Equivalency: Misconceptions and Instructional Strategies

Key Concepts

Numerical expressions are equivalent when they produce the same result, and an equal sign is a symbol denoting that the two expressions are equivalent.

Numerical expressions are not equivalent when they do not produce the same result, and an equal sign with a slash through it (≠) is a symbol denoting that the two expressions are not equivalent.

When using a balance model, the representations of the addition or subtraction expressions are manipulated until there is an identical representation on both sides of the balance. When using a balance scale, the objects on the scale are manipulated until the scale is level.

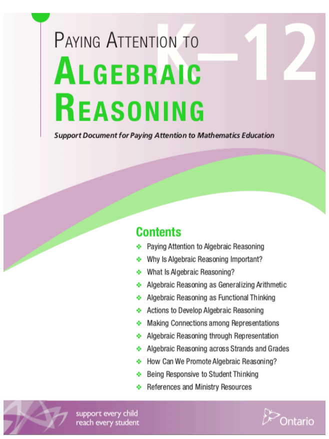
When numbers are decomposed, the sum of the parts is equivalent to the whole.

The same whole can result from different parts.

Many mathematical concepts are based on an underlying principle of equivalency.

The commutative and associative properties of addition are founded on equivalency: the commutative property (e.g., 1 + 2 = 2 + 1) / the associative property: (e.g., (8 + 9) + 2 is the same as 8 + (9 + 2)

Paying Attention to Algebraic Reasoning



Algebraic reasoning permeates all of mathematics and is about describing patterns of relationships among quantities – as opposed to arithmetic, which is carrying out calculations with known quantities. In its broadest sense, algebraic reasoning is about generalizing mathematical ideas and identifying mathematical structures. (pg. 3)

It has been well documented that students do not recognize that the equal sign denotes equality...Most students see the equal sign as a signal to do something - to carry out a calculation and put the answer after the equal sign. (pg. 6)

When students work with equations, it is imperative that they understand that the equal sign represents a relation between quantities...Students who develop this understanding can compare without having to carry out the calculations. They can focus on the equivalence..(algebraic reasoning) rather than comparing...answers (arithmetic reasoning). (pg. 7)

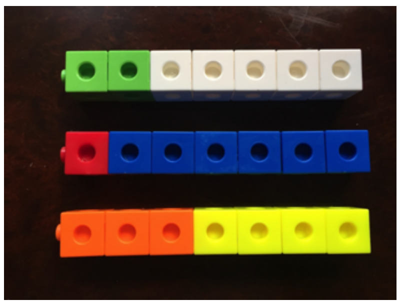
Common Errors and Misconceptions	Suggested Strategies
Students often incorrectly **interpret the equality sign**.	Students need to participate in conversations about what the equal sign means; that is, both sides of the equation represent different names for the same amount. It also helps if students see a variety of equation types. For example: 8+7 = 15 15+7 = 8 6+9 = 10+5 (Making Math Meaningful, M.Small, p.386) When reading equations to students use the language "is the same as" when reading the equal sign. This reinforces that this is a relationship sign rather than a sign signaling an operation. (Teaching Student Centered Mathematics,Van de Walle, p.291)
Balance Scale: The most commonly employed model for algebra, used to develop an understanding of equations, is the balance pan. But the use of this model can be complicated by the fact that children do not necessarily have a deep enough understanding of conservation of weight for this model to make sense (Piaget 1954). - Fosnot "Trades, Jumps, and Stops"	 Helping students understand the idea of equivalence can and must be developed concretely. Kinesthetic approaches, tactile objects, and visualizations can reinforce the "balancing" notion of the equal sign.

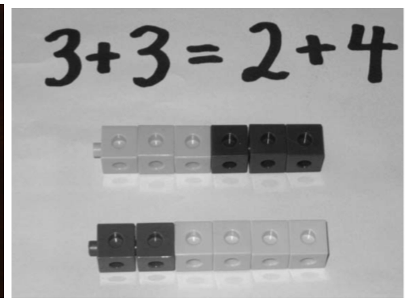
Instructional Strategies for Teaching Equivalency

(From "Guide to Effective Instruction in Mathematics (Patterning and Algebra)

GRADE 1:

- providing opportunities to explore concepts related to equality and inequality. For example, students might investigate how adding the same quantity to (or removing the same quantity from) both sides of a balance maintains equality
- providing many experiences in demonstrating equality, using concrete materials (e.g., create different number combinations for 8, such as 1 and 7, 2 and 6, 4 and 4, and so on, using two colours of interlocking cubes)





- providing experiences in exploring the concept of 0 (e.g., identify objects of which there are 0 in the classroom; take away objects from a set to show the concept of 0)
- prompting students to make generalizations about number properties (e.g., "If you add 0 to a number, the number stays the same")
- providing opportunities to view and discuss equations written in a variety of formats (e.g., 3=3, 4=3+1, 4+1=5).

GRADE 2:

- providing opportunities to explore concepts related to equality and inequality. For example, students might investigate how adding the same quantity to (or removing the same quantity from) both sides of a balance maintains equality
- providing many experiences in demonstrating equality, using concrete materials (e.g., show that a combination of 18 counters and 2 counters is equivalent to a combination of 13 counters and 7 counters)
- providing opportunities to discuss the meaning of equations (e.g., have students explain whether given number sentences, such as 2+3=5+1 and 7–7=0+7, are true or false)
- providing opportunities to view and discuss equations written in a variety of formats (e.g., 8=5+3, 4+2=2+4, 12+3+3=15+3)
- discussing and demonstrating number properties (e.g., use counters to demonstrate the commutative property of addition)
- providing opportunities to solve problems involving unknown quantities in addition and subtraction situations
- providing opportunities to find the missing number in equations involving addition and subtraction to 18 (e.g., find the missing number in 6+3=4+□, using counters).

GRADE 3:

- providing many experiences in demonstrating equality, using concrete materials (e.g., show that a 3x4 array of tiles is equal to a 2x6 array of tiles)
- providing opportunities to view and discuss equations written in a variety of formats (e.g., 19=12+7, 41+2=2+41, 4+23=4+20+3)
- having them generate a variety of equations for a given number (e.g., 8=2+2+2+2, 8=4 x 2, 8=2 x 4, 8=8 x 1, and so on)
- discussing and demonstrating number properties (e.g., use counters to demonstrate the associative property of addition)
- providing experiences in using the associative property to facilitate mental computations with whole numbers (e.g., to find 5+4+16, add 4+16 to get 20, and then add 5+20)
- providing many experiences in exploring the properties of 0 and 1 in multi- plication (e.g., use counters to show that any number multiplied by 0 equals 0; use a calculator to show that any number multiplied by 1 equals the original number)
- providing opportunities to solve problems involving unknown quantities in addition and subtraction situations
- providing opportunities to find the missing number in equations involving the addition and subtraction of one- and two-digit numbers (e.g., find the missing number in 26 – 3=12+□, using concrete materials).

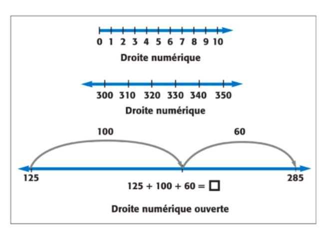
Ontario Mathematics Glossary:

https://www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics/context/glossary#D

Vocabulary in this Unit:
Equal
Equality
Equivalent
Array
Equation
Relationship
Balance
Combination
Double Number Line
Open Number Line

Mathematical Models:

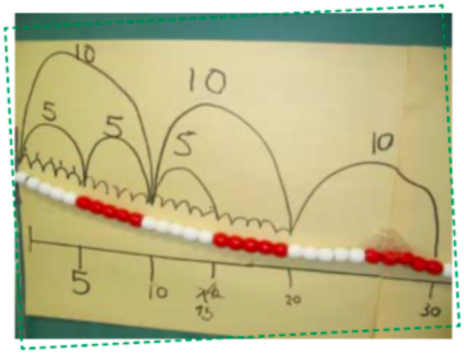
Using the open number line as a model for algebra:

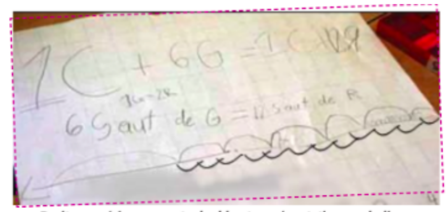


- This model is a powerful tool in developing a deep understanding of equations. It supports children in exploring associativity and commutativity and in constructing rules for simplifying equations (such as cancellation and substitution). In upper grades the model helps students develop strategies for solving simultaneous equations with two unknowns.

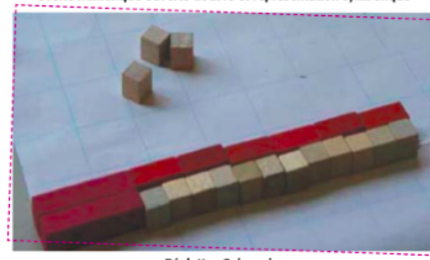
- Using connecting cubes to explore equivalence (model of the situation)

- Double number line (model of children's strategies)





Droite numérique ouverte double et représentation symbolique



Réglettes Cuisenaire