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B.Sc. (Hons.) Mathematics (Semester : 1st)
CALCULUS-1
Subject Code: BMATS1121
Paper ID: 22131201

Time: 03 Hours **Maximum Marks: 60**

Instructions for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consists of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consists of 3 questions of 10 marks each. The student has to attempt any 2 questions out of it.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. If $\lim_{x\to a} f(x) = 0$ and g(x) is bounded in a deleted neighborhood of a, then prove that

$$\lim_{x\to a} f(x)g(x) = 0$$

b. If [x] denotes greatest integer function, then show that $f(x) = \begin{cases} [x]+[-x] & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$ is discontinuous at x = 0. Also write the kind of discontinuity.

c. Let $f(x) = \begin{cases} x^2 + 3x + a & ,x \geq 1 \\ bx + 2 & ,x < 1 \end{cases}$ for what values of a and b, f(x) is differentiable at x=1.

d. Examine the curve $y = x^4 - 2x^3 + 1$ for concavity upwards, concavity downward and points of inflection.

e. Find the asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$

f. If $Z = \log(x^2 + xy + y^2)$, then prove that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = 2$

g. $If\ u = xf\left(\frac{y}{x}\right) + y\phi\left(\frac{y}{x}\right),\ \text{Prove that}\ x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$

h. If $u = \frac{x}{y-z}, v = \frac{y}{z-x}, w = \frac{z}{x-y},\ \text{Show that}\ \frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$

i. Examine for minimum and maximum values : sinx + siny + sin(x+y)

j. In what direction from (3,1,-2) is the directional derivative of $\varphi = x^2y^2z^4$ maximum and what is its magnitude.?

Section – B **(5 marks each)**

Q2. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, Prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

Q3. Show that the asymptotes of the cubic curve x3 – xy2 – 2xy + 2x – y - 1 = 0 cut the curve in at most three points which lie on the line 3x – y – 1 = 0.

Q4. If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$, prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$$

Q5. Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

Q6. Show that Curl(curl $\overrightarrow{V}$) = grad div $\vec{V}$ - $\nabla^2(\overrightarrow{V})$

Section – C **(10 marks each)**

Q7. Trace the curve y2(a + x) = x2 (3a – x), a>0

Q8. Find the minimum value of x2 + y2 + z2 , given that ax + by + cz = p

Q9. If y = sin(m.sin-1x), prove that (1-x2)yn+2 –(2n+1)xyn+1 – (n2 – m2)yn = 0

Deduce that $y_n(0) = \begin{cases} 0 \text{ , when n is even} \\ m(1^2 - m^2)(3^2 - m^2).......((n-2)^2 - m^2), \text{ when n is odd} \end{cases}$