

## **AP Calculus AB**

Summer Work 2025 (verified May 1, 2025)

due date: Monday, August 26 (first day of class)

estimated time: 8 hours (for planning purposes only; work until you finish)
 target goal for iXL: minimum 5 problems per section AND minimum score of 80

Dear AP Calculus AB students,

This assignment is designed for you to review all of your algebra, geometry, and precalculus skills and make sure you are well prepared for the start of AP Calculus AB in the fall. The assignment is in two parts: written problems in this packet, and problems on iXL.com. See the next page for detailed instructions. We anticipate that the work will take about 8 hours to complete.

Both the written work and the online work are due on the first day of class. At that time your teacher will go over any questions you may have. On about the third day of class you will have a short quiz with problems taken directly from both this packet and the iXL sections listed on the next page. A portion of the quiz grade will be based on the completeness of your summer work.

We are really looking forward to the upcoming school year and hope you are, too! Please email any of us if you have questions.

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### **On-Line Work**

- Log in to <u>iXL.com</u> with the username and password you were given in your math class.
  - Students new to Steward: your parents will receive an email with your username and password.
- In order to receive credit for summer work, your account must show activity between the dates of June 1 and August 25, 2025. Two criteria will be evaluated:
  - A minimum of 5 problems from each section must be completed.
  - A current minimum score of 80 must be reached.
- Work the problems listed below.
  - No matter what, complete 5 problems in each section even if you have already done the section before. This should be done regardless of your original score in each section. Teachers will be looking only at work completed between June 1 and August 25 (the first day of school).
  - After doing 5 problems, continue to do problems until you have a final score of at least 80. If your score is lower than 80 after doing five problems then keep working more problems until your score is over 80. If your score is higher than 80 after doing five problems, you can stop.

Even if you have worked in a particular section at an earlier date (and even if you already have a minimum score in that section), you must complete the above two requirements on or after June 1, 2025. This is extremely important. Teachers can filter out work completed before June 1, 2025 and that work/score will not count towards summer requirements.

- If you find that the name of the section does not match the numbers below, go by the section name.
- If you are having difficulty of any sort with iXL, reach out to a teacher at one of the email addresses above.

1. click this tab	2. find this section	3. work these problem sets	
Pre-Calculus	Families of functions	C.1 - Function transformation rules C.2 - Translations of functions C.3 - Reflections of functions C.4 - Dilations of functions C.5 - Transformations of functions C.6 - Describe function transformations	
	Limits This is a new topic for you. Use iXL to explore. Take guesses, make mistakes and read explanations. See what you can learn!	HH.1 Find limits using graphs HH.2 Find one-sided limits using graphs HH.3 Determine if a limit exists HH.4 Find limits at vertical asymptotes using graphs HH.5 Determine end behavior using graphs HH.6 Find limits using addition, subtraction, and multiplication laws HH.7 Find limits using the division law HH.8 Find limits using power and root laws HH.9 Find limits using limit laws HH.10 Find limits of polynomials and rational functions HH.11 Find limits involving factorization and rationalization	

### **Written Work**

A note before you begin: This assignment as a whole asks you to review the algebra and geometry skills that you've been taught over many years – and that's a lot! Please don't get too discouraged! Please do plan to spend some time on this. You are welcome to use any printed resource at your disposal (old notes, the PreCal textbook, Internet sites, etc.), and everyone will probably need to look up something (distance formula, quadratic formula, etc.). This assignment, however, is designed to assess if **you** are ready, **not** if your <u>friends</u> are ready for Calculus, so please understand that it is hoped that you will do the right thing and work mostly by yourself. If you need help, feel free to email any math teacher – see the cover letter for their email addresses.

A note on calculator use: Although you'll probably have your calculator handy as you work, please know that most of the first semester of Calculus is done mostly **without** a calculator of any kind. In fact, **two-thirds** of the AP Exam is done without a calculator. Please show algebra work as you do most of these problems because that will be key to success next year.

#### Good luck!

#### Instructions:

- 1. Complete <u>each problem</u> on the following pages.
- 2. Do the problems on separate paper. Do the problems in order and number them clearly.
- 3. Please show all of your work as was required of you throughout the entire school year.
  - o Remember: no work, no credit.

1. Simplify.

(a) 
$$\frac{x^3 - 9x}{x^2 - 7x + 12}$$

(b) 
$$\frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$$

(c) 
$$\frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$$

(d) 
$$\frac{9-x^{-2}}{3+x^{-1}}$$

2. Write the conjugate of each expression. (For example, the conjugate of x-y is x+y)

(a) 
$$(3+2i)$$

(b) 
$$\left(-\sqrt{3}-\sqrt{7}\right)$$

(c) 
$$\left(\sqrt{x+h} - \sqrt{x}\right)$$

3. Simplify. Write answers using positive exponents.

(a) 
$$\frac{\left(2a^2\right)^3}{b}$$

(b) 
$$\sqrt{9ab^3}$$

(c) 
$$\frac{a\left(\frac{2}{b}\right)}{\frac{3}{a}}$$

(d) 
$$\frac{ab-a}{b^2-b}$$

(e) 
$$\frac{a^{-1}}{\left(b^{-1}\right)\sqrt{a}}$$

4. Solve for x. Do not use a calculator.

(a) 
$$\frac{1}{3} = 3^{2x+2}$$

(b) 
$$\log_2 x = 3$$

(c) 
$$\log_3 x^2 = 2\log_3 4 - 4\log_3 5$$

5. Simplify as much as possible.

(a) 
$$3^{2\log_3 5}$$

6. Simplify as much as possible.

(a) 
$$\log_{10} \left(10^{\frac{1}{2}}\right)$$

(b) 
$$\log_{10}\left(\frac{1}{10^x}\right)$$

7. Solve the following equations for the indicated variables.

(a) solve for a: 
$$V = 2(ab + bc + ca)$$

(b) solve for P: 
$$A = P + nrP$$

(c) solve for d: 
$$2x - 2yd = y + xd$$

(d) solve for x: 
$$\frac{2x}{4\pi} + \frac{1-x}{2} = 0$$

8. Factor completely.

(a) 
$$x^6 - 16x^4$$

(b) 
$$2x + 3x + 1$$

(c) 
$$\frac{1}{2}$$
  $-\frac{1}{2}$ 

(d) 
$$x^4 - 1$$

9. Find <u>all</u> real solutions. (Note that these equations are similar to the expressions in #8)

(a) 
$$x^6 - 16x^4 = 0$$

(b) 
$$2x + 3x + 1 = 0$$

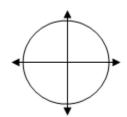
(c) 
$$\frac{1/2}{3x} - \frac{1}{2} = 0$$

10. Solve for *x* algebraically.

(a) 
$$3\sin^2 x = \cos^2 x$$
;  $0 \le x < 2\pi$ 

(b) 
$$\cos^2 x - \sin^2 x = \sin x; -\pi < x \le \pi$$

11. Draw a complete trig unit circle, labeling the coordinates at every "special" angle. Then answer:



(a) 
$$\cos 210^{\circ}$$

(b) 
$$\sin \frac{5\pi}{4}$$

(c) 
$$\tan^{-1}(-1)$$

(d) 
$$\sin^{-1}(-1)$$

(e) 
$$\cos \frac{9\pi}{4}$$

(f) 
$$\sin^{-1}\frac{\sqrt{3}}{2}$$

(g) 
$$\tan \frac{7\pi}{6}$$

(h) 
$$\cos^{-1}(-1)$$

12. Sketch the graph of each function by hand, labeling critical points.

(a) 
$$y = \sin\left(\frac{x}{2}\right)$$

(b) 
$$y = \cos x$$

(c) 
$$y = \tan x$$

13. Solve the equations algebraically.

(a) 
$$4x^2 + 12x + 3 = 0$$

(b) 
$$2x+1=\frac{5}{x+2}$$

(a) 
$$4x^2 + 12x + 3 = 0$$
 (b)  $2x + 1 = \frac{5}{x+2}$  (c)  $\frac{x+1}{x} - \frac{x}{x+1} = 0$ 

14. Solve the inequalities.

(a) 
$$\frac{2x-1}{3x-2} \le 1$$

(b) 
$$x^2 + 2x - 3 \le 0$$

15. Solve for x.

(a) 
$$|-x+4| \le 1$$

(b) 
$$|5x-2|=8$$

- 16. Determine equations of the following lines, in **point-slope form**.
  - (a) the slope is -4 and the y-intercept is 5
  - the slope is 8 and the line passes through  $\left(-6,123\right)$ (b)
  - the line that passes through  $\left(-1,3\right)$  and  $\left(2,-4\right)$ (c)
  - the line that passes through (-1,2) and is perpendicular to the line 2x-3y+5=0(d)
  - (e) the line that passes through (2,3) and the midpoint of the line segment from (-1,4) to (3,2)
- 17. Find the point of intersection of the lines by hand.

$$3x - y - 7 = 0$$

$$x + 5y + 3 = 0$$

18. Find the domain of the functions by hand.

(a) 
$$g(x) = \frac{5x-3}{2x+1}$$

$$(b) f(x) = 7$$

(c) 
$$f(x) = \frac{3x+1}{\sqrt{x+2}}$$

19. Write as a piecewise function.

(a) 
$$f(x) = |x|$$

(b) 
$$f(x) = \frac{|x|}{x}$$

$$\underline{f(x+h)-f(x)}$$

20. Simplify  $\frac{f(x+h)-f(x)}{h}$  , where

(a) 
$$f(x) = 2x + 3$$

(b) 
$$f(x) = x^2$$

$$(c) f(x) = \frac{1}{x+1}$$

21. Sketch the graphs of the following functions by hand, labeling important points (intercept, vertex, etc.)

(a) 
$$f(x) = -x + 6$$

(b) 
$$g(x) = x^2$$

(c) 
$$h(x) = x^3$$

(d) 
$$j(x) = \sqrt{x}$$

(e) 
$$k(x) = \sqrt[3]{x}$$

(f) 
$$l(x) = \frac{1}{x}$$

(g) 
$$m(x) = e^x$$

$$(h) n(x) = e^{-x}$$

(i) 
$$p(x) = \ln x$$

(j) 
$$q(x) = \log_5 x$$

(k) 
$$r(x) = -2(x+3)^2 - 1$$

$$(1) s(x) = |x-4|$$

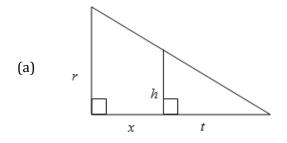
22. Find the inverse of the functions.

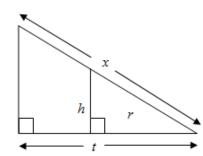
(a) 
$$f(x) = 2x + 3$$

(b) 
$$f(x) = \frac{x+2}{5x-1}$$

23. Express *x* in terms of the other variables in the picture. In part (b), "r" refers to the hypotenuse.

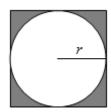
(b)



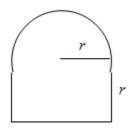


## 24. Answer the following.

(a) Find a formula for the shaded area inside the square but outside the circle.



(b) Find a formula for the perimeter of a window in the shape depicted.



- (c) A water tank has the shape of a cone (like an ice cream cone without the ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep in the middle, what is the surface area of the top of the water?
- (d) Two cars start moving from the same point. One travels south at 100km/hr, the other west at 50 km/hr. How far apart are they two hours later?
- (e) A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight.)
- 25. List the following Fundamental Trigonometric Identities.
  - (a) the six (6) Reciprocal Identities
  - (b) the two (2) Ratio/Quotient Identities
  - (c) the three (3) Pythagorean Identities
  - (d) the Double-Angle Identities for sine and cosine
- <u>26. Use the above identities</u> to verify the following identities. Show work/steps in an organized fashion.
  - (a)  $\cot^2 \theta + \cos^2 \theta + \sin^2 \theta = \csc^2 \theta$
  - (b)  $\frac{\sin \theta}{\csc \theta} + \frac{\cos \theta}{\sec \theta} = \sin \theta \csc \theta$

- 27. Write the following formulas from geometry.
  - (a) area of a circle
  - (b) circumference of a circle
  - (c) area of a triangle
  - (d) volume of a cube
  - (e) surface area of a cube
  - (f) volume of a cone
  - (g) volume of a sphere
  - (h) surface area of a sphere
  - (i) area of an equilateral triangle
- 28. Describe how to find where a rational function has a vertical asymptote.
- 29. Describe how to find out if a rational function has a horizontal asymptote and, if it does, the equation of the horizontal asymptote.
- 30. Describe how to find out if a rational function has a "hole" in the graph.
- 31. For the function  $f(x) = \frac{x^2 5x + 4}{x 1}$ , describe what happens to the *y*-values
  - (a) as the x-values get really close to x = 1
  - (b) as the *x*-values get really large in the positive direction
  - (c) as the *x*-values get really large in the negative direction
- 32. Find the equation of the circle with center at (1,2) that passes through the point (-2,-1)
- 33. For the circle  $x^2 + y^2 + 6x 4y + 3 = 0$ , find
  - (a) the center and radius
  - (b) the equation of the line that is tangent to the circle at the point (-2, 5)

# **Selected Answers**

1a)	$\frac{x(x+3)}{x-4}$	1d)	$\frac{3x-1}{x}$
2b)	$(-\sqrt{3}+\sqrt{7})$		
3b)	$ 3b \sqrt{ab}$		
4b)	x = 8	4c)	$x = \frac{4}{25}, -\frac{4}{25}$
7a)	$a = \frac{V - 2bc}{2(b+c)}$	7d)	$x = \frac{-\pi}{1-\pi}$
8a)	$x^4(x+4)(x-4)$	8b)	(2x+1)(x+1)
9b)	$x = -2, -\frac{1}{2}$	9c)	x = -2
10a)	$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	10b)	$x = \frac{-\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$
11c)	$-\frac{\pi}{4}$	11f)	$\frac{\pi}{3}$
11h)	π		
13a)	$\frac{-3\pm\sqrt{6}}{2}$	13c)	$-\frac{1}{2}$
14b)	[- 3, 1]	15a)	[3, 5]
16d)	$(y-2) = -\frac{3}{2}(x+1)$	16e)	y = 3
18a)	$(-\infty, -\frac{1}{2})\cup(-\frac{1}{2}, \infty)$	19b)	2x + h
23b)	$x = \frac{rt}{\sqrt{r^2 - h^2}}$		
24b)	$r(4 + \pi)$	24d)	223.607 km.