

CVMS Math League  
Probability (Part B)

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Note: All people are distinct unless specified otherwise

★ = Difficult

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\*Probability = # of desired outcomes/number of possible outcomes

1. If a fair 6-sided dice is rolled 3 times, what is the probability that the sum showing on the faces is less than 17?
2. A circle is inscribed within a square. If a point is randomly selected inside the square, what is the probability that it is not in the circle (express your answer in terms of  $\pi$ )?
3. If 5 fair coins are flipped what is the probability that there is more than 1 head showing and less than 5 heads showing?
4. Alice and Bob are playing a game. Each round Alice has a  $\frac{1}{2}$  chance of winning and Bob has a  $\frac{1}{3}$  chance of winning. If no one wins the game moves onto the next round. What is the probability that the game ends in less than 5 turns?
5. Integers  $a$ ,  $b$ ,  $c$ , and  $d$ , not necessarily distinct, are chosen independently and at random from 0 to 2007, inclusive. What is the probability that  $ad - bc$  is even?
6. William is taking the 25-question, multiple choice American Mathematics Competition. Each question has five answer choices. William guesses random answers for the last four questions. What is the probability that he will get at least one of these final four questions right?
7. Two integers are relatively prime if they have no common factors other than 1 or  $-1$ . What is the probability that a positive integer less than or equal to 30 is relatively prime to 30? Express your answer as a common fraction.
8. Annie and Xenas each arrive at a party at a random time between 2 : 00 and 4 : 00 . Each stays for 45 minutes and then leaves. What is the probability that Annie and Xenas see each other at the party?
9. Given that  $a$  and  $b$  are real numbers such that  $-3 \leq a \leq 1$  and  $-2 \leq b \leq 4$ , and values for  $a$  and  $b$  are chosen at random, what is the probability that the product  $a \cdot b$  is positive? Express your answer as a common fraction. (Source: Mathcounts)

### Challenge Problems

10. ★ An o-Pod MP3 player stores and plays entire songs. Celeste has 10 songs stored on her o-Pod. The time length of each song is different. When the songs are ordered by length, the shortest song is only 30 seconds long and each subsequent song is 30 seconds longer than the previous song. Her favorite song is 3 minutes, 30 seconds long. The o-Pod will play all the songs in random order before repeating any song. What is the probability that she hears the first 4 minutes, 30 seconds of music - there are no pauses between songs - without hearing every second of her favorite song? Express your answer as a common fraction. (Source: Mathcounts)
11. ★ Two numbers,  $x$  and  $y$  are selected at random from the interval  $(0, 3)$ . What is the probability that a triangle with sides of length 1,  $x$ , and  $y$  exists?
12. ★★ Consider the L-shaped region formed by three unit squares joined at their sides, as shown below. Two points  $A$  and  $B$  are chosen independently and uniformly at random from inside the region. The probability that the midpoint of  $\overline{AB}$  also lies inside this L-shaped region can be expressed as  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ . (Source: AIME)

