Roll No…….. **Total No. of Printed Pages: 2**
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B.Sc. (Hons.) Maths (Semester – 6th)
MATHEMATICAL MODELLING
Subject Code: BMATS1-603
Paper ID: [19131228]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. What is mathematical modeling? Write any two limitations.
b. Define Malthus population model..
c. Find the Laplace transform of $\cos 2t$
d. State Bessel's equation and write the formula for Bessel function of first kind.
e. Define Laplace transform with the help of an example.
f. What is Chebyshev polynomial. Write any one property of this polynomial.
g. What is middle square method. Explain.
h. Write any ecological model and explain the terms.
i. What do you mean by the term canonical form in LPP.
j. Explain morning rush hour.

Section – B **(5 marks each)**

Q2. Consider a small harbor with with unloading facilities for ships. Only one ship can be unloaded at any one time. Ships arrive for unloading of cargo at the harbor, and the time between the arrival of successive ships varies from 15 to 145 mins. The unloading time required for a ship depends on the type and amount of cargo and varies from 45 to 90 mins. Answers to the following questions:

a) What is the average and maximum times per ship in the harbor.
b) If the waiting time for a ship is the time between its arrival and the start of unloading, what are the average and maximum waiting times per ship.

Q3.Use Monte carlo simulation, write an algorithm to calculate that part of the volume of an ellipsoid $\frac{x^2}{2}+\frac{y^2}{4}+\frac{z^2}{8}\le 16,$ that lies in the first octant x > 0, y > 0, z > 0.

Q3. Find the generating function for Bessels function.

Q4. Using Laplace transform method to find the solution of IVP

$$\frac{\partial u}{\partial x}+x\frac{\partial u}{\partial t}=xt, u(x,0)=0 \ and \ u(0,t)=t.$$

Q5. Use the linear congruence method to generate

(a) 10 random numbers using a = 5, b = 1, and c = 8.
(b) 15 random numbers using a = 1, b = 7, and c = 10.

(c) 20 random numbers using a = 5, b = 3, and c = 16.
(d) Comment about the result of each sequence. Was there cycling. If so, when did it occur.

Section – C **(10 marks each)**

Q6. Find the solution of Legendre's equation and also find out Legendre's function of first and second kind.

Q7. An insulated rod of length l has its ends A and B maintained at 0ºC and 100ºC respectively until steady state conditions prevail. If B is suddenly reduced to 0ºC and maintained at 0ºC, find the temperature at a distance x from A at time t. Also find the temperature if the change consists of raising the temperature of A to 20ºC and reducing that of B to 80ºC.

Q8. Solve the LPP by Simplex method

$$Z = x_1 - 3x_2 + 3x_3,$$

subject to $3x_1 - x_2 + 2x_3 \le 7, 2x_1 + 4x_2 \ge -12,$

$-4x_1 + 3x_2 + 8x_3 \le 10,\ x_1, x_2, x_3 \ge 0.$