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B. Sc. (Hons.) Maths (Semester – 4th)

DIFFERENTIAL EQUATIONS-II

Subject Code: BMATS1-421

Paper ID: [22131215]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Define the order and degree of the partial differential equation with an example.

b) Formulate a partial differential equation by eliminating the constants a and b from $2z=(ax+y)^2+b$.

c) Show that the equations $xp=yq$ and $z(xp+yq)=2xy$ are compatible.

d) Classify the partial differential equation $xyr-\left(x^2-y^2\right)s-xyt+py-qx=2\left(x^2-y^2\right)$.

e) Find the complementary function of $\frac{\partial^4 z}{\partial x^4}-\frac{\partial^4 z}{\partial y^4}=e^{x-y}$.

f) Solve the partial differential equation $r-2s+t=\sin(2x+3y)$.

g) Find the P.I. of $(D^3-D'^3)z=x^3y^3$.

h) Solve the partial differential equation $x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=0$.

i) Write the Laplace equation and all its possible solutions.

j) By using the method of separation of variables, solve the differential equation $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$, where $u(x,0)=6e^{-3x}$.

Section – B **(5 marks each)**

Q2. Find the integral surface of the partial differential equation $(x-y)y^2\,p+(y-x)x^2\,q=(x^2+y^2)z$ passing through the curve $xz=a^3, y=0$.

Q3. Use Charpit's method to find the complete integral of the partial differential equation $2(z+xp+yq)=yp^2$.

Q4. Solve the partial differential equation $(D-3D'-2)^2 z=2e^{2x}\tan(3x+y)$.

Q5. Solve the partial differential equation $x^2r+2xys+y^2t=0$ by using Monge's method.

Q6. Derive the solution of one dimensional diffusion equation by method of separation of variables.

Section – C **(10 marks each)**

Q7. Find the surface which intersects the surface of the system $z(x+y)=c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1$, $z=1$.

Q8. Solve the partial differential equation $p^2x+q^2y=z$ by Jacobi's method.

Q9. A tightly stretched flexible string has its ends fixed at $x=0$ and $x=l$. At time $t=0$, the string is given a shape defined by $F(x)=\mu x(l-x)$, where μ is a constant, and then released. Find the displacement of any point *x* of the string at any time $t>0$.