ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2020

S4

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **1 September 2020 Tuesday**

2 hours

Marking Scheme

1 The roots of the quadratic equation $2x^2 - 6x + 1 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Find the value of $\dfrac{\alpha^2 + \beta^2}{\alpha\beta}$. [1]

(iii) Form a quadratic equation with roots $\dfrac{\alpha}{\beta} + 2$ and $\dfrac{\beta}{\alpha} + 2$. [3]

1(i)

$\alpha + \beta = -\left(\dfrac{-6}{2}\right) = 3$

$\alpha\beta = \dfrac{1}{2}$

M1 for both roots

$(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\alpha^2 + \beta^2 = (3)^2 - 2\left(\dfrac{1}{2}\right)$

M1

$\alpha^2 + \beta^2 = 8$

A1

(ii)

$\dfrac{\alpha^2 + \beta^2}{\alpha\beta} = 8 \div \dfrac{1}{2} = 16$

B1

(iii)

$\dfrac{\alpha}{\beta} + 2 + \dfrac{\beta}{\alpha} + 2 = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} + 4$

$\dfrac{\alpha}{\beta} + 2 + \dfrac{\beta}{\alpha} + 2 = 16 + 4 = 20$

M1 for sum of roots

$\left(\dfrac{\alpha}{\beta} + 2\right)\left(\dfrac{\beta}{\alpha} + 2\right) = 1 + \dfrac{2\alpha}{\beta} + \dfrac{2\beta}{\alpha} + 4$

$\left(\dfrac{\alpha}{\beta} + 2\right)\left(\dfrac{\beta}{\alpha} + 2\right) = 5 + \dfrac{2\alpha^2 + 2\beta^2}{\alpha\beta}$

$\left(\dfrac{\alpha}{\beta} + 2\right)\left(\dfrac{\beta}{\alpha} + 2\right) = 5 + 2(16) = 37$

M1 for product of roots

$x^2 - (20)x + 37 = 0$

$x^2 - 20x + 37 = 0$

A1

2	The mass, m grams, of a radioactive substance, present at time t days after first being observed, is given by the formula $m = 30\,e^{-0.025t}$. **(i)** Find the mass remaining after 30 days. [2] **(ii)** Find the number of days required for the mass to drop to half of its value at $t = 0$. Give your answer correct to the nearest integer. [2] **(iii)** State the value m approaches when t becomes large. [1]	
2(i)	$m = 30\,e^{-0.025(30)}$ $= 14.171$ $= 14.2$ The remaining mass after 30 days is 14.2g.	M1 A1 – 0 mark for omission of unit in answer
(ii)	$15 = 30\,e^{-0.025t}$ $e^{-0.025t} = \frac{1}{2}$ $-0.025t = \ln\frac{1}{2}$ $t = 27.726$ $t = 28$ The number of days required is 28 days.	B1 A1
(iii)	As $t \to \infty$, $30\,e^{-0.025t} \to \infty$, the value of m approaches 0.	A1

3

(a) Find, in ascending powers of x, the first three terms in the expansion of $(2-x)^7$. [2]

Hence, find the value of the constant a for which the coefficient of x^2 in the expansion of $(a-x)(2-x)^7$ is 616. [3]

(b) In the expansion of $\left(x^2-\frac{1}{2x^4}\right)^n$ in descending powers of x, the sixth term is independent of x. Find the value of n and the term independent of x. [4]

3(a)(i)

$(2-x)^7 = 2^7 - \binom{7}{1}(2^6)x + \binom{7}{2}(2^5)x^2 + \ldots$ M1

$= 128 - 448x + 672x^2 + \ldots$ A1

M1

$(a-x)(2-x)^7$

$= (a-x)(128 - 448x + 672x^2 + \ldots)$

(ii) $= \ldots + 672ax^2 + 448x^2 + \ldots$ M1

coefficient of x^2: $672a + 448 = 616$ A1

$a = \frac{1}{4}$

(b)

$\left(x^2-\frac{1}{2x^4}\right)^n$

$T_6 = \binom{n}{5}\left(x^2\right)^{n-5}\left(-\frac{1}{2x^4}\right)^5$ M1

$= \binom{n}{5}\left(x^{2n-10}\right)\left(-\frac{1}{2}\right)^5 x^{-20}$

$= \binom{n}{5}\left(x^{2n-30}\right)\left(-\frac{1}{2}\right)^5$ A1

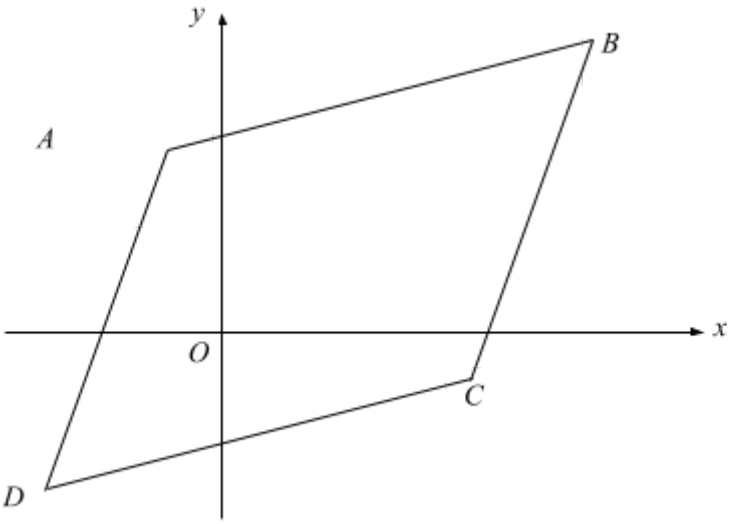
As it is independent of x

$2n - 30 = 0$ M1

$n = 15$

A1

	Value of the term $= \binom{15}{5}\left(-\frac{1}{2}\right)^5$ $= -\frac{3003}{32}$ or $-93\frac{27}{32}$	

4	**Solutions to this question by accurate drawing will not be accepted.**  The diagram shows a parallelogram $ABCD$ in which A (– 2, 6), B (6, 8) and C (4, – 1) are the coordinates of its vertices. Find the **(i)** equation of AD, [2] **(ii)** coordinates of D, [2] **(iii)** equation of the perpendicular bisector of the line AD, [2] **(iv)** area of the parallelogram $ABCD$, [2] **(v)** acute angle the line AB makes with the y-axis. [2]	
4(i)	Gradient of AD = Gradient of BC $= \frac{8+1}{6-4}$ $= \frac{9}{2}$ Equation of AD : $\frac{y-6}{x+2} = \frac{9}{2}$ $y = \frac{9}{2}x + 15$	M1 A1
(ii)	Coordinates of $D = (-2-2, 6-9)$ $= (-4, -3)$	M1 A1

(iii)	Gradient of Perpendicular Bisector $= -\frac{2}{9}$ Midpoint of $AD = \left(\frac{-2+(-4)}{2}, \frac{-3+6}{2}\right)$ $= \left(-3, \frac{3}{2}\right)$ Equation of Perpendicular Bisector $y - \frac{3}{2} = -\frac{2}{9}(x+3)$ $y = -\frac{2}{9}x + \frac{5}{6}$	M1 A1
(iv)	Area of $ABCD$ $= \frac{1}{2}\begin{vmatrix} -2 & -4 & 4 & 6 & -2 \\ 6 & -3 & -1 & 8 & 6 \end{vmatrix}$ $= \frac{1}{2}\left[(6+4+32+36)-(-24-12-6-16)\right]$ $= 68$ units^2	M1 clockwise and -68 (max 1 mark) A1
(v)	Angle made by line AB with y-axis $= \tan^{-1}\left(\frac{6+2}{8-6}\right)$ $= 76.0°$ (1 d.p.)	M1 A1 accept both degrees and radians

5	**(i)** Show that $\frac{d}{dx}\left(\tan^3 5x\right) = 15\sec^4 5x - 15\sec^2 5x$. [3] **(ii)** Use your answers to part **(i)**, find $\int \sec^4 5x \, dx$. [4]	
(i)	$\frac{d}{dx}\left(\tan^3 5x\right) = 3\left(\tan^2 5x\right)\left(\sec^2 5x\right)(5)$ $= 15\left(\tan^2 5x\right)\left(\sec^2 5x\right)$ $= 15\left(\sec^2 5x - 1\right)\left(\sec^2 5x\right)$ $= 15\sec^4 5x - 15\sec^2 5x$	M1 – power M1 – chain rule differentiate tan $5x$ M1

(ii)	$\frac{d}{dx}\left(\tan^3 5x\right) = 15\sec^4 5x - 15\sec^2 5x$ Integrate w.r.t. x $\tan^3 5x + C_1 = \int\left(15\sec^4 5x - 15\sec^2 5x\right) dx$ $\tan^3 5x + C_1 = 15\int \sec^4 5x\, dx - 15\int \sec^2 5x\, dx$ $\tan^3 5x + C_1 = 15\int \sec^4 5x\, dx - 15\left(\frac{\tan 5x}{5}\right)$ $\tan^3 5x + 3\tan 5x + C_1 = 15\int \sec^4 5x\, dx$ $\int \sec^4 5x\, dx = \frac{1}{15}\tan^3 5x + \frac{1}{5}\tan 5x + C_2$	M1 M1 M1 A1 If everything is C, minus 1 mark for overall presentation

6

(i) Given that $y = \dfrac{3x}{\sqrt{5-4x}}$, express $\dfrac{dy}{dx}$ in the form $\dfrac{ax+b}{\sqrt{(5-4x)^n}}$ where a, b and n are real constants. [4]

(ii) Hence find the equation of the normal to the curve $y = \dfrac{3x}{\sqrt{5-4x}}$ at the point on the curve where $x = 1$. [2]

6(i)

$$y = \frac{3x}{\sqrt{5-4x}}$$

$$\frac{dy}{dx} = \frac{\sqrt{5-4x}\,\frac{d}{dx}(3x) - (3x)\frac{d}{dx}\sqrt{5-4x}}{\left(\sqrt{5-4x}\right)^2}$$ M1 – Quotient Rule M1 – Chain Rule

$$= \frac{\sqrt{5-4x}\,(3) - (3x)\frac{1}{2}(5-4x)^{-\frac{1}{2}}(-4)}{5-4x}$$

$$= \frac{\sqrt{5-4x}\,(3) + \frac{(3x)2}{\sqrt{5-4x}}}{5-4x}$$

$$= \frac{\sqrt{5-4x}\,(3) + \frac{6x}{\sqrt{5-4x}}}{5-4x} \times \frac{\sqrt{5-4x}}{\sqrt{5-4x}}$$ M1

$$= \frac{(5-4x)(3)+6x}{(5-4x)\sqrt{5-4x}}$$

$$= \frac{15-6x}{\sqrt{(5-4x)^3}}$$ A1

(ii)

$x = 1,\ y = \dfrac{3(1)}{\sqrt{5-4(1)}} = 3$

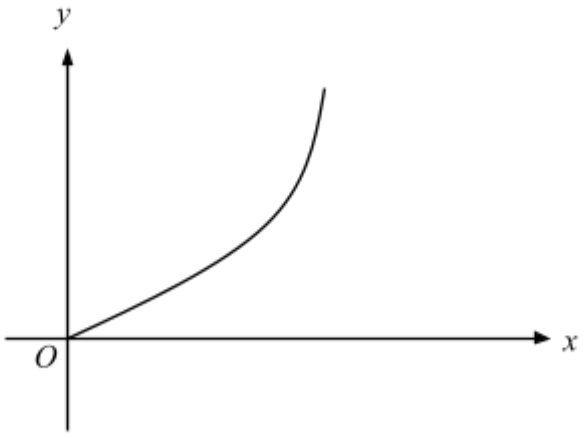
M1 – find gradient of tangent and normal

Gradient of the tangent $= \dfrac{15-6(1)}{\sqrt{(5-4(1))^3}}$

$= 9$

Gradient of the normal $= -\dfrac{1}{9}$

Equation of normal

	$y-3=-\frac{1}{9}(x-1)$ $y=-\frac{1}{9}x+\frac{28}{9}$ $9y+x=28$	A1
7	(i) On the same diagram, sketch the graphs of $y=16x^{\frac{5}{3}}$ and $y=\frac{9}{\sqrt[3]{x}}$ for $x>0$. [3] (ii) Find the $x-$ coordinate of the intersection point of the two graphs. . [2]	
		B1 – For shape of $y=\frac{9}{\sqrt[3]{x}}$ B1 – For asymptotes B1 – For shape of $y=16x^{\frac{5}{3}}$ and pass through origin
(ii)	$\frac{9}{\sqrt[3]{x}}=16x^{\frac{5}{3}}$ $\frac{9}{16}=x^{\frac{5}{3}}\times\sqrt[3]{x}$ $x^{\frac{5}{3}}\times x^{\frac{1}{3}}=\frac{9}{16}$ $x^2=\frac{9}{16}$ $x=\frac{3}{4}\quad(x>0)$	M1 A1

8 Given that $y = \text{cosec } x \tan x$,

(i) show that $\frac{dy}{dx} = \sin x \sec^2 x$. [2]

(ii) determine where y is decreasing for $0 \le x \le 2\pi$. [2]

8(i)

$y = \text{cosec } x \tan x$

$= \frac{1}{\sin x} \times \frac{\sin x}{\cos x}$

$= \frac{1}{\cos x}$

$= (\cos x)^{-1}$ M1

$\frac{dy}{dx} = (-1)(\cos x)^{-2}(-\sin x)$

$= \frac{1}{\cos^2 x}(\sin x)$ M1 – chain rule and differentiation of trigonometric functions

$= \sec^2 x \sin x$

Or

$y = \text{cosec } x \tan x$

$= \frac{1}{\sin x} \times \tan x$

$= (\sin x)^{-1} \tan x$

$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \sec^2 x + \tan x\left(-\sin^{-2} x \cos x\right)$

$= \frac{1}{\sin x} \cdot \sec^2 x - \frac{\sin x}{\cos x}\left(\frac{\cos x}{\sin^2 x}\right)$ M1

$= \frac{1}{\sin x} \cdot \sec^2 x - \frac{1}{\sin x}$

$\frac{dy}{dx} = \frac{\sec^2 x - 1}{\sin x}$

$= \frac{\tan^2 x}{\sin x}$

$= \frac{1}{\sin x}\left(\frac{\sin^2 x}{\cos^2 x}\right)$ M1

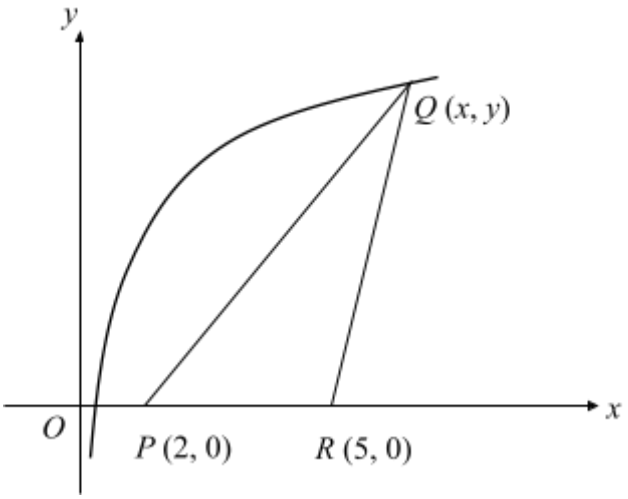
$= \frac{1}{\cos^2 x}(\sin x)$

$= \sin x \sec^2 x$

	Or $y = \cos ecx \tan x$ $= \frac{\tan x}{\sin x}$ $\frac{dy}{dx} = \frac{\sin x \sec^2 x - \tan x \cos x}{\sin^2 x}$ $= \frac{\sin x \sec^2 x - \sin x}{\sin^2 x}$ $= \frac{\sin x \left(\sec^2 x - 1\right)}{\sin^2 x}$ $= \frac{\sin x \left(\tan^2 x\right)}{\sin^2 x}$ $= \frac{1}{\sin x}\left(\frac{\sin^2 x}{\cos^2 x}\right)$ $= \frac{1}{\cos^2 x}(\sin x)$ $= \sin x \sec^2 x$	M1 M1
(ii)	For a decreasing function, $\frac{dy}{dx} < 0$ For $0 \le x \le 2\pi$, $\sec^2 x > 0,\ x \ne \frac{\pi}{2}, \frac{3\pi}{2}$, $\sin x < 0$ for $\pi < x < 2\pi$ Hence $\frac{dy}{dx} < 0$ for $\pi < x < 2\pi$, $x \ne \frac{3\pi}{2}$	B1 B1 - if didn't write $x \ne \frac{3\pi}{2}$, its acceptable.

9

The diagram shows the curve $y = \ln\left(x^2\sqrt{3x+1}\right)$ and three points $P(2,\ 0)$, $Q(x,\ y)$ and $R(5,\ 0)$. The point $Q(x,\ y)$ lies on the curve.



(i) Show that the area, A units^2, of the triangle PQR is given by

$$A = 3\ln x + \frac{3}{4}\ln(3x+1)$$

[2]

(ii) Given that x is increasing at a rate of 0.2 units/s, find the rate at which the area, A, is changing at the instant when $x = 15$ units. [3]

9(i)

$$A = \frac{1}{2}\times PR\times y = \frac{1}{2}\times 3\times \ln\left(x^2\sqrt{3x+1}\right)$$

$$= \frac{3}{2}\ln\left(x^2\sqrt{3x+1}\right)$$

$$A = \frac{3}{2}\left(\ln x^2 + \ln\sqrt{3x+1}\right)$$

$$= \frac{3}{2}\left(2\ln x + \ln(3x+1)^{\frac{1}{2}}\right)$$

$$= \frac{3}{2}\left(2\ln x + \frac{1}{2}\ln(3x+1)\right)$$

$$= 3\ln x + \frac{3}{4}\ln(3x+1)$$

M1

M1 – apply the laws for logarithms

(ii)

$$\frac{dx}{dt} = 0.2 \text{ m/s} \quad \text{and} \quad x = 15$$

$$A = 3\ln x + \frac{3}{4}\ln(3x+1)$$

M1 – differentiation of logarithms

	$\frac{dA}{dx} = 3\left(\frac{1}{x}\right) + \frac{3}{4}\left(\frac{1}{3x+1}\right)(3)$ $= \frac{3}{x} + \frac{9}{4(3x+1)}$ $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $= \left(\frac{3}{(15)} + \frac{9}{4(3(15)+1)}\right) \times (0.2)$ $= 0.0498 \text{ units}^2 / \text{s}$ The area is increasing at 0.0498 units^2/s.	M1 A1

10

H

EFGH is a plot of land that comprises two smaller plots, triangle *EFG* and triangle *EGH*. *EF* and *EH* are perpendicular, angle $FEG = \theta$, $EH = 42$ m, $EG = 55$ m and $EF = 48$ m.

(i) Show that the area, A m^2, of *EFGH* can be expressed as

$$A = 1320\sin\theta + 1155\cos\theta.$$

[2]

(ii) Express A in the form in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Find the value of θ if the area is 1231 m^2. [2]

10(i)

$$\text{Area of triangle } EFG = \frac{1}{2} \times EF \times EG \sin \angle FEG$$
$$= \frac{1}{2} \times 48 \times 55 \sin\theta$$
$$= 1320\sin\theta$$

M1 – area of triangle

$$\text{Area of triangle } EGH = \frac{1}{2} \times EH \times EG \sin \angle FEG$$
$$= \frac{1}{2} \times 42 \times 55 \sin(90° - \theta)$$
$$= 1155(\sin 90°\cos\theta - \cos 90°\sin\theta)$$
$$= 1155\cos\theta$$

M1 – use of trigonometry

$$A = \text{Area of triangle } EFG + \text{Area of triangle } EGH$$
$$= 1320\sin\theta + 1155\cos\theta$$

(ii)

$$1320\sin\theta + 1155\cos\theta = R\sin(\theta + \alpha) = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$$

$$R = \sqrt{1320^2 + 1155^2} = \sqrt{3076425}$$

$$\tan\alpha = \frac{1155}{1320}, \qquad \alpha = \tan^{-1}\left(\frac{1155}{1320}\right) = 41.1859°$$

M1 – finding R and α

$$A = \sqrt{3076425}\sin(\theta + 41.1859°)$$

A1

Or

$$A \approx 165\sqrt{113}\sin(\theta + 41.2°)$$

A1

(iii)	$\sqrt{3076425}\sin(\theta+41.1859°)=1231$ $\sin(\theta+41.1859°)=\dfrac{1231}{\sqrt{3076425}}$ $\text{Basic angle}=\sin^{-1}\left(\dfrac{1231}{\sqrt{3076425}}\right)$ $=44.57449°$ $\theta+41.1859°=44.57449°,\ 180-44.57449°$ $\theta=3.3885°,\ 94.2397°(\text{NA})$ $\theta=3.4°$	M1 A1

11	**(a)** The variables x and y are related in such a way that when $\frac{x}{y}$ is plotted against $\frac{1}{x}$, a straight line is obtained. The line passes through (2, 9) and (5, 3). Find an expression for y in terms of x. [4]	
11(i)	Let $Y = \frac{x}{y}, X = \frac{1}{x}$ Gradient = $\frac{9-3}{2-5} = -2$ $\therefore Y - 3 = -2(X - 5)$ $Y = -2X + 13$ $\frac{x}{y} = -\frac{2}{x} + 13$ $\frac{x}{y} = \frac{13x - 2}{x}$ $y = \frac{x^2}{13x - 2}$	M1 M1 M1 A1

(b) The table shows experimental values of two variables, x and y.

x	2	4	6	8
y	8.48	5.99	4.90	4.24

It is known that x and y are related by the equation $x^n y = k$, where n and k are constants. Draw a suitable straight line graph to represent the above data and use it to estimate the values of n and k. [6]

(ii)

$x^n y = k$

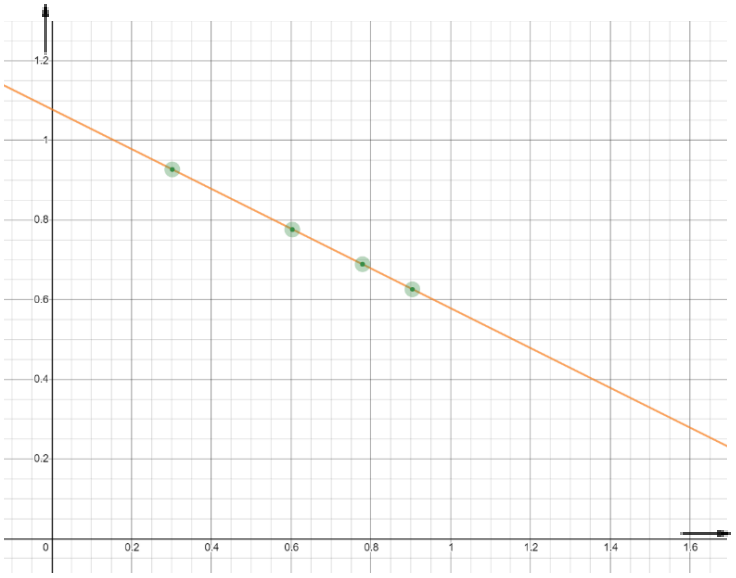
$\lg y = -n \lg x + \lg k$

M1

x	2	4	6	8
y	8.48	5.99	4.90	4.24
$\lg x$	0.301	0.602	0.778	0.903
$\lg y$	0.928	0.777	0.690	0.627

B1

$\lg y$



$\lg x$

G1 – correct points

G1 – y-intercept

Deduct 1 mark for labels

$\lg k = 1.08 \quad (1.06 - 1.10)$

$k = 12.0 \quad (11.5 - 12.6)$

A1

$-n = \dfrac{1.00 - 0.60}{0.15 - 0.95}$

$= -0.500$

$\therefore n = 0.500 \quad (0.45 - 0.55)$

A1

12	Solve the equation $3\text{cosec}^2 x \sin x = 5(\cos x + \sin x)$, giving the principal values of x, in radians. [5]	
	$3\text{cosec}^2 x \sin x = 5(\cos x + \sin x)$ $3\text{cosec}^2 x \sin x = 5\cos x + 5\sin x$ $3\text{cosec}^2 x = \dfrac{5\cos x + 5\sin x}{\sin x}$ $= \dfrac{5\cos x}{\sin x} + \dfrac{5\sin x}{\sin x}$ $= 5\cot x + 5$ $3\text{cosec}^2 x = 5\cot x + 5$ $3(\cot^2 x + 1) - 5\cot x - 5 = 0$ $3\cot^2 x + 3 - 5\cot x - 5 = 0$ $3\cot^2 x - 5\cot x - 2 = 0$ $(3\cot x + 1)(\cot x - 2) = 0$ $\cot x = -\dfrac{1}{3}$ or $\cot x = 2$ $\tan x = -3$ or $\tan x = \dfrac{1}{2}$ $x = -1.25$ or $x = 0.464$	M1 – use of $\cot x = \dfrac{\cos x}{\sin x}$, $\text{cosec}\, x = \dfrac{1}{\sin x}$, $\tan x = \dfrac{\sin x}{\cos x}$ etc M1 – use identity to change into an equation with single trigonometric term M1 – reach quadratic equation and factorize, or use formula A2 – deduct one mark if more than the principal values are given.