Whiteboard: 1/60, f5.6, iso 640, **WB Use Grey Card**, Manual Audio at level 11, focal length ½ way to ∞. H4n **INPUT** @ **95%** gain.

BBB 1/60, f5.6, iso 320, **WB Use Grey Card**, Manual Audio at level 11, focal length ½ way to ∞. H4n **INPUT** @ **95%** gain.

Title: AP Physics C: Momentum, Impulse and Center of Mass Review (Mechanics)

Mr.p: {AP Disclaimer} Good morning. Today we are going to review Momentum, Impulse, Collisions and Center of Mass as a part of the AP Physics C, mechanics curriculum. [intro] Bo, what is the equation for momentum?

- Bo: Momentum equals mass times velocity.
- Bobby: We should point out momentum is a lowercase p, Power is an uppercase P.
- Billy: And momentum and velocity are vectors.
- Bobby: And the units for momentum are kilogram meters per second.

- Billy: And the units have no special name, it's just kilogram meters per second. Unlike newtons which are kilograms meters per second squared.
- Bo: Sure.

Mr.p: [$\vec{p} = m\vec{v} \Rightarrow \frac{kg \cdot m}{s} \neq \frac{kg \cdot m}{s^2} = N$] Billy, what is newton's second law in terms of momentum?

- Billy: The sum of the forces acting on an object or system of objects equals the derivative of the momentum of that object or system of objects with respect to time.
- Bo: Don't forget force and momentum are vectors.
- Billy: Sure.

Mr.p: [$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(m\vec{v}\right)$] Because momentum equals mass times velocity, we can use the product rule to get [$= \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt} = \frac{dm}{dt}\vec{v} + m\vec{a}$] The net force equals the derivative of mass with respect to time multiplied by velocity plus mass times the derivative of velocity with respect to time. The derivative of velocity with respect to time is acceleration. Therefore, Bobby, in the past when we have used net force equals mass times acceleration as newton's second law, what were we assuming?

- Bobby: Well, that would mean the derivative of mass with respect to time was zero.

- Mr.p: (right, but what does that mean about the object?)
- Bobby: Oh, that means the mass of the object is not changing. I get it. Net force equals mass times acceleration assumes the mass of the object is constant.

Mr.p: [$\Rightarrow \sum \vec{F} = m\vec{a}$ assumes constant mass] Billy, when can we use conservation of momentum?

- Billy: We can use conservation of momentum when all the forces are internal to the system.
- Mr.p: (That is correct. Can you prove it?)
- Billy: Sure. … If all the forces are internal to the system, then the net force acting on the system

equals zero and the derivative of the momentum of the system with respect to time equals zero. And if the derivative of the momentum of the system with respect to time equals zero, then the momentum of the system is not changing as a function of time so the sum of the initial momenta equals the sum of the final momenta.

- Bo: It's also useful to realize momentum is conserved during all collisions and explosions.
- Bobby: Because all the forces are internal to the system during collisions and explosions.

Mr.p: [$\sum \vec{F} = \frac{d\vec{p}}{dt} = 0 \Rightarrow \sum \vec{p}_i = \sum \vec{p}_f$ all forces are internal to the system. True during collisions and explosions.]

Now let’s derive the equation for impulse, which is often called the impulse-momentum theorem. [$\sum \vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \sum \vec{F} dt = dp$] We start by looking a the net force acting on an object and using the equation for newton’s second law we were just working with, we can rearrange it to get the net force times dt equals dp and then take the definite integral of the whole equation. [$\Rightarrow \int_{t_i}^{t_f} \sum \vec{F}\, dt = \int_{p_i}^{p_f} dp$] The limits when taking the definite integral with respect to time are from time initial to time final and the limits when taking the definite integral with respect to momentum are from momentum initial to momentum final. [$\vec{p}_f - \vec{p}_i \Rightarrow \Delta\vec{p} = \int_{t_i}^{t_f} \sum \vec{F}\,dt = \vec{J}$] The definite integral of 1 with respect to momentum is momentum final minus momentum initial which is change in momentum. In other words, the change in momentum of the object equals the definite integral with respect to time of the net force acting on the object which is called the Impulse acting on the object. The symbol is a capital J and, just like momentum, impulse is a vector. [Vector!] Because impulse is force times time, the units for impulse are Newton seconds. And yes, Billy, the units for impulse, Newton seconds, are the same as the units for momentum: kilograms meters per second.

{hear Billy} I know.

Billy: (Mr. p) (I was going to ask that.)

Mr.p: [$\vec{J} = \int_{t_i}^{t_f} \sum \vec{F}\, dt \Rightarrow N \cdot s = \left(\frac{kg \cdot m}{s^2} \right) s = \frac{kg \cdot m}{s}$] Remember, integrals are the area "under" a curve, therefore Impulse is the area between a net force as a function of time curve and the horizontal time axis. Where the area above the time axis is positive and the area below the time axis is negative. Also, [$W = \int_{x_i}^{x_f} F_x\, dx$] do not confuse impulse with work, which is the definite integral of force with respect to *position* not *time*. The two equations are very close to one another. … Bo, what is the "impulse approximation"?

Bo: The impulse approximation is where we can say the force of impact during the collision is so large that it overshadows all other forces and therefore the net force is approximately equal to the force of impact during the collision. That is true with large forces of impact which usually last for a short time interval.

Mr.p: Absolutely. [Impulse approximation: $\sum \vec{F} \approx \vec{F}_{impact} \Rightarrow \vec{F}_{impact} = \frac{d\vec{p}}{dt}$] Sadly, Impact force and Impulse also often get confused. They both start with Imp, perhaps that's why. [$\vec{J} = \vec{F}_{average}\Delta t$] If we can consider the force of impact to be constant or we have the average impact force, the impulse is equal to the average force of impact times the change in time during the collision. This creates a rectangle with the same area as the definite integral impulse equation. [Rectangle with same area as $\vec{J} = \int_{t_i}^{t_f} \sum \vec{F}\, dt$] Now let's review the types of collisions. [draw table] Bobby, please explain this table.

- Bobby: This table lists two types of collisions, elastic and inelastic. Momentum is conserved during all types of collisions, so the difference between elastic and inelastic collisions is whether kinetic energy is conserved. And … kinetic energy *is* conserved during *elastic* collisions and kinetic energy is *not* conserved during *inelastic* collisions.

- Billy: What about *perfectly inelastic* collisions?

Mr.p: [Perfectly Inelastic → Stick. Also, “completely” or “totally” inelastic] Yes. A “perfectly” inelastic collision is just an inelastic collision where the two objects stick to one another afterwards rather than bouncing off of one another. I’ve also seen a perfectly inelastic collision referred to as a completely or totally inelastic collision. [“hard sphere” collisions are “nearly” elastic & most collisions are inelastic] Be aware that collisions between “hard spheres” are generally considered to be *elastic* in physics classes. And, most real world collisions are actually *inelastic*, where kinetic energy is converted to heat and sound during the collision. Next let's talk about center of mass. The position of the center of mass of a system of particles equals [System of Particles, Center of Mass: $x_{cm}=\frac{\sum m_i x_i}{\sum m_i}$] the sum of the mass of each particle times the location of each particle all divided by the sum of the masses of the particles. [$=\frac{m_1 x_1+m_2 x_2+\ldots}{m_1+m_2+\ldots}$] In other words, if we are solving for the center of mass of the system of particles in the x direction, it equals the mass of object one times the x position of object one relative to a zero-reference line which is often the origin, plus the mass of object two times the x positon of object two relative to the zero-reference line. Repeat that for however many objects are in the system, all divided by the total mass of the system of particles. [$v_{cm} = \frac{dx_{cm}}{dt} = \frac{d}{dt}\left(\frac{\sum m_i x_i}{\sum m_i}\right)$] The velocity of the center of mass of a system of particles is the derivative of the position of the center of mass of the system of particles and because the masses of the objects do not change as a function of time and the derivative of position with respect to time is velocity, [$= \frac{\sum m_i}{\sum m_i}\frac{dx_i}{dt} = \frac{\sum m_i v_i}{\sum m_i}$] the equation for the velocity of the center of mass of a system of particles is almost identical to the equation for the position of the center of mass of a system of particles except position is replaced with velocity. … And we can do the same thing to determine the acceleration of the center of mass of a system of particles. [$a_{cm} = \frac{dv_{cm}}{dt} = \frac{d}{dt}\left(\frac{\sum m_i v_i}{\sum m_i}\right) = \frac{\sum m_i a_i}{\sum m_i}$] Because the derivative of velocity with respect to time is acceleration the equation for the acceleration of the center of mass of a system of particles is almost identical to the equation for the velocity of the center of mass of a system of particles except velocity is replaced with acceleration. Realize the equations for the velocity and acceleration of the center of mass of a system of particles are not on the AP Equation sheet, however, they are easy to derive and memorize. [Is on ES, Not on ES] Now let's talk about finding the center of mass of a rigid system of many particles or what is more commonly called an object with shape. [$r_{cm} = \frac{1}{m_{total}} \int r\,dm$] The position of the center of mass of an object with shape equals one over the total mass of the object times the integral with respect to mass of the position of all of the infinitesimally small pieces of the object, which are called dm, relative to a zero-reference line.

- Billy: Could you please say that again?
- Bo: Yes.

- Bobby: Yeah. And what is “r”?

Mr.p: Sure. I’ll start with “r”. Just like the position vector “r” we used in kinematics, this r represents the generic position of the object, it could be in 2 or 3 dimensions, often it is just in one dimension and we can replace it with a direction we are more familiar with, like x. [$x_{cm} = \frac{1}{m_{total}} \int x\, dm$] So in this case “x” is the same thing it was in equation for the center of mass of a system of particles, only instead of having definite particles, we take the object and look at infinitesimally small pieces of the object which we call dm. And because these pieces dm are infinitesimally small, we need an infinite number of them to add up to the object.

Bobby: That's right. That's what an integral is.

- Mr.p: Exactly. That's what an integral is. And this particular integral is not on your AP equation sheet. I don't know why. It's just not there.
- Bo: (that's not helpful.)
- Mr.p: I know. I know. Okay, on to more things which are not on the AP equation sheet.

Mr.p: [Volumetric Mass Density: $\rho = \frac{m}{\forall}$] Volumetric mass density equals the mass of the object divided by its volume. The symbol is the lowercase Greek letter Rho and just so you know, I put a slash through my V for volume to distinguish Volume from velocity. [Surface mass density: $\sigma = \frac{m}{A}$] Surface mass density equals the mass of the object divided by its surface area. The symbol is the lowercase Greek letter sigma. [Linear mass density: $\lambda = \frac{m}{L}$] And linear mass density equals the mass of the object divided by its length. The symbol is the lowercase Greek letter lambda.

- Bobby: Did he say none of those are on the equation sheet?
- Billy: Yep.
- Bo: Oh boy.

Mr.p: These equations for density tend to be helpful for determining the center of mass of objects with shape. Although, sigma, the surface mass density is rarely used in AP Physics C mechanics. I just thought I would throw it in the mix because surface charge density gets used in AP Physics C Electricity and Magnetism and y’all should start getting used to it.

BBB: Thanks. Thank you? Sure.

Mr.p: That completes my review of momentum, impulse, collisions and center of mass; next, feel free to enjoy my AP Physics C review of rotational kinematics or visit my AP Physics C review webpage. Thank you very much for learning with me today, I enjoyed learning with you.