5 Logic & Inference

PURPOSE & HIGHLIGHTS

This activity is designed to **apply logic** to solve a puzzle delivered in "word problem" form; and **analyze logical fallacies**.

Highlights: Many students greatly enjoy solving the "Who is the Thief?" puzzle. Examining mistakes by (fictional) ChatGPT responses underlines the importance of valid arguments.

LESSON OVERVIEW

Plan for the lesson

Introduction and Teamwork Preparation (10 min): Refresh familiarity with the rules of inference. Assign groups and roles.

Core Activity: Looking for the Thief (25-30 min). With the theme of statements about finding the thief, students work on shared surfaces (e.g., standing at whiteboards) to: (1) Translate statements into symbolic form; (2) Arrange the statements in logically valid sequences; (3) Draw a conclusion. In the final 10 minutes, selected team Reporters share solutions.

Extension Activity: ChatGPT Fallacies (15 min). Students examine computer generated logical arguments as answers to the core activity. They turn the ChatGPT narratives into symbolic forms, identify potential errors, and then answer: Which steps were justified? Which ones were not? Teams share their answers with the class.

Formative Activity (10-15 min): Students discuss the pros and cons of using symbolic logic and/or natural language to construct and validate arguments.

Wrap-up (5 min):
Conclude with student survey or further reflection.

Class Time Estimate

- Core + Formative Activities: 50 min
- Core + Extension + Formative Activities: 70 to 75 min

Key Ideas for Instructors

Core Activity has some built-in logical blind alleys/dead ends. It's OK if student answers are not maximally efficient, as finding the Thief efficiently is NOT a goal. Getting there *eventually* is a goal.

Student Prerequisites

The activity assumes the students have seen some variation of *rules of inference* or *valid argument forms* (see page 3 of the instructor roadmap for an example of a table). Works well after covering truth tables and the contrapositive.

LOGIC RESOURCES

- Instructor Roadmap
- Student Handout
- Team Roles Handout

User Guide

Logic & Inference

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Logic & Inference

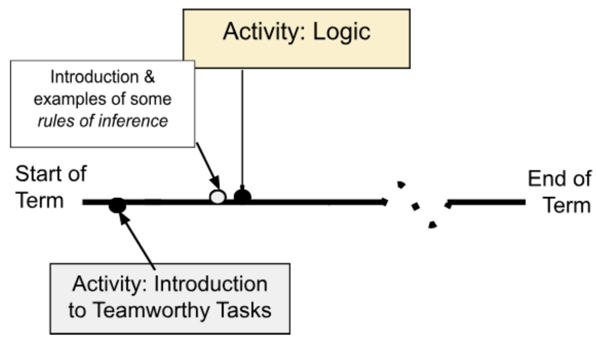
Table Of Contents

Purpose

This activity is designed to have students

- **build** upon their skills at working in teams;
- **apply** logic to solve a puzzle delivered in "word problem" form;
- **analyze** logical fallacies by examining fallacious arguments.

Lesson Fit and Student Prerequisites



The activity focuses on syllogistic reasoning.
It assumes students already:

- have had some ***practice with truth tables***, including ***identification of tautologies***.
- are familiar with basic symbolic representations and their meanings*:*

  ***P → Q*:** If *P* is true, then *Q* is true
  ***P ^ Q*:** Both P and Q are true
  ***P* v *Q*:** At least one of P or Q is true
  ~*P*: not *P* is true
  ∴ (triangle of dots) means *therefore*

- have seen and used some variation of ***Valid Argument* Forms** or ***Rules of Inference***:

Rule	Assumption(s)	Conclusion
Modus Ponens	$P \to Q$ and P are both true	Q is true
Modus Tollens	$P \to Q$ and $\sim Q$ are both true	$\sim P$ is true
Elimination	$P \lor Q$ and $\sim Q$ are both true	P is true
Transitivity	$P \to Q$ and $Q \to R$ are both true	$P \to R$ is true
Cases	$P \lor Q$ and $P \to R$ and $Q \to R$ are all true	R is true
Contradiction	$\sim P$ being true implies a contradiction	P is true

Instructor Preparation

- Calculate the number of groups based on class size. We estimate groups of about 4 people, given that each team gets a set of 4 clue statements.
- Prepare paper handouts of the Student Activity and of the Clue Strips with one set for each group.
- Make enough copies of the four slips of clue statements so that each team gets one of the sets of clues (see last page of this document).
  Make sure to prepare paper copies of the clue strip sets before class.

Lesson Overview

Set-up [before class]	**Set-up:** Move desks away from boards to make space for students to walk around. Post static cling whiteboards as needed to increase workspace. **Students work in groups of 4. Each group gets all four clue strips (each person has a different strip). *Suggestion:* Make copies of clues page (page 11) and cut out strips to distribution.**
Launch ~10 min	1. Remind students of symbols and rules for inference. 2. **Announce "Roles Rules" for teams** (Team Roles handout) 3. Students count off or otherwise form teams of 4 (3 if needed). If there are 3 students on a team, ask the team to share the information on the extra clue strip. 4. Have teams move to their shared board space. Remind teams to record role assignments on their boards AND the handout (**you may find it useful to collect one handout per team at the end of class**). 5. For each team, distribute the Student Activity Handout and 1 set of clues **on paper:** have *Facilitators get handouts, pens and **clue strips** from you*. 6. In teams, students do the introduction activity.
Activity 1 Searching for the Thief ~25 min	[~15 minutes] Students work in teams of 4 to translate clue statements into symbolic logic and draw a conclusion using logical rules. [~10 minutes] Reporters from selected groups share the group's thinking and solution(s) with the whole class. **If your class is 50 minutes long, end with the Wrap-up after Activity 1.**
Activity 2 Finding Fallacies ~20 min	[~10 minutes] Teams analyze invalid ChatGTP-generated arguments. Sam ChatGPT, Ashley ChatGPT, and Phuong (foo-aang) ChatGPT. *Suggestion: Assign different arguments to different teams (e.g., Teams 1 and 4 get Sam, Teams 2 and 5 get Ashley, Teams 3 and 6 get Phuong).* [~10 minutes] Share-out by team Reporters.
Wrap-up ~15 min	[~10 minutes] Students reflect on the activity and discuss in their teams as a formative task. Follow up for this or for uncompleted work could be assigned as homework, possibly to be discussed in the next class. [~5 minutes] Students complete the anonymous feedback survey.

Detailed Timetable for the Lesson (~70 minutes)

This lesson contains two main activities. The first is the core of the lesson. Ideally, you will also do the second activity. In either case, use a formative assessment activity (an alternative formative assessment activity is included right after Activity 1 if you use only Activity 1).

Introduction (5 minutes)

The goal for the introduction is to have the students refresh their familiarity with the rules of inference and get some idea of the sense-making value of the rules. It is also used to plant some techniques and nudges for when students jump into the activities.

Example to recall the rules of inference and the justification format

Q. Suppose that the following are true:

If I am a flower, then I am a plant.

If I am a plant, then I am alive.

Can we conclude that if I am a flower, then I am alive? If so, by what rule of inference?

A. There are three declarative statements implicit in the question:

P = “I am a flower”
Q = “I am a plant”
R = “I am alive”

Valid Argument Form:

Step	Justification/rule of inference
1) $P \rightarrow Q$	Given
2) $Q \rightarrow R$	Given
3) $P \rightarrow R$	By Transitivity using 1) and 2).

Comment that can be used to motivate the activity

It turns out that when we reason about things in real life we are using logic, so in some sense we are already logicians! Getting practice with “real life logic” in terms of formal “math logic” (and vice versa) gives practice with how logic can be represented in different situations. In the working world, we will come to teamwork with different sets of knowledge. Some will be similar to our colleagues and some different. This is represented by the slips with different true statements that your team will communicate about and use to do this activity.

Team Preparation (5 minutes)

Split the students into **groups of 4** students. Remind students of Team Roles and provide the Roles Rules you want them to use. Try to have as many groups of 4 as possible. See the **User Guide** for details on forming and managing student teams.

Activity 1 (Core Activity): Looking for the Thief (~25 minutes, including share-out)

After students break out into groups:

- Share the Student Activity Handout (preferably on paper, or in LMS/Canvas, or slides).
- Distribute the clue strips of paper to the Facilitator: one person on each team gets (1), another person gets (2), a third person gets (3), and fourth person (4).
- Let students know that Activity 1 calls for communicating about and representing the premise statements on the team’s part of the board, documenting the argument symbolically, translating it into English and they have about 15 minutes.
- The instructor goes from group to group to observe and nudge them to complete their task.
- After **15 minutes** (or better choice of stopping time based on observations of student progress) call the students back for discussion.

***Italics* are text from the Student Activity Handout. The plain text is for you, as an instructor.**

Problem Situation:

Context*: You are an agent of L.O.G.I.C., a crime fighting organization based in Texas. Carolina Ramos is the hottest Texan pop music star and stealing her crystal necklace is a goal of the criminal group, E.V.I.L. A fellow L.O.G.I.C. agent has confirmed that a super thief is going to show up at Ramos’ 22nd birthday party to steal her necklace (but you do not know who the thief is). You and your fellow agents have all obtained true (but incomplete) information, and your goal is to put your minds together to find the identity of the thief.*

NOTE: MAKE COPIES OF THE LAST PAGE OF THIS DOCUMENT AND THEN CUT TO CREATE CLUE STRIPS. Students work in groups of 4. Each group gets all four strips 1, 2, 3, and 4 so each team member has a different strip.

Each of the four slips of paper contains a subset of the following six true statements. Thus, each team member comes to the activity with SOME of the information needed to complete it, but team members must communicate with each other to gather all the information needed to understand and solve the problem (as in real life!):
Givens:

(a) If the Ramos house is wiretapped, then Rafael Evergarden is not the thief.
(b) If E.V.I.L. agents are in the house, then the thief must be Rafael Evergarden.
(c) The Ramos house is wiretapped.
(d) The E.V.I.L. agents are in the house or Carmen Wong is the thief.
(e) If Kit Luna is the thief, then E.V.I.L. agents will not be in the house.
(f) If the Ramos house is not wiretapped, then Kit Luna is not the thief.

Clue Strips: The four key statements are (a), (b), (c), and (d). The following explains which of the key statements is missing from each slip.

Strip (1) – (b) is missing: If E.V.I.L. agents are in the house, then the thief must be Rafael Evergarden.

Strip (2) – (a) is missing: If the Ramos house is wiretapped, then Rafael Evergarden is not the thief.

Strip (3) – (c) is missing: The Ramos house is wiretapped.

Strip (4) – (d) is missing: The E.V.I.L. agents are in the house or Carmen Wong is the thief.

Note that some students may try to use (e) and (f) in their explanations, which is harmless but not necessary.

Translate into Mathematical Logic form (5 min)

Step 1. *Work with your team to translate the statements into symbolic form. Agree on letters to replace phrases. Write down what each letter represents.*

Example Translations:

(a) W → (not R), W = “the Ramos house is wiretapped”,
R = “Rafael Evergarden is the thief”

(b) E → R, E = “E.V.I.L. agents are in the house”

(c) W

(d) E or C, C = “Carmen Wong is the thief”

(e) K → (not E), K = “Kit Luna is the thief”

(f) (not W) → (not K)

Valid Argument Form (5 min)

Step 2. *Find the thief! Discuss and decide how to use the rules of inference in the Table of Valid Argument Forms (previous page) to draw a conclusion by organizing the statements in Step 1.*

Example Solution: Valid Argument Form

Argument	Justification/rule of inference
(1) W is true	Given [statement (c)]
(2) not R	Since W is true and, by Modus Ponens and (a), "W → not R"
(3) not E	Since (not R) by (2) and, by Modus Tollens, using (b) "E → R"
(4) C	Since (not E) by (3) and, by Elimination, using (d) "E or C"

Connect symbolic solution to statements in English (5 min)

Step 3. *Translate your argument in Step 2 into English, without math symbols. Write it clearly so others can read it and be able to understand your reasoning.*

Example Solution (using the Translations previously shown in Step 1):

Statement (c) says the Ramos house is wiretapped, so by Modus Ponens and Statement (a) [which says that If the Ramos house is wiretapped, then Rafael Evergarden is not the thief], Rafael Evergarden is not the thief.

This means E.V.I.L. agents are not in the house by Modus Tollens and (b), [which says, If E.V.I.L. agents are in the house, then the thief must be Rafael Evergarden.]).

Therefore, Carmen Wong must be the thief by Elimination and (d) [which says that the E.V.I.L. agents are in the house or Carmen Wong is the thief.], and the fact that E.V.I.L. agents are not in the house.

Discussion/Presentation (10 min)

Choose either a class discussion or student presentations.

- The aim of discussions / presentations is to cover:
  - Ways to translate and represent the **statements** using symbols
  - Ways to represent the **argument** using symbols
  - Ways to translate the symbolic argument into **human-readable** form
  - Additional observations.
- For student presentations, have Reporters from randomly selected groups **describe** their answers and give **one highlight** from their thinking processes. See the **User Guide** for tips on orchestrating whole class discussion.
- Additional talking points:
  - The symbolic forms are a useful set of tools to be able to use to represent ideas when things get complicated.
  - Rules of inference (Valid Argument Forms) are useful because valid reasoning is represented concisely.
  - Connect to truth tables, if taught previously, or foreshadow the connections if doing truth tables in the future.
  - Why two clues were not useful. What could have happened if you used them?

Formative Assessment after Activity 1 (Use only if class ends here) (10 min)

NOTE: To be used to conclude class if it appears that there won't be enough time for Activity 2. Each team reflects and some teams share out about the following:

- What are the pros and cons of using symbolic logic versus using everyday language narrative in validating or finding errors in a conclusion based on a series of arguments?

Activity 2: Finding Fallacies (~20 minutes)

Start Activity 2 by assigning each of the three arguments in the handout to at least one team (Sam ChatGPT, Ashley ChatGPT, Phuong (foo-aang) ChatGPT). For example, if there are 6 groups, Teams 1 and 4 get Sam, Teams 2 and 5 get Ashley, Teams 3 and 6 get Phuong. If a team finishes their assigned argument early, they can be encouraged to work on another argument.

In this activity, your goal is to identify some common errors made in logical arguments. Such errors are called logical fallacies. *Here are some answers that other students, Sam ChatGPT, Ashley ChatGPT, and Phuong ChatGPT, gave for Activity 1. Turn their narratives back into argument form. Which steps were justified? Which ones were not?*

Sam's argument:

(1) Given: if the Ramos house is not wiretapped, then Kit Luna is not the thief.
(2) (1) is the same as [if the Ramos house is wiretapped, then Kit Luna is the thief].
(3) Given: the Ramos house is wiretapped.
(4) (2) and (3) together tell us that Kit Luna is the thief.

Answer for Sam, assuming the notation:

W = "the Ramos house is wiretapped"
R = "Rafael Evergarden is the thief."
E = "E.V.I.L. agents are in the house"
C = "Carmen Wong is the thief"
K = "Kit Luna is the thief"

Argument	Justification/rule of inference
(not W) → (not K)	Given
W → K	INVERSE ERROR
W is true	Given
K is true	(2) and (3) and Modus Ponens

Ashley's argument:

(1) We are given 'The Ramos house is wiretapped.
(2) From (1) and the given 'if the Ramos house is wiretapped, then Rafael Evergarden is not the thief' we know (2) Rafael Evergarden is not the thief.
(3) From (2) and the contrapositive of the given: ' If E.V.I.L. agents are in the house, then the thief must be Rafael Evergarden' we know (3) The E.V.I.L. agents are in the house.
(4) From (3) and the converse of 'If Kit Luna is the thief, then E.V.I.L. agents will not be in the house.' We know (4) Kit Luna is the thief.

Answer for Ashley, assuming the notation:

W = "the Ramos house is wiretapped"
R = "Rafael Evergarden is the thief
E = "E.V.I.L. agents are in the house"

C = “Carmen Wong is the thief”
K = “Kit Luna is the thief”

Argument	Justification/rule of inference
(1) W	Given
(2) Not R	By Step 1 and a Given (W → Not R)
(3) Not E	By Step 2 and by the contrapositive of the Given E → R; Contrapositive is Not R → Not E
(4) K	By Step 3 and by the converse of the Given K → Not E; Converse is Not E → K (CONVERSE ERROR)

Phuong’s argument:

(1) A given statement tells us that if E.V.I.L. agents are in the house, then the thief must be Rafael Evergarden.
(2) By Elimination, the given statement “The E.V.I.L. agents are in the house or Carmen Wong is the thief.” is the same thing as saying that if the thief is not Carmen Wong, then the E.V.I.L. agents are in the house.
(3) By (1) and (2), if the thief is not Carmen Wong, then it must be Rafael Evergarden.
(4) This means that Rafael Evergarden is the thief.

Answer for Phuong, assuming the notation:

W = “the Ramos house is wiretapped”
R = “Rafael Evergarden is the thief
E = “E.V.I.L. agents are in the house”
C = “Carmen Wong is the thief”
K = “Kit Luna is the thief”

Argument	Justification/rule of inference
(1) E → R	Given
(2) (Not C) → E	Elimination, using Given “E or C” (ERROR in use of Elimination)
(3) (Not C) → R	Transitivity/Hypothetical Syllogism
(4) R	Does not follow from anything. (POSSIBLE ERROR: sometimes people interpret A→B as “A and B are both true]”)

Presentation for Activity 2
Select three teams to present to the class, one for each “argument.”

Formative Assessment if class did BOTH Activity 1 and Activity 2 (10 minutes)

Each team reflects, talks about, and makes notes about the following:

What are the pros and cons of using symbolic logic versus using natural language in validating or finding errors in a conclusion based on a series of arguments?

Let students know how to share their notes about the discussion (e.g., if there is time, call on teams to share with the class or ask for notes on paper or through an LMS assignment).

Student Survey
To gather student feedback, give the link to the following survey. Note that student handout is formatted to include QR code https://forms.gle/t98BA78VxqFRysoaA

Quiz Questions/Assessment

Answers coming soon

NOTE: these questions refer to the Valid Argument Forms table. Make sure to include the table when using the questions.

Table of Common Valid Argument Forms also known as *Rules of Inference*

Rule	Assumption(s)	Conclusion
Modus Ponens	$P \rightarrow Q$ and P are both true	Q is true
Modus Tollens	$P \rightarrow Q$ and $\sim Q$ are both true	$\sim P$ is true
Elimination	$P \vee Q$ and $\sim Q$ are both true	P is true
Transitivity	$P \rightarrow Q$ and $Q \rightarrow R$ are both true	$P \rightarrow R$ is true
Cases	$P \vee Q$ and $P \rightarrow R$ and $Q \rightarrow R$ are all true	R is true
Contradiction	$\sim P$ being true implies a contradiction	P is true

1. Use the rules of inference to show that the following collection of premises (a) to (d) imply the conclusion as given.
   (a) $q \rightarrow s$
   (b) $\neg p \vee q$
   (c) $\neg r$
   (d) $\neg p \rightarrow r$
   Therefore, s.

2. *[For use with students with existing programming experience]*
   There is a bug in your program. You perform various debugging tests to determine the location of the bug. You know the following statements are true:
   a. If the message "Value of var1: 10" is displayed, then the bug is not in method foo.
   b. If Barney wrote the program, then the bug is in method foo.
   c. The message "Value of var1: 10" is displayed.
   d. Barney wrote the program, or the bug is in method stuff.

   2.1. Translate the structures of the statements (a) to (d) into logical form, using letters to replace phrases. Specify what each propositional variable represents.

   2.2. Use the rules of inference in the Valid Argument Form to draw a conclusion from the statements you wrote out in part 2.1). Show your work. Where is the bug in the program?

3. Use the rules of inference provided in the Valid Argument Forms table to deduce the conclusion from the premises shown below. Give a reason for each step.

r v s
s -> p
r^q -> t
~p
~s -> r^q
Therefore, t

MAKE COPIES OF THIS PAGE AND THEN CUT TO CREATE CLUE STRIPS.

Students work in groups of 4. Each group gets all four strips 1, 2, 3, 4 and each team member has a different strip.

1

- If the Ramos house is wiretapped, then Rafael Evergarden is not the thief.
- The Ramos house is wiretapped.
- The E.V.I.L. agents are in the house or Carmen Wong is the thief.
- If Kit Luna is the thief, then E.V.I.L. agents will not be in the house.

2

- If E.V.I.L. agents are in the house, then the thief must be Rafael Evergarden.
- The Ramos house is wiretapped.
- The E.V.I.L. agents are in the house or Carmen Wong is the thief.
- If the Ramos house is not wiretapped, then Kit Luna is not the thief.

3

- If E.V.I.L. agents are in the house, then the thief must be Rafael Evergarden.
- If the Ramos house is wiretapped, then Rafael Evergarden is not the thief.
- The E.V.I.L. agents are in the house or Carmen Wong is the thief.
- If Kit Luna is the thief, then E.V.I.L. agents will not be in the house.

4

- If the Ramos house is wiretapped, then Rafael Evergarden is not the thief.
- If E.V.I.L. agents are in the house, then the thief must be Rafael Evergarden.
- The Ramos house is wiretapped.
- If the Ramos house is not wiretapped, then Kit Luna is not the thief.
