavior while playing the Centipede Game illustrates this point, the majority of games played did not end in the first node; hence allowing altruists to gain. The Centipede Game research provides foundational evidence behind employing this strategy as the game consists of a lack of complete information: altruists still achieved higher payoffs even without a reputation of being an altruist, as well as players never playing each other more than once which isolates the repeat customer phenomenon seen in business environments. Altruists can almost always receive higher payoffs than offered at the initial node, and business owners can choose to employ this strategy in customer-based interactions as it proves to be a simple formula which can guarantee success even in a brutishly competitive and unadorned model such as the centipede game.

The centipede game has also been given the context of two players being business associates writing references for each other. Empirically, it is seen that cooperation is likely among the two players (Pulford, Krockow, Colman, Lawrence, 2016). This seems fitting as both parties will benefit from cooperation by receiving a better job than they had before. In Mark Granovetter's study: "Getting a Job" (1995), which explored the link between connections and careers, a startling finding was that acquaintances or "weak ties" provided more value in acquiring employment than closer relationships or "strong ties". Of the new jobs acquired from the participants, 56% of the respondents learned about these jobs from acquaintances, and only 17% from a close tie. This was due to acquaintances leading drastically different lives than the individual and their respective strong ties, thus extending more diverse employment opportunities . What becomes interesting from a game theorists' perspective is the level of co-operation among these acquaintances. Weak ties may allow for greater employment opportunities; however, an acquaintance may feel a lesser obligation to provide the individual access to these networks in comparison to strong ties. Such interactions present themselves as being similar to the centipede game described as a "scratch my back and I'll scratch yours" interaction (Pulford et al, 2016). However, since they are less obliged to each other, their preferences should more closely resemble that of a rational agent, seeking to maximize their own payoff. From this perspective, rationally interacting with one another entails that one would take advantage of an opportunity, which would then leave the other acquaintance left to their own devices. Of course, when applying the concept to these interactions, there is a dissonance between not only the experimental data, but in these real-world experiences where 56 % of respondents acquired a job through a weak tie. Co-operation again appears to be a strong determinant in such game play, and yet again thwarts the subgame perfect nash equilibrium prediction. One reason may be that interactions between individuals tend to be more cooperative than interactions between groups, “and this is usually attributed to greater fear and greed in intergroup relative to interindividual interactions” (Wildschut, Pinter, Vevea, Insko, & Schopler, 2003) . This indicates that when searching for a job through an acquaintance, individuals recognize that gaining access to such a resource is much more tit for tat than competing for access to a resource. Other-regarding preferences come into play once again: “Numerous studies across several ideas of research in experimental games have indicated…human decision makers…appear also to be motivated by other-regarding preferences.” In the case of network enhancing interactions, the individuals involved must know that in order to gain from the interaction, it would be beneficial to keep the other person in mind. In other words, achieving a “win, win” situation would be optimal. One could of course manipulate the other in order to acquire a job or a network, but would do so at the detriment of attaining future access to the individual, their network, and any possibly enhanced employment opportunities that may accompany. As is evident, playing the S.P.N.E limits a person's potential career progression (larger payoffs in the centipede game), while not bringing anything to the table for the other player would also achieve a similar result. Since it is in every individual’s best interest to provide value for the other player, that they should be there to cooperate, and although this real-world application of the game remains one of incomplete information, knowing the above two facts makes it so that a large probability can be assigned to cooperation in the game.

Conclusion

The centipede game has provided a lack of harmony between theory and real model behavior, which acts as a necessary reality check for the study which regards rationality as the be all and end all for human interactions regarding scarce resources. The backwards induction solution is rarely seen in experimental data conducted among participants outside of and including our class. This has been explained by altruism existing in game play, and players thus mimicking altruist game play in order to achieve more desirable payoffs than the Subgame Perfect solution can offer. This result, however, has been shown in both experimental data from the real world as well as our class to converge to a more backwards inductive type solution when higher payoffs are concerned. We have explained our class' differences as being due to Mental Accounting and the framing fallacies that Prospect Theory has presented. We have shown that in real world environments, complete information is all but impossible and therefore interactions which are comparable to centipede games such as networking for a prospective job, maintaining a strong business reputation and trying to prosper in customer-based transactions as a business owner all rely on humanizing to compensate for these incomplete information environments. For a business owner, the altruist algorithm proves to be a relatively simple and profitable. A reputation for good business and strong connections alleviates uncertainty and fosters profitable interactions, as well as networking in a co-operative manner with another individual to both boost each own's prospective employment opportunities. The former has been demonstrated by a cultural phenomenon in China, which has articulated this phenomenon into the term “Guanxi”. What occurs in environments of incomplete information without this reputation to compensate can be exemplified in the recent decline in the country’s shadow lending industry, where cases of investor money being run off with are rampant. The latter has been found in a study concerning individuals’ acquisitions of jobs, where 56 percent of respondents reported having found their employment through a weaker alliance, showing that cooperation is prevalent even among less obligatory connections. Game theory has provided an excitement in its promise to predict that has long since faded. It has been said that “the test of science is its ability to predict”(Richard P. Feynman). The crisis of the study has been in its inability to predict; will economics continue its unyielding commitment to rationality that will result in its own undoing, or will human behaviour be incorporated into models so that the study can meet “the test of science”, and as a result: the test of time.

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