Aleph-3 Middle School Problem-Solving Round

Do not open the test until instructed to do so.

Information:

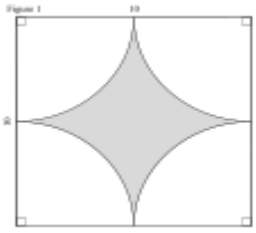
- The Problem-Solving Round contains 10 questions for which you will be given 40 minutes to answer.
- There will be an 11th estimation question that will be used as a tiebreaker.
- There is no penalty for guessing, so make sure to give an answer to any of the questions, even if you are unsure.
- This test will be used along with the Speed Round to decide who is seeded in the buzzer round. Each question is worth 3 points counting towards this score.
- Do not expect to be able to solve every question. Likewise, do not be afraid to skip questions. The questions are only roughly, but not strictly, in order of difficulty.
- Answers **do not** need to be integers. They can be numbers, words, coordinate points, or even dates.

Instructions:

- When starting the test, it is suggested to rip off the front page that contains the answer sheet.
- On the answer sheet, make sure to write your name, ID, and school.
- Only answers on the answer sheet will be graded, not answers in the test booklet.
- Answer with fractions reduced to simplest terms and with radicals having no squares in them.
- Units are not needed in the final answer, but make sure the number itself is in the correct units that is specified in the question.
- Good luck and have fun! If you have any questions, raise your hand.

Middle School Problem-Solving Round

Aleph-3



1. Given that Figure 1 depicts a square of side length 10 and four circular arcs centered at its vertices, what is the area of the shaded region?

2. Mary says that she is thinking of two prime numbers that sum up to 55. What is the absolute difference between these two prime numbers?

3. A die is weighted so that the number on each face is proportional to the probability of rolling that number (e.g. Rolling a 3 is three times as likely as rolling a 1. A 6 is two times as likely as rolling a 3). What is the probability of rolling a prime number?

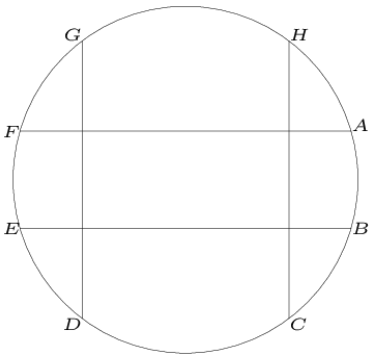
4. There are some number of cardinals and goldfinches sitting on a branch. When 14 goldfinches and 3 cardinals land on the branch, 2/5 of the birds are cardinals. After 6 cardinals leave and 16 goldfinches arrive, 5/7 of the birds are now goldfinches. How many cardinals were there originally?

5. $\frac{\sqrt[3]{x^{10}}+\sqrt[3]{x}}{\sqrt[3]{x}} = 9$. Find x.

6. What is the smallest positive integer N which the following relation holds true: $\frac{1}{d_1} + \frac{1}{d_2} + \ \ldots\ \frac{1}{d_n} \ = \ 2$ where $d_1, d_2, d_3 \ldots d_n$ are all of the positive divisors of N, including N?

7. How many positive integers less than 1001 are divisible by either 3,5, or 8?

8. Define $a \bmod b$ to give the remainder when a is divided by b. For example, $5 \bmod 3 \ = \ 2$. Find the smallest possible value of the number n, if $n \bmod 8 \ = \ 7, n \bmod 7 \ = \ 6$, and $n \bmod 6 \ = \ 5$.



9. In the diagram to the right, a circle of radius 25 is intersected with 4 lines. CH || DG, AF || BE, and AF ⊥DG. If CH = DG = 40 and AF = BE = 48, what is the area of the rectangle in the center?

10. Call a 2-digit number ab spectacular if it is equal to $a^2 + 2b + 1$. For instance, 43 is not spectacular because $4^2 + (2)(3) + 1 = 23 \neq 43$. There are two spectacular 2-digit numbers. What is their sum?

11. TIEBREAKER: How many digits would $\lfloor e^{\pi^{e^{\pi}}} \rfloor$ have? Answer with an integer that does not have any operations on it (e.g. no exponents or logarithms). ⌊x⌋ is defined as the greatest integer smaller than x. This is an estimation question that is likely not possible to be solved by hand. It will not be used to affect your score except for the event of a tiebreaker.