THESIS INFORMATION

Thesis title: Some properties of generalized local homology modules
Speciality: Algebra and Number theory
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1. SUMMARY

The thesis studies some properties of the generalized local homology modules. Specifically, we study some of conditions for the finiteness of coassociated prime ideals of the generalized local homology module to a pair of modules. In addition, through the duality theorems we will give the results on the finiteness of the set of associated prime ideals of the local cohomology modules. Another generalization of the local cohomology module is the formal local cohomology module introduced by P. Schenzel. By duality, we introduce the concept of formal local homomology modules and study some basic properties of this modules in the case of linearly compact modules. Simultaneously, the vanishing, non-vanishing properties and finiteness of these modules were also studied.

2. NOVELTY OF THESIS

For the generalized local homology module to a pair of modules, we obtain the following important results:

- Let M be a finitely generated R-module and N a semi-discrete linearly compact R-module such that $N/(\bigcap_{t>0} I^t N)$ is an artinian R-module. Let s be a non-negative integer. If $H_i^I(M,N)$ is minimax for all $i<s$ and G is a closed submodule of $H_s^I(M,N)$ such that $H_s^I(M,N)/G$ is minimax, then G/IG is artinian. In particular, $\mathrm{Coass}(G)$ is a finite set.
- Let M be a finitely generated R-module and N a linearly compact R-module. If N and $H_i^I(N)$ satisfy the finiteness condition for coassociated primes for all $i<k$, then $\mathrm{Coass}_R(H_k^I(M,N))$ is finite.
- Let M be a finitely generated R-module and N a semi-discrete linearly compact R-module. Let s be a non-negative integer. If $H_i^I(M,N)$ satisfy the finiteness condition for coassociated primes for all $i<s$, then $H_s^I(M,N)/IH_s^I(M,N)$ also satisfy the finiteness condition for Coass. In particular, $\mathrm{Coass}(H_s^I(M,N))$ is finite.
- Let M be a finitely generated R-module and N a semi-discrete linearly compact R-module such that $(0:_N I)\neq 0.$ If $t=\mathrm{Width}_I(N)$ and $h=\mathrm{tor\_}(M,H_t^I(N))$, then $\mathrm{Width}_{I+\mathrm{Ann}(M)}(N)=t+h.$ In particular, $\mathrm{Coass}_R(H_{h+t}^I(M,N))$ is a finite set.

Through the duality theorems, we obtain some results about the finiteness of the associated prime ideals of the generalized local cohomology module:

- Let M be a finitely generated R-module and N a semi-discrete linearly compact R-module such that $\Gamma_I(N)$ is finitely generated R-modul. Let s be a non-negative integer. If $H_I^i(M,N)$ is minimax for all $i<s$ and G is a closed submodule of $H_I^s(M,N)$, then $0:_{H_I^s(M,N)/G} I$ is finitely generated. In particular, the set $\mathrm{Ass}(H_I^s(M,N)/G)$ is finite.
- Let M be a finitely generated R-module and N a semi-discrete linearly compact R-module. Let s be a non-negative integer. If $H_I^i(M,N)^*$ satisfy the finiteness condition for all $i<s$, then $0:_{H_I^s(M,N)} I$ is weakly Laskerian. In particular, $\mathrm{Ass}(H_I^s(M,N))$ is a finite set.

For the formal local homology module, we obtain the following results:

- Let M be a R-module. Them

$$H_I^i(\mathrm{F}_{j,J}^I(M)) \cong \begin{cases} 0 & ,i \neq 0 \\ \mathrm{F}_{j,J}^I(M) & ,i=0 \end{cases}$$

  for any integer j.
- Let $(R,\mathfrak{m})$ be a complete ring and M a linearly compact R-module. Then $\mathrm{F}_i^I(M^*) \cong \mathrm{F}_I^i(M)^*$ và $\mathrm{F}_I^i(M^*) \cong \mathrm{F}_i^I(M)^*$ for all i.
- Let M be a semi-discrete linearly compact R-module and non-zero . Then
  i) $\mathrm{Ndim}(0:_M I) = \max\{i \mid \mathrm{F}_i^I(M) \neq 0\}$ if $0 \leq \mathrm{Ndim}(0:_M I) \neq 1$.
  ii) $\mathrm{Ndim}(0:_{\Gamma_\mathfrak{m}(M)} I) = \max\left\{i \mid \mathrm{F}_i^I(M) \neq 0\right\}$ if $\mathrm{Ndim}(0:_{\Gamma_\mathfrak{m}(M)} I) \neq 0$.
- Let M be a artinian R-module and s a positive integer. Then the following statements are equivalent:
  i) $\mathrm{F}_i^I(M)$ is a finitely generated $\hat{R}$-module for all $i<s$;
  ii) $I \subseteq \sqrt{0:_R \mathrm{F}_i^I(M)}$ for all $i<s$.
- Let M be an artinian R-module with $\mathrm{Ndim}\, M = d$. Then $\mathrm{F}_d^I(M)$ is a noetherian $\hat{R}$-module.

3. APPLICATIONS/ APPLICABILITY/ PERSPECTINE

The thesis has given new results on the finiteness of the coassociated prime ideal of the generalized local homology module to a pair of modules and the study of the formal local homomology module class. These results have theoretical significance, which can be applied to the study of the local homology module. We will continue to study the properties of the formal local homology modulus class, specifically in terms of vanishing, non-vanishing and finite.

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