Analysis of Luck Scales in Dominions 5

By SirToma

After seeing a large amount of recent discussion on the value of luck scales across a number of dominions discords I decided to take a look and see if I could calculate an approximate value for them.
The ‘TLDR Summary’ contains just the basic results, the ‘Method’ section details how I came to the numbers displayed, and the ‘Results’ section contains further data that I was able to extract.
If you have any feedback, suggestions, or ideas to improve this feel free to contact SirToma#8442 on discord.

To Do List:

- Currently nothing

1. TLDR Summary

- Luck scales are slightly better than order and production as a pure income scale when accounting for raw gold, income from discovered sites (e.g. gold mines), and free infrastructure.
    - Note that without the free infrastructure, which may not always be worth its face value, it is approximately equivalent to order and production.
    - This income is heavily weighted to luck 2 and 3, with luck 1 providing minimal benefit.

- Underwater the value of luck scales is higher due to a better pool of events. Luck 2+ appears better for income than order and production even before considering additional infrastructure.

- Additionally luck scales generate a significant amount of additional gems at luck 2+ and average an additional const 4-6 item every 14 turns.

- By contrast misfortune is the cheapest income scale in the game to tank.
    - I've assumed an independent attack costs 1 turns income and the cost to replace 6pd. Unrest is accounted for based on the formula in the manual. I assume you take the income penalty for 10 turns. If you are already patrolling or can patrol cheaply misfortune is slightly cheaper.
    - Furthermore, the lost income from destroyed infrastructure also imposes costs on research and recruitment beyond just their gold value.
    - The main contributors to the gold loss from misfortune are permanent income reductions (both raw income and pop based) along with increases to unrest.

- The addition of magic scales decreases the payoff of luck, though not significantly.
    - This is due to profitable events being unlocked at lower luck levels increasing the baseline income (as the analysis compares luck X to luck 0).

- [Opinion warning] Depending on your needs for expansion it may be worth putting scale points into luck rather than order or production if you can get to 2+ and either have solid heroes, good uses for gems, good use for random infrastructure, or low magic diversity.
    - Furthermore your ability to win early wars is also relevant as being 20% bigger and having an additional cap will outweigh the benefit of luck in most cases.

For the tables below income_pc and income_w_infra_pc refer to the percentage increase in income taking a certain luck scale would have on our sample empire. The additional gems and additional item columns represent the expected gems and const 4-6 items per turn respectively.

luck	income_pc	income_w_infra	additional_gems	additional_item4
-3	-0.048	-0.053	-0.600	0.000
-2	-0.026	-0.028	-0.381	0.000
-1	-0.013	-0.014	-0.260	0.000
0	0.000	0.000	0.000	0.000
1	0.020	0.024	0.689	0.000
2	0.056	0.068	2.600	0.000
3	0.096	0.114	5.430	0.073

Expected value of luck scales in a sample empire with growth 3. Other scales are neutral.

luck	income_pc	income_w_infra_pc	additional_gems	additional_item4
-3	-0.046	-0.051	-0.722	0.000
-2	-0.025	-0.027	-0.437	0.000
-1	-0.013	-0.013	-0.282	0.000
0	-0.000	-0.000	0.000	0.000
1	0.019	0.023	0.701	0.000
2	0.053	0.064	2.515	0.000
3	0.090	0.108	5.169	0.067

Expected value of luck scales in a sample empire with growth 3 and magic 3. Other scales are neutral.

2. Method

In order to calculate an expected value for luck scales a number of assumptions had to be made due to unavailable information. This section walks through the process I took to produce results and highlights these assumptions.

My main sources of information were the:

- Dominions 5 Mod Inspector (https://larzm42.github.io/dom5inspector/)
- Dominions 5 Event Modding Manual (http://www.illwinter.com/dom5/eventmanual.html)

Initial Data

The initial event data was taken from the Dominions 5 Mod Inspector. The gamedata folder contains a csv (events.csv) which holds all game events. The data was filtered down to events with rarities ±1 and ±2 as these are the random events according to the event manual.
Note that this filtering means all events that are part of a chain have been excluded from this analysis.

```
w_evs[lambda x: ~x.effects.isna()][lambda x: x.effects.str.contains('lab')].iloc[0]
```

```
name                                A renegade wizard was found an
rarity                                                          -1
description     A renegade wizard was found and staked in a bo...
requirements                                          lab 0|swamp 1
effects                                              lab 1|2d6vis N
Name: 84, dtype: object
```

An example event.

The data was then processed in order to extract the requirements and effects into a usable format.

Following this the effects were used to calculate the amount of gold, future gold income, infrastructure value, gems, and items the event provides.
For gems the average of the rolls were used in all cases and for gold the ±50% variance was ignored. Items were broken down into the categories shown in the event manual and excludes specific item rewards.

Calculating Expected Values

Given the above event information it should be trivial to calculate the expected value of having an event in a given province. One simply needs to:

- Determine which events are possible.
- Determine the weighting of each possible event.
- Divide the weighted sum of the desired property (e.g. gems) and divide by the total weight.

Step one simply requires that we define a province state and then check our state against the event requirements. Step three is also trivial with the information we have.

Unfortunately step two in the above process is very opaque and I struggled to find any information about the relative chances of events of different rarities or how luck scales impact the balance of positive and negative events.

My assumptions were:

- Rarity ±1 events had a base weight of 0.95 and rarity ±2 events had a base weight of 0.05
    - Changing this balance slightly (0.9/0.1, 0.975/0.025, and 1.0/0.0) did not materially impact the results.
- Luck scales multiply the weight of an event by $1 - 0.1 * LuckScale * Rarity$
    - As the rarity is negative for positive events this means that positive luck will increase the weighting of positive events and vice versa for negative events or misfortune scales.
    - Luck scales also make events more likely but I decided to account for this later in the process.

The Sample Empire

We are now able to calculate the expected value of having a single event in a given province. This in of itself is not particularly useful information as other scales provide percentage based increases to income so making comparisons is difficult without further assumptions.

In order to generate comparable numbers I decided to create a sample empire snapshot. This empire is 20 provinces large. It has 5 forts with labs and temples (cap, two on farms, one on a plains, and one on a mountain) and an otherwise standard distribution of terrain based on the MapNuke defaults. All forts have commanders and researchers in them while all other provinces are empty except for 6 province defence. Finally I have assumed that the scales of the empire are fully in all provinces. This is not entirely true, but it should at least allow a comparison to the impact of other scales which are also rarely perfectly spread.
This sample empire allowed me to account for the fact that many events have requirements based on terrain, infrastructure, and commanders.

```
_EMPIRE_STATE = [
    ('cap', cap_state.copy()),
    ('swamp', swamp_state.copy()),
    ('waste', waste_state.copy()),
    ('mountain1', mountain_1_state.copy()),
    ('mountain2', mountain_2_state.copy()),
    ('sea1', sea_state.copy()),
    ('sea2', sea_state.copy()),
    ('farm1', farm_1_state.copy()),
    ('farm2', farm_1_state.copy()),
    ('farm3', farm_2_state.copy()),
    ('cave', cave_state.copy()),
    ('forest1', forest_state.copy()),
    ('forest2', forest_state.copy()),
    ('forest3', forest_state.copy()),
    ('fort', fort_state.copy()),
    ('plains1', plains_state.copy()),
    ('plains2', plains_state.copy()),
    ('plains3', plains_state.copy()),
    ('plains4', plains_state.copy()),
    ('plains5', plains_state.copy()),
]
```

Generating Results

Unfortunately to actually generate usable results further assumptions were required.

- An event is equally likely to occur in any province
    - This allows the calculation of the expected value of an event occuring in the entire empire by taking a simple mean.

At this point it is possible to simply pass in a set of scales and calculate the expected payoff of a single event in the sample empire.

luck	gems	gold	income	infra	item4
-3	0.93	5.39	-0.83	-5.30	0.00
-2	1.07	7.44	-0.52	-2.59	0.00
-1	1.18	9.24	-0.23	-1.07	0.00
0	1.37	11.41	0.04	-0.72	0.00
1	1.64	24.32	0.28	3.46	0.00
2	2.43	47.33	0.72	11.01	0.00
3	3.55	80.70	0.68	17.41	0.03

Unfortunately the amount of events per turn is variable, as is the income of an empire, so we need to make some further assumptions about these values in order to have comparable values to the other income scales.

For the values in this document I have assumed that an average of 2 events occur per turn increased by 5% per luck/misfortune scale and that the average province has an income of 90g while the capital has an income of 500g. All results are linear with respect to both event frequency and total income, so scaling the results to your preferred values for these parameters is trivial.

3. Results

In addition to the results in the initial TLDR I also tested a number of other sets of scales to ensure the final values weren't overdependent on them.

Random Land Empire 1

Below are the results for the scales I ran in a game as MA Marignon (Order 0, Prod 2, Heat 3, Growth 3, Magic 3). These are in line with the initial results in the TLDR.

luck	income_pc	income_w_infra_pc	additional_gems
-3	-0.050	-0.055	-0.819
-2	-0.027	-0.029	-0.538
-1	-0.014	-0.014	-0.393
0	0.000	0.000	0.000
1	0.020	0.024	0.685
2	0.056	0.068	2.334
3	0.095	0.115	4.948

Random Land Empire 2

Below are the results for the scales I ran in a game as MA Ulm (Order 0, Prod 3, Cold 1, Growth 3, Drain 3). These are in line with the initial results in the TLDR.

luck	income_pc	income_w_infra_pc	additional_gems
-3	-0.052	-0.058	-0.761
-2	-0.028	-0.030	-0.526
-1	-0.015	-0.015	-0.387
0	0.000	0.000	0.000
1	0.022	0.026	0.676
2	0.061	0.073	2.556
3	0.104	0.124	5.221

Random Land Empire 3

Below are the results for the scales I saw in a game for an MA Caelum player (Order 3, Prod 0, Cold 3, Growth 3, Magic 3). These are in line with the initial results in the TLDR.

luck	income_pc	income_w_infra_pc	additional_gems
-3	-0.051	-0.055	-0.806
-2	-0.029	-0.031	-0.530
-1	-0.015	-0.015	-0.341
0	0.000	0.000	0.000
1	0.020	0.023	0.696
2	0.053	0.063	2.476
3	0.090	0.108	4.801

Hellbless Empire

Below are the results for an empire that has tanked their scales other than magic (Turmoil 3, Sloth 3, Cold 3, Death 3, Magic 3). These are in line with the initial results in the TLDR.

luck	income_pc	income_w_infra_pc	additional_gems
-3	-0.058	-0.062	-0.854
-2	-0.035	-0.037	-0.559
-1	-0.019	-0.019	-0.361
0	0.000	0.000	0.000
1	0.023	0.026	0.712
2	0.060	0.070	2.465
3	0.102	0.119	4.779

UW Empire

As many events can only occur on land (or underwater) the value of luck scales changes significantly for UW nations. Below are the results for an UW empire with 3 land provinces and

17 sea provinces. They still have 5 fort/temple/lab'd provinces including the capital. I am assuming that events occur just as frequently underwater despite the lower event pool.

luck	income_pc	income_w_infra_pc	additional_gems
-3	-0.050	-0.050	-0.907
-2	-0.031	-0.031	-0.607
-1	-0.016	-0.016	-0.413
0	0.000	0.000	0.000
1	0.025	0.030	0.819
2	0.071	0.086	2.939
3	0.126	0.150	5.185

Interestingly it appears that luck is noticeably more valuable underwater and likely the second best income scale for those nations.