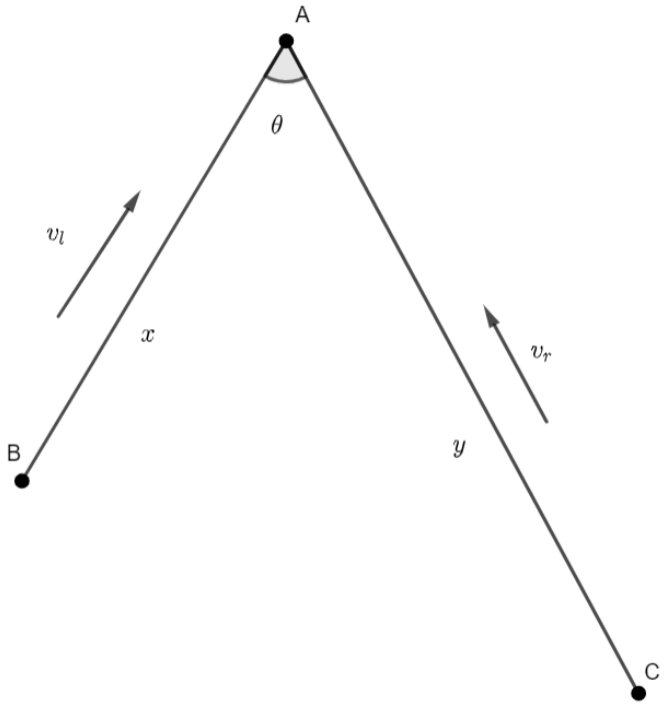
Renovating a Castle - Capstone Project Finale

Things are almost finished in the castle. There are just a few details that need to be addressed, beginning with the mechanism in the clock tower. Once these are done and the paint is applied and dried, the contract is finished and the castle is yours!

Brown Brick Building Under Blue Sky by Splendid Photography licensed with Unsplash Open License found at https://unsplash.com/photos/lGOgrP7G734

(a) In the clocktower, the clock has a baroque mechanism that you have never seen before that activates on the hour. Two weights (B and C) move along tracks to a fixed point (A). The two tracks make a fixed angle of θ. The velocities are constant and given by v_l for the left track and v_r for the right track. Find the distance between the points B and C when B is x cm from the point A and C is y cm from the point A. ***(LO 4.5.3)***



(b) For the mechanism from part (a), if $\theta = \pi/3$, what is the rate at which the distance between points B and C is changing if $v_l = 3\ cm/s$, $v_r = 5\ cm/s$, $x = 10\ cm$, and $y = 15\ cm$? ***(LO 4.5.3)***

Baroque Mechanism, created by William Lindsey, CC-BY

(c) If θ is smaller than $\pi/3$ but the other parameters from part (b) are the same, would you expect the rate of change of the distance to be smaller or larger? Explain your answer. ***(LO 4.5.3)***

(d) One of the plates in the clock has a shape that takes the form of one petal of the polar curve $r = 4\cos 3\theta$. To line up the mechanism, you need to know where to balance the plate. Assuming the density is constant, find the center of mass of the plate. ***(LO 5.3.2, 5.3.3, 5.3.4)***

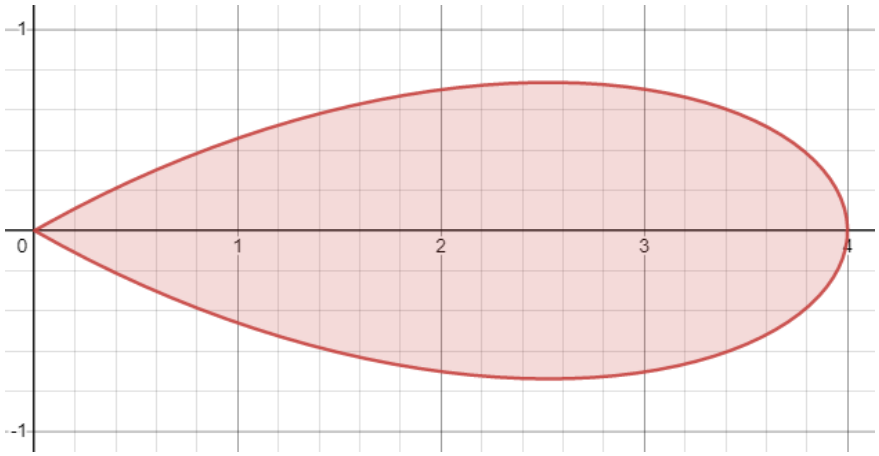


Plate in Clock, created by William Lindsey, CC-BY

(e) There is a portion of the clock tower that needs to be resurfaced. The shape looks like a cone where the flat surface was curved in a particular way. One model for this is the portion of the cone $z^2 = x^2 + y^2$ with $z \geq 0$, but only the portion of the cone that fits between the planes $z = 1$ and $z = 4$, with the distances being measured in meters. Unfortunately, the resurfacing material is quite expensive, so an accurate measure of the surface area is key. ***(LO 6.6.1, 6.6.3, 6.6.6)***

Part a: Find parameterizations for the curved part of the surface. (See Example Calculating Surface Area in the textbook for an example on how to start.)

Part b: Write down the surface integral to find the area for the curved part of the surface.

Part c: Resolve the integral. Find the three areas.

Part d: If the resurfacing material is $1500 per square meter and the curved portion needs two coats, how much would the resurfacing cost?

Conclusion

With the last few drops of paint and curtains hung, the castle is now in pristine condition. There were a lot of challenges along the way, but you can be proud of the work you put in. The people from the nearby town are ecstatic for the tourism that they are sure the castle will bring in. You have your final inspection from the agency and the contract was deemed fulfilled. Now the only question that is left is to figure out who this relative was that left you the castle!