Defining Core Practices in Early Mathematical Modeling

Jennifer Suh, George Mason University, USA
Cynthia O. Anhalt, University of Arizona, USA

The Need for Core Practices in Mathematical Modeling in the Early Grades

Traditionally, mathematical modeling (MM) has been implemented primarily in secondary schools, but recent research examines using this approach with elementary students to promote their problem solving and problem-posing abilities (English, 2010). MM provides the opportunity for students to solve genuine problems and to construct significant mathematical ideas and processes instead of simply executing previously taught procedures and is important in helping students understand the real world (English, 2010). Teaching through MM is ambitious instruction and a change in our teachers traditional approach to teaching. This paradigm shift requires much time and reflective practice to move one's practice to a model of teaching oriented differently than one's prior learning experiences (Lampert, 2001). To develop a common understanding of the concept of a core practice, Grossman, Hammerness, et al. (2009) offered a preliminarily list of criteria that all core practices might share: Practices that occur with high frequency in teaching; Practices that novices can enact in classrooms across different curricula or instructional approaches; Practices that novices can actually begin to master; Practices that allow novices to learn more about students and about teaching; Practices that preserve the integrity and complexity of teaching, and Practices that are research-based and have the potential to improve student achievement (p. 277). McDonald, et al (2013) in their article *Core Practices and Pedagogies of Teacher Education: A Call for a Common Language and Collective Activity* state that having this set of criteria for identifying core practices challenges scholars to avoid a reductionist approach in which core practices become nothing more than the simple selection of specific moves or a list of best practices. In fact, having an in depth look at core practices can help educators define what it takes to enact these high leverage practices and provide structures that support the enactment of these practices,

> "While the core practice of eliciting student thinking is improvisational in nature, the instructional activity of sourcing documents involves many structured supports to help novices create opportunities to elicit student thinking, enact a plan for elicitation, and use his or her enactment as a learning tool for further professional development" (p 383).

Suh et al., (In press) reported ways in which researchers are collaborating with teacher designers to develop personally relevant and rigorous MM tasks for elementary students. These core design practices for teachers designers included : 1) Leveraging problem posing routines: When posing a MM problem, teacher-designers adopted instructional routines for problem posing and worked on developing teacher and student questioning competence; 2) Connecting familiar context that engages students: Teachers, as designers, looked for situational features that warranted mathematizing and searched for contexts that were relevant and important to support students engagement in modeling. In addition, teachers elicited students to think about how their solution was shareable, reuseable, or generalizable, in order to evaluate whether a systematic model was created; 3) Connecting context with content: Teachers connected the need for mathematics in a modeling task with the curricular objectives of their grade level; 4) Considering categories of MM tasks: The modeling tasks tended to fall into four general categories where a mathematical solution or model could be used to describe, predict, optimize, and make decisions about real world situations. This paper will focus on identifying core practices for mathematical modeling and the natural connection between computational thinking and modeling practices. Weintrop (2016) introduced an initial set of computational thinking skills that are essential in fostering the practices of mathematical modeling. These include: the ability to deal with open-ended problems; creating abstractions for aspects of problem at hand; persistence in working through challenging problems;

reframing problem into a recognizable problem; confidence in dealing with complexity; assessing strengths/weaknesses of a representation of data/representational system; representing ideas in computationally meaningful ways; generating algorithmic solutions; breaking down large problems into smaller problems; recognizing and addressing ambiguity in algorithms. For this current study, we analyzed cases of two lesson studies that focused on planning a field trip and solving a school supply dilemma. Through the research lessons from these lesson studies, we wanted to begin to define the core teaching practices that support modeling in the early grades by examining practices that occur with high frequency, practices that teachers new to modeling can begin to enact; practices that allow teachers to learn more about students and teaching through modeling and practices that are research based and have significant impact on improving student learning.

Methods

A qualitative case study approach was selected for this study as our intent was to gain insight and understanding of elementary school teachers' enactment of core teaching practices related to implementing mathematical modeling . The boundaries of the case were two lesson study teams with teachers who were in-service, K-6 following a PD on mathematical modeling. The two topics included the Field Trip Task and the School Supply Task. We were interested in gaining insights into how teachers' pedagogical practices and knowledge emerged as they enacted MM in the elementary school and how inquiry into their own practice helped them negotiate the adoption of MM. We were not looking to do a detailed study of the individual teachers as that would entail separate case studies. Instead, our unit of analysis was the similarities and differences across these teachers' experiences in implementing MM.

Research Questions. In order to understand what core teaching practices are important to mathematical modeling in the elementary grades, we addressed these research questions: 1) What core teaching practices were essential to successfully implementing mathematical modeling in the elementary school classroom? 2) How did teachers create opportunities to elicit these core MM practices as they implemented modeling in the elementary grades?

Data Sources. Planning, enacting and debriefing episodes of individual Lesson Study session and Lesson Study group presentation at a final symposium were captured on video. Data sources also included individual teacher reflections, researcher memos for each Lesson Study and symposium presentation, exit passes from the summer institute and the final symposium, and semi structured group interviews of the lesson study teams. The artifacts collected from the Lesson Study cycle included the planning agendas, actual lesson plans, student work samples, the analysis of student work, and teacher reflections. Each of these factors contributed to compiling a comprehensive picture of teachers' experiences with MM.

Data Analysis. We used the observational memos from lessons with analytic researcher memos that allowed us to not only describe our teacher~~'s~~ planning and enactment of the mathematical modeling lessons but also summarize our thoughts about potential ways in which certain skills, and knowledge contributed to the teachers' ability to facilitate learning through MM. Interviews, videoclips of classroom episodes and artifacts from lessons including lesson plans, and student artifacts also helped identity the development of the MM processes in the elementary classroom and the various models that emerged from the MM tasks.

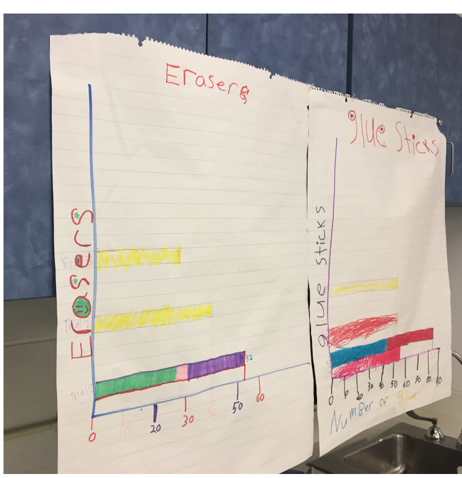
Findings and Discussion

In detailing the teachers' enactments of the MM lessons and reflections on the MM process using several of the episodes in these cases, we found four main categories of mathematical modeling core teaching practices that emerged as being central to the success of enacting mathematical modeling in the elementary classroom: a) Questioning practices: Developing student questioning competence; b) Data Practices: Connecting relevant data with formulating the problem and eliciting student thinking about important variables and assumption in a problem situation; c) Modeling Practices: Building a solution that can be communicated to others through uses of records of student work, concrete tools, written and verbal explanations, number sentences and pictorial representations; d) Analytic and Interpretive Practices: Facilitating productive analysis of a model for the purpose of refining the model.

a) **Questioning practices:** A core practice essential to launching a modeling task involved questioning practices. This entailed problem formulation (OECD, 2018) which involves identifying the mathematical aspects of the problems contained in the real context situation and the important variables and breaking down problems into smaller, manageable parts that helps simplify a situation or problem in order to make it amenable to mathematical analysis. In our work with teachers, we noticed teachers developing routines that promoted students' questioning competence. Across the two lesson studies analysis, teachers elicited questions about the problem situation. They used photo elicitation by showing a photo of a field trip and asking what they noticed and wondered about the situation. When students offered the questions, a teacher color coded and sorted questions that related to mathematical endeavors and other questions that were not mathematically related. In their questioning process, students were already starting to think about some important variables such as cost, time, distance, and means for transportation.

General Wonderings	Mathematical Wonderings
What about weather conditions? What if it rains/storms? Will weather affect our trip/ travel time? What are we going to do when we get there? What will we want to explore? What is the date? What kind of clothing should we wear?	M- Where in the locations are we going? M- What is the budget? M- How long would it take to get there? How long are we staying there? When do we have to wake up? What time will we leave the school? How many hours will we spend at the location? M- How much will gas cost? M- How many students are going? M- What kind of bus are we taking? How many people can fit on the bus?

b) **Data Practices**: A core practice essential to modeling that connect to computational thinking include data practices, which includes gathering data, analyzing data, and representing data through graphic representations. Teaching through MM means supporting students to gather relevant data when formulating the problem and eliciting student thinking about important variables and assumption in a problem situation to describe, predict and prescribe a solution. In the case of the School Supply Dilemma, students needed to determine the rate in which students used the school supply before they could describe the trend of use, predict the usage, and then prescribe the appropriate use rate to ensure they did not run out. It also involved helping students learn how to depict and organize data in appropriate graphs, charts, words, or images. A teacher reflects on facilitating the core practice of using data to build a solution.



"At the beginning of the year, we presented our classes with the above question: What is the best way to organize and maintain classroom supplies to last until the end of the year? We then led students through finding variable and making assumptions. During this step in the process, students formed groups in charge of different supplies, took inventory of their supplies, and came up with usage guidelines for the class. In the weeks following, students revisited their inventory and collected more data points as time went on. As students collected these various

data points, they began to form charts to show trends in their data. Once students had several data points, they looked for patterns and predicted how many supplies they would have left at the end of the year. After analyzing and predicting, some students found that they would run out of supplies before the end of the year. These students then revised their problem and came up with a new problem: How can we make our supplies that we have last us throughout the year? As students revised their supply guidelines, they reported these back to the class."

c) **Modeling Practices:** MM promotes students computational thinking by using mathematics to make important decisions. Some of the solutions lead to mathematical models that described, predicted, optimized situations that helped one make decisions. By building a solution that can be communicated to others through uses of records of student work, concrete tools, written and verbal explanations, number sentences and pictorial representations, teachers can develop students modeling practices. In our research lessons, teachers annotated their lesson plan outlining the components they expect the students' models to include such as food costs, transportation costs, event costs, and the number of people. They defined this MM task would yield a *useful model that wi*ll be able to represent the total cost of their planned field trip, and that it can be "reuseable" in that it could be applied to any future trips.

d) **Analytic and Interpretive Practices**: Finally, another common core practices is providing students the time and space to analyze their solutions and models to critique and refine their models. This requires important discursive practices from both the teacher and students including, communicating one's model, justifying their solutions and critiquing one's thinking. Facilitating productive conversation around the analysis of a model is a critical component in refining the model and validating the result back to the real issue that it set out to address.

Conclusion

Through the oral presentation, we hope to share these research lessons and begin to define the core teaching practices that supported mathematical modeling in the early grades by examining practices that occurred with high frequency, practices that teachers new to modeling could begin to enact; practices that allowed teachers to learn more about students and teaching through modeling and practices that were research based and improved student learning.

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