

BRIDGE COURSE STUDY MATERIAL – CLASS XII (MATHEMATICS)

S.N O	TOPIC	CONTENT	LEARNING OBJECTIVES
1	SETS	Sets and their representations. Types of sets. Subsets of a set. Venn diagrams. Union and Intersection of sets.	Students will be able to use these contents in solving questions from the topic probability and day today life situational problems
2	RELATION AND FUNCTIONS	Ordered pairs. Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the set of reals with itself ($R \times R$ only). Definition of relation, domain, co-domain and range of a relation. Function as a special type of relation. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs.	Students will be able to identify: <ol style="list-style-type: none"> 1) Difference between relation and function 2) Types of functions 3) Domain and range of functions 4) Graphs of different types of functions
3	TRIGONOMETRIC FUNCTIONS	Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x . Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing $\sin(x \pm y)$ and $\cos(x \pm y)$ in terms of $\sin x$, $\sin y$, $\cos x$ & $\cos y$ and their simple applications. Deducing identities like the following: $\tan(x \pm y)$, $\cot(x \pm y)$, $\sin \alpha \pm \sin \beta$, $\cos \alpha + \cos \beta$, Identities related to $\sin 2x$, $\cos 2x$, $\tan 2x$, $\sin 3x$, $\cos 3x$ and $\tan 3x$.	<ol style="list-style-type: none"> 1. Angles 2. Trigonometric Functions 3. Sum and Difference of Two Angles 4. Trigonometric Equations <ul style="list-style-type: none"> • Measurement of an angle: The measure of an angle is the amount of rotation from the initial side to the terminal side. • Right angle: If the rotating ray starting from its initial position to final position, describes one quarter of a circle, then we say that the measure of the angle formed is a right angle. • If in a circle of radius r, an arc of length l subtends an angle of θ radians, then $l = r\theta$ • Radian measure = $\frac{\pi}{180} \times \text{Degree measure}$ • Degree measure = $\frac{180}{\pi} \times \text{Radian measure}$ <p>Students should know the following basic formulae :</p>

$$\blacklozenge \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\blacklozenge \cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\blacklozenge \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\blacklozenge \sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\blacklozenge \sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\blacklozenge \sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\blacklozenge \cos\left(\frac{\pi}{2} + x\right) = -\sin x \quad \sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\cos(\pi - x) = -\cos x$$

$$\sin(\pi - x) = \sin x$$

$$\cos(\pi + x) = -\cos x$$

$$\sin(\pi + x) = -\sin x$$

$$\cos(2\pi - x) = \cos x$$

$$\sin(2\pi - x) = -\sin x$$

\blacklozenge If none of the angles x , y and $(x \pm y)$ is an odd multiple of $\frac{\pi}{2}$, then

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\blacklozenge \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

\blacklozenge If none of the angles x , y and $(x \pm y)$ is a multiple of π , then

$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$$

$$\blacklozenge \cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

$$\blacklozenge \sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$\blacklozenge \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\blacklozenge \sin 3x = 3 \sin x - 4 \sin^3 x$$

$$\blacklozenge \cos 3x = 4 \cos^3 x - 3 \cos x$$

$$\blacklozenge \tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

$$\blacklozenge \text{ (i) } \cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\text{ (ii) } \cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\text{ (iii) } \sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\text{ (iv) } \sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\blacklozenge \text{ (i) } 2 \cos x \cos y = \cos(x+y) + \cos(x-y)$$

$$\text{ (ii) } -2 \sin x \sin y = \cos(x+y) - \cos(x-y)$$

$$\text{ (iii) } 2 \sin x \cos y = \sin(x+y) + \sin(x-y)$$

$$\text{ (iv) } 2 \cos x \sin y = \sin(x+y) - \sin(x-y).$$

$$\blacklozenge \sin x = 0 \text{ gives } x = n\pi, \text{ where } n \in \mathbf{Z}.$$

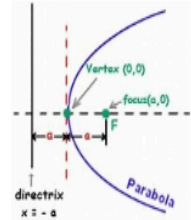
$$\blacklozenge \cos x = 0 \text{ gives } x = (2n+1) \frac{\pi}{2}, \text{ where } n \in \mathbf{Z}.$$

$$\blacklozenge \sin x = \sin y \text{ implies } x = n\pi + (-1)^n y, \text{ where } n \in \mathbf{Z}.$$

4	LINEAR INEQUALITIES	Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Graphical method of finding a solution of system of linear inequalities in two variables.	<p>Students will be able to find:</p> <ol style="list-style-type: none"> Inequalities - Algebraic Solutions and Graphical Representation Graphical Solution of Linear Inequalities Solution of System of Linear Inequalities
5	PERMUTATIONS AND COMBINATIONS	Fundamental principle of counting. Factorial n . $(n!)$ Permutations and combinations, formula for nPr and nCr , simple applications.	<p>Fundamental Principle of Counting</p> <ul style="list-style-type: none"> • Addition Law: If there are two operations such that they can be performed independently in m and n ways respectively, then either of the two operations can be performed in $(m + n)$ ways. • Multiplication: If one operation can be performed in m ways and if corresponding to each of the m ways of performing this operation, there are n ways of performing a second operation then the number of ways of performing two operations together in $m \times n$. • Factorial Notation: The continued product of first n natural numbers is called the 'n factoria' and is denoted by $n!$. • $0! = 1$ • Permutations: The number of permutations of n different things taken r at a time, where repetition is not allowed, is denoted by nP_r and is given by ${}^nP_r = \frac{n!}{(n-r)!}$ <p>where $0 \leq r \leq n$.</p> $n! = 1 \times 2 \times 3 \times \dots \times n$ $n! = n \times (n - 1)!$ <ul style="list-style-type: none"> • The number of permutations of n different things, taken r at a time, where repetition is allowed, is n^r. • The number of permutations of n objects taken all at a time, where p_1 objects are of first kind, p_2 objects are of the second kind, ..., p_k objects are of the k^{th} kind and rest, if any, are all different is $\frac{n!}{p_1! p_2! \dots p_k!}$.

			<p>Combinations:</p> <ul style="list-style-type: none"> The number of combinations of n different things taken r at a time, denoted by nC_r is given by ${}^nC_r = \frac{n!}{r!(n-r)!}, 0 \leq r \leq n.$ ${}^nC_0 = 1$ ${}^nC_n = 1$ ${}^nC_r = {}^nC_{n-r}$ ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ ${}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$ $n \cdot {}^{n-1}C_{r-1} = (n-r+1) {}^nC_{r-1}$ Division into Groups: The number of ways $m + n$ things can be divided into two groups containing m and n things respectively = ${}^{m+n}C_m = \frac{(m+n)!}{m!.n!}$
6	STRAIGHT LINES	<ol style="list-style-type: none"> Slope of a Line Various Forms of the Equation of a Line General Equation of a Line and Distance of a Point From a Line 	<p>Students will be able to find:</p> <ol style="list-style-type: none"> Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point -slope form, slope-intercept form, two-point form, intercept form and normal form. General equation of a line. Distance of a point from a line.

			<p>Various forms of equations of a line:</p> <ul style="list-style-type: none"> • Two points form: Equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ • Slope-Intercept form: The point (x, y) on the line with slope m and y-intercept c lies on the line if and only if $y = mx + c$. • If a line with slope m makes x-intercept d. Then equation of the line is $y = m(x - d)$. • Intercept form: Equation of a line making intercepts a and b on the x- and y-axis, respectively, is $\frac{x}{a} + \frac{y}{b} = 1$. • Normal form: The equation of the line having normal distance from origin p and angle between normal and the positive x - axis ω is given by $x \cos \omega + y \sin \omega = p$ • General Equation of a Line: Any equation of the form $Ax + By + C = 0$, with A and B are not zero, simultaneously, is called the general linear equation or general equation of a line. • Working Rule for reducing general form into the normal form: <ol style="list-style-type: none"> (i) Shift constant 'C' to the R.H.S. and get $Ax + By = -C$ (ii) If the R.H.S. is not positive, then make it positive by multiplying the whole equation by -1. (iii) Divide both sides of equation by $\sqrt{A^2 + B^2}$. <p>The equation so obtained is in the normal form.</p> <ul style="list-style-type: none"> • Parametric Equation (Symmetric Form): $\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r$ • Equation of a line through origin: $y = mx$ or $y = x \tan \theta$. • The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \frac{ Ax_1 + By_1 + C }{\sqrt{A^2 + B^2}}$ • Distance between the parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, is given by $d = \frac{ C_1 - C_2 }{\sqrt{A^2 + B^2}}$
7	CONIC SECTIONS	Sections of a cone: circles, ellipse, parabola, hyperbola. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle	Students will be able to find and identify standard forms of parabola, ellipse, hyperbola and circle equations and their curve sketching.

			<p>PARABOLA</p> <ul style="list-style-type: none"> • A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane. In geometric, Parabola is a locus of the point which moves so that its distance from a fixed point is equal to the distance from moving point to a fixed straight line. • Standard Equation: The equation of the parabola with focus at $(a, 0)$ $a > 0$ and directrix $x = -a$ is $y^2 = 4ax$.  <ul style="list-style-type: none"> • Focus: The given points are known as Focus. • Directrix: The fixed straight line is known as Directrix. • Axis: Any line passing through the focus and perpendicular to the directrix is known as the axis of parabola. • Vertex: The point of intersection of the axis and the parabola is known as Vertex. • Latus Rectum: Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola. • Length of the latus rectum of the parabola $y^2 = 4ax$ is $4a$. • Double Ordinate: A chord passing through P (any point on the parabola) and perpendicular to the axis of parabola is called the Double Ordinate through point P. • Focal Chord: Any chord passing through the focus is known as Focal Chord. • Four standard forms of Parabola: (i) $y^2 = 4ax$, (ii) $y^2 = -4ax$, (iii) $x^2 = 4ay$, (iv) $x^2 = -4ay$. <p>Ellipse , Hyperbola :</p> <p>Standard forms of equations, Major axis, Minor axis, length of Latus rectum, Eccentricity, Vertices, Directrices.</p> <p>Equation of circle in standard and general form its centre and radius.</p>
8	INTRODUCTION TO 3D GEOMETRY	Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.	Students will be able to use below formulae to solve problems:

			<ul style="list-style-type: none"> • Planes: The three planes determined by the pair of axes are the coordinate planes, called XY, YZ and ZX planes. xy-plane i.e., $z = 0$ yz-plane i.e., $x = 0$ zx-plane i.e., $y = 0$ • Octants: The three coordinate planes divide the space into eight parts known as octants. • Points in 3D: The coordinates of a point P in three dimensional geometry is always written in the form of triplet like (x, y, z). Here x, y and z are the distances from the YZ, ZX and XY Any point on XY \rightarrow plane (x, y, 0) Any point on YZ \rightarrow plane (0, y, z) Any point on ZX \rightarrow plane (x, 0, z) • Distance formula between two points: Distance between two points P (x₁, y₁, z₁) and Q (x₂, y₂, z₂) is $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ • Section Formula: The co-ordinates of R which divides a line segment joining the points P (x₁, y₁, z₁) and Q (x₂, y₂, z₂) internally and externally in the ratio m : n respectively Internally: $R\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$
9	LIMITS AND DERIVATIVES	Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions. Derivative introduced as rate of change both as that of distance function and geometrically. Definition of Derivative, relate it to slope of tangent of the curve, derivative of sum,	Students should have basics of the following to understand further concepts:

		<p>difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions</p>	<p>Derivatives</p> <p>The derivative of a function f at a is defined by</p> $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ <p>Derivative of a function f at any point x is defined by</p> $f'(x) = \frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ <p>For functions u and v the following holds:</p> $(u \pm v)' = u' \pm v'$ $(uv)' = u'v + uv' \quad \Rightarrow \quad \frac{d}{dx} (uv) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$ $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \quad \Rightarrow \quad \frac{d}{dx} \left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$
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			<p>Following are some of the standard derivatives</p> $\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\tan x) = \sec^2 x$ $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$ $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$
10	PROBABILITY	Random experiments; outcomes, sample spaces (set representation). Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events, Probability of an event, probability of 'not', 'and' and 'or' events	<p>Students should have the following basic concepts of probability:</p> <ol style="list-style-type: none"> 1. Coin: On tossing a coin there are two possibilities either head may come up or tail may come up. 2. Die: A die is a well balanced cube with its six faces marked with numbers (dots) from 1 to 6, one number on the one face. The plural of die is dice. 3. Cards: A pack of cards consists of four suits i.e., Spades, Hearts, Diamonds and Clubs. Each suit consists of 13 cards, nine cards numbered 2, 3, 4,, 10 and an Ace, a King, a Queen and a Jack or Knave. Colour of Spades and Clubs is black and that of Hearts and Diamonds is red. Ace, King, Queen and Jack cards are called Face cards. 4. Random Experiments : An experiment, whose outcomes cannot be predicted in advance is called a Random

			<p>experiment. For example, on tossing a coin, we cannot predict whether head will come up or tail will come up.</p> <p>5. Event : Every subset of a sample space is called an Event.</p> <p>6. Types of Events:</p> <ul style="list-style-type: none"> • Simple Event: Single element of the sample space is called a Simple event. It is denoted by S. • Compound Event: Compound event is the joint occurrence of two or more events. • Sure Event: In a sure event, a set of all the favorable outcomes is the sample event itself. Its probability is always 1. • Impossible Event: If E is an impossible event, then $S \cap E = \phi$ and the probability of impossible event is 0. • Equally Likely Events: Two events are said to be equally likely, if none of them is expected to occur in preference to the other. For example, if we toss a coin, each outcome head or tail is equally likely to occur. • Mutually Exclusive Event: Two events E_1 and E_2 are said to be mutually exclusive if $E_1 \cup E_2 = \phi$. On tossing a coin two events are possible, (i) coming up a head excludes coming of a tail, (ii) coming up a tail excludes coming of a head. Coming of a head and coming of a tail are mutually exclusive events. • Independent Events: Occurrence of one event does not depend on the occurrence of other. For example, on tossing two coins simultaneously occurrence of one toss does not depend upon the occurrence of the second one. • Exhaustive Events: Exhaustive events consist of all possible outcomes. • Complement of an Event: The complement of an event E with respect to the sample space S is the set of all elements of S, which are not in E. The compliment of E is denoted by E' or \bar{E}. • Probability of an event
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