Roll No…………………….. **Total No. of Printed Pages: [01]**

Total No. of Questions: [09]

B. Sc. (Hons.) Maths (Semester – 4th)

DIFFERENTIAL EQUATIONS-II

Subject Code: BMATS1-421

Paper ID: [22131215]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Define the order and degree of the partial differential equation with an example.

b) Formulate a partial differential equation by eliminating the constants a and b from $2z = (ax + y)^2 + b$.

c) Show that the equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible.

d) Classify the partial differential equation $xyr - \left(x^2 - y^2\right)s - xyt + py - qx = 2\left(x^2 - y^2\right)$.

e) Find the complementary function of $\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4} = e^{x-y}$.

f) Solve the partial differential equation $r - 2s + t = \sin(2x + 3y)$.

g) Find the P.I. of $(D^3 - D'^3)z = x^3 y^3$.

h) Solve the partial differential equation $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$.

i) Write the Laplace equation and all its possible solutions.

j) By using the method of separation of variables, solve the differential equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x,0) = 6e^{-3x}$.

Section – B **(5 marks each)**

Q2. Find the integral surface of the partial differential equation $(x - y)y^2 p + (y - x)x^2 q = (x^2 + y^2)z$ passing through the curve $xz = a^3, y = 0$.

Q3. Use Charpit's method to find the complete integral of the partial differential equation $2(z + xp + yq) = yp^2$.

Q4. Solve the partial differential equation $(D - 3D' - 2)^2 z = 2e^{2x} \tan(3x + y)$.

Q5. Solve the partial differential equation $x^2 r + 2xys + y^2 t = 0$ by using Monge's method.

Q6. Derive the solution of one dimensional diffusion equation by method of separation of variables.

Section – C **(10 marks each)**

Q7. Find the surface which intersects the surface of the system $z(x+y)=c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1$, $z=1$.

Q8. Solve the partial differential equation $p^2x+q^2y=z$ by Jacobi's method.

Q9. A tightly stretched flexible string has its ends fixed at $x=0$ and $x=l$. At time $t=0$, the string is given a shape defined by $F(x)=\mu x(l-x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at any time $t>0$.