Chiral Transverse Electromagnetic Waves with $\mathbf{E} \parallel \mathbf{H}$ to study Circular Dichroism

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Abstract

It is shown that a general class of transverse electromagnetic waves with $\mathbf{E} \parallel \mathbf{H}$ can be obtained. These waves possess magnetic helicity and chirality. This condition is important to excitation of nano molecules when it is necessary consider a global factor as the product of the parameter of optical chirality with the inherent enantiometric properties of the material. The absorption of a chiral molecule in a chiral electromagnetic field is proportional to the imaginary part of mixed electric-magnetic dipole polarizability of the molecules, which determines the circular dichroism, CD of molecules. Chiral fields with different handedness can be used to obtain basic information from the interaction fields-molecules with high optical chirality, having chiral hot spots in nodes of stationary waves with parallel components of electric and magnetic fields.

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I. Introduction

Circular dichroism (CD), the difference in absorption of right and left circularly polarized light, has long been used to distinguish chiral objects (enantiomers) which lack a mirror symmetry in their point group [1]. Recently, a proposal by Tang and Cohen [2] has triggered an intense activity in understanding and enhancing such enantioselective signals for plane waves which are intrinsically weaker than non chiral signals [3-5] by a factor a/λ where a is a molecule size and λ the optical wavelength, this factor is typically 10^{-2} to 10^{-3}. The circular dichroism signal is defined as the difference of the signals from two circularly polarized plane waves, a LCPL (left circularly polarized light) and a RCPL (right CPL):

$$A_{CD} = A_L - A_R / (1/2)(A_L + A_R) \qquad (1)$$

with $A_L(\omega)$ and $A_R(\omega)$ being the absorption of left and right CPL at frequency ω respectively. Under some assumptions, Tang and Cohen have shown that A_{CD} can be factorized into a product of the chirality of the matter and the local chirality of the field

$$A_{CD} = (R / |\mathbf{p}|^2)(C / \omega U_e) . \qquad (2)$$

The material quantity that creates the CD signal depends on the rotatory strength $R = \mathrm{Im}(\mathbf{p}\cdot\mathbf{m})$ of the molecule, where $\mathbf{p}$ and $\mathbf{m}$ are the electric and magnetic dipoles respectively. U_e is the time average electric energy density and $\overline{C}$ is the optical field chirality

$$\overline{C} = -\frac{1}{8\pi}\mathrm{Im}(\mathbf{e}^{*}\cdot\mathbf{h}) \tag{3}$$

where $\mathbf{e}$, $\mathbf{h}$ are the complex vectors of real $\mathbf{E,H}$. This relationship has permitted the design of hyperchiral fields that amplify chiral signals beyond the a/λ factor that applies to plane waves [6]. However, this simple picture is limited to the electric and magnetic dipole approximation of matter which may be insufficient when working with fields that strongly vary across a molecule or a nanostructure. Moreover, optimizing the local field chirality $\overline{C}$ creates an enhanced chirality only at one point, where the electric field amplitude is small. to obtain superchirality near of $\mathbf{E}_{\min}$ we need to a field configuration with $\mathbf{E}\perp\mathbf{B}$ which is analyzed in next section working with $\mathbf{e}\perp\mathbf{h}$

II. Chiral Transverse Electromagnetic Waves with $\mathbf{e}\perp\mathbf{h}$

It is generally believed that in transverse electromagnetic waves electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ are always perpendicular to each other. In this section we show that, however, a general class of transverse electromagnetic waves with $\mathbf{E}\perp\mathbf{H}$ exists in a chiral media. We show how to obtain these waves in general and give example in vacuum and plasma . All these waves carry magnetic helicity. In a cold collision less chiro-plasma, the magnetostatic mode [7-9] of this class becomes the more familiar force-free field, $\nabla\times\mathbf{h} = \mathrm{k}\mathbf{h},\ \mathrm{k} = \sqrt{\mathrm{k_0}^{2} + \beta^{-2}}$, where β is the chiral parameter. We consider transverse electromagnetic waves in a uniform medium. These transverse waves can be described by considering $\mathbf{E} = \mathrm{Re}(\mathbf{e}\exp(\mathrm{i}\omega t))$, $\mathbf{H} = \mathrm{Re}(\mathbf{h}\exp(\mathrm{i}\omega t))$,

$$\mathbf{h} = \nabla\times\mathbf{F}, \tag{4}$$

$$\mathbf{e} = -\frac{1}{\mathrm{c}}\frac{\partial\mathbf{F}}{\partial t}, \tag{5}$$

in which the vector potential $\mathbf{F}$ satisfies $\nabla\cdot\mathbf{F} = 0$ and the wave equation

$$\nabla\times\nabla\times\mathbf{F} + \frac{1}{\mathrm{c}^2}\frac{\partial^2\mathbf{F}}{\partial t^2} = \frac{4\pi}{\mathrm{c}}(\mathbf{j} + \beta\nabla\times\mathbf{j}) \tag{6}$$

Here

$$\mathbf{j} = \overleftrightarrow{\sigma}\cdot\mathbf{e}, \tag{7}$$

where $\overleftrightarrow{\sigma}$ is the conductivity tensor operator of the medium under consideration, After Fourier analysis in time, we have

$$\nabla\times\nabla\times\mathbf{F}-\left(\omega^2/c^2\right)\overleftrightarrow{K}(\omega)\cdot\mathbf{F}=0 \tag{8}$$

with the dielectric tensor

$$\overleftrightarrow{K}(\omega)=\overleftrightarrow{I}-4\pi\overleftrightarrow{\sigma}(\omega)/i\omega. \tag{9}$$

For simplicity, we consider only cases where $\overleftrightarrow{K}(\omega)$ is independent of wavelength. we first look at the vacuum case. In vacuum $\sigma=0,\beta=0$ and Eq. (4) becomes

$$\left(\nabla^2+k_0^2\right)\mathbf{A}_k=0 \tag{10}$$

with $\omega^2=k_0^2c^2$. This waves equation allows the well known linear polarized plane waves with $\mathbf{e}\perp\mathbf{h}$ [10]. For every solution of Eq. (10), it is straightforward to show that

$$\mathbf{F}_k=\mathbf{A}_k+k_0^{-1}\nabla\times\mathbf{A}_k \tag{11}$$

satisfies not only Eq. (10) but also

$$\nabla\times\mathbf{F}_k=k\mathbf{F}_k. \tag{12}$$

For those vector potentials $\mathbf{F},$ we have $\beta k_0=1$ satisfying Eq. (11), the electric field $\mathbf{e}$ and magnetic field $\mathbf{h}$ are parallel to each other and both are perpendicular to the vector $\mathbf{k}_0$. Therefore, for every plane wave solution, a wave solution with $\mathbf{e}\rightarrow\mathbf{h}$ or $\mathbf{E}\rightarrow\mathbf{H}$ can be constructed. If we consider

$$\mathbf{k}=k_0(0,0,1), \tag{13}$$

$$\mathbf{F}=A(\sin k_0z,\cos k_0z,0)\cos\omega t \tag{14}$$

the electromagnetic field is given by

$$\mathbf{E}=(\omega A/c)(\sin k_0z,\cos k_0z,0)\sin\omega t, \tag{15}$$

$$\mathbf{B}=k_0A(\sin k_0z,\cos k_0z,0)\cos\omega t \tag{16}$$

This solution corresponds to two circularly polarized waves [10-13] propagating opposite to each other in such a way that their Poynting vectors are cancelled out. The time-averaged magnetic helicity density is related to the energy density ε by

$$\langle\mathbf{A}\cdot\mathbf{F}\rangle=\frac{2\pi}{k}\left\langle\left(\frac{E^2}{4\pi}+\frac{H^2}{4\pi}\right)\right\rangle=\frac{2\pi}{k}\varepsilon \tag{17}$$

Therefore, a single helical photon with energy $\hbar\omega$ carries a magnetic helicity of $\frac{hc}{}$.

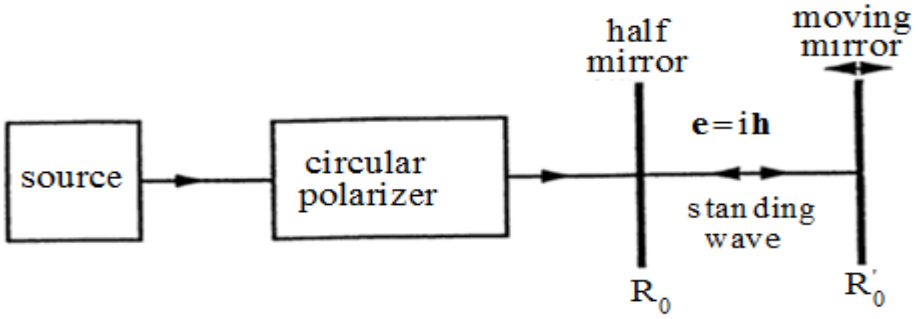


Figure 1. Schematic for the generation of parallel electromagnetic fields using two counter-propagating, orthogonally polarized light.

In figure 1. we show how can be obtained the condition $\mathbf{E} \perp \mathbf{H}$ with equations (15) and (16). $R_0, R_0^{'}$ are the reflectivity of mirror to obtain stationary condition with a minimum of electric field.

Figure 2, shows the standing wave of $\mathbf{E} = \mathrm{Re}(\mathbf{e}\exp(i\omega t))$ for different times $t_1 \quad t_2 \cdots$. Chiral absorption is obtained when the nano molecule is placed in a minimal electric field.

The main condition is obtained when $\beta k_0 = 1$.

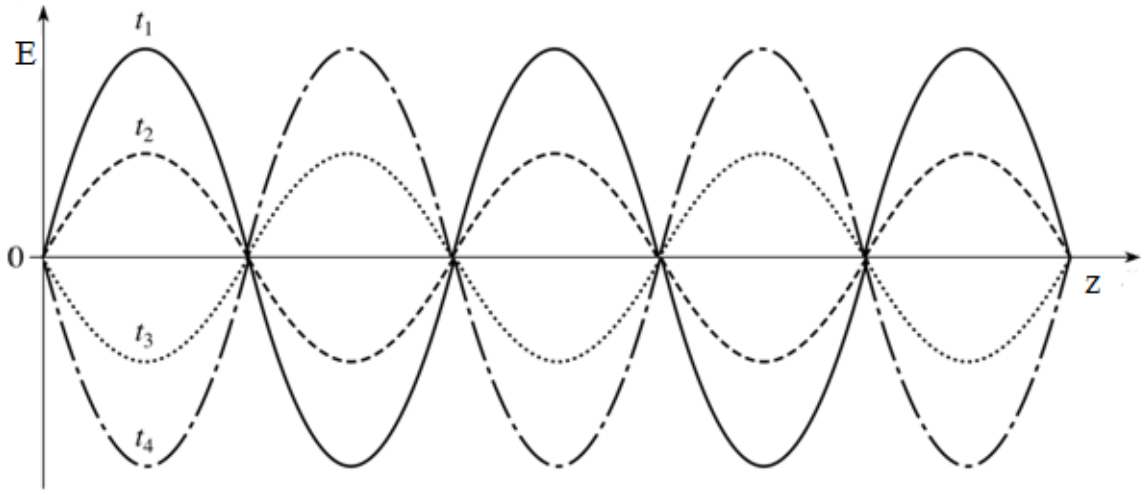


Figure 2, Illustrate the standing wave of $\mathbf{E} = \mathrm{Re}(\mathbf{e}\exp(i\omega t))$ for different times $t_1 \quad t_2 . t_3 .$

The main condition is obtained when $\beta k_0 = 1$, so $\mathbf{e} = i\mathbf{h}$.

In this case we have a stationary wave with a minimum which can be founded very close of the chiral sample with $\mathbf{E}$ co-linear with $\mathbf{H}$. Here, we have $\mathbf{e} = i\mathbf{h}$ (CGS units), the electric field is parallel to the magnetic field with a time-phase delay of 90° like [13] . The chirality density in complex notation (3), is

$$\overline{C} = -\frac{1}{8\pi}\mathrm{Im}(\mathbf{e}^* \cdot \mathbf{h}) = -\frac{1}{8\pi}\mathrm{Im}(\mathbf{e}^* \cdot (-i\mathbf{h})) = \frac{1}{8\pi}(\mathbf{e}^* \cdot \mathbf{e}) = \frac{1}{8\pi}|\mathbf{e}|^2 \qquad (18)$$

where vectors $\mathbf{e}$ and $\mathbf{h}$ are complex amplitudes of the electric and magnetic fields. In the vicinity of the electric nodes $kz = \pi(n+1/2),\ n = 0,1,2....$ .., one has pronounced resonances which can be interpreted as localized regions of energy.
In this situation we have true chirality even when the electric field is co-linear with the magnetic field. Here, we note that the condition imposed by Barron to have true chirality is not necessary. The effect of optical chirality was applied recently for experimental detection and characterization of biomolecules, [3, 5, 6] . This result is important to excitation of chiral molecules when it is necessary consider a global factor as the product of the parameter of optical chirality with the inherent enantiometric properties of the material. The approximate absorption of a chiral molecule in a chiral electromagnetic field is [2, 3]

$$A_{abs} \propto \frac{1}{8\pi} G'' \omega \, \mathrm{Im}(\mathbf{e}^* \cdot \mathbf{h}) \qquad (19)$$

where G'' is the imaginary part of mixed electric-magnetic dipole polarizability of the molecules, which determines the different absorption (circular dichroism, CD) of molecules to chiral fields with different handedness. The basic information obtained from equation (19) is that for fields with non-zero optical chirality, there should be parallel components of electric and magnetic fields having chiral “hot” spots.

III. Conclusion

Through our chiral approach described in section II. Superchirality was studied when electromagnetic field interacts with a chiral molecular sample. The field configuration obtained is a standing wave field with minimum of energy density in the vicinity of the chiral sample. True chirality was found even when the electric field is co-linear with the magnetic field noting that the condition imposed by Barron to have true chirality is not necessary. The main condition to obtain $\mathbf{e} \parallel \mathbf{h}$ or $\mathbf{E} \parallel \mathbf{H}$ is $\beta k_0 = 1$.

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