ELMWOOD PARK PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

PRE-CALCULUS HONORS

Grades 10, 11, and 12

Prerequisites: Algebra II Honors; Teacher's Recommendation

5 credits

ABSTRACT

This full-year, five-credit course is to prepare students for *Advanced Placement (AP) Calculus* at the high school level or a college *Calculus* course appropriate for engineering, science or mathematics majors. This course is designed to serve as a comprehensive and in-depth review of skills previously acquired and to both refine and extend those abilities. The study of trigonometry, analytic geometry and elementary functions constitute most of this curriculum. Attention is also given to mathematical rigor, career opportunities, and the attitude with which mathematics must be approached in order to experience success at higher levels. There is an intense focus on word problems and real world application of higher mathematics.

Pre-Calculus Honors helps prepare students for the Scholastic Aptitude Test (SAT). Extensive use is made of graphing calculators and computer laboratory time is assigned. Various projects must be completed and significant time and independent effort are expected outside the classroom. This course is a prerequisite for *Advanced Placement (AP) Calculus AB*. In preparation for the state assessments, students will be given formative and summative assessments throughout the course.

UNIT #: *Unit Title* Number of Days	**Unit 1:** *Polynomial and Rational Functions* 20 days	**Unit 2:** *Exponential and Logarithmic Functions* 15 days	**Unit 3:** *Trigonometric Functions and Identities* 35 days
STAGE 1: DESIRED RESULTS ***What will students understand as a result of the unit? What are the BIG ideas?***			
ESTABLISHED GOALS: *(NJSLS-Mathematics)*	**Algebra** A-APR.A.1 A-APR.B.2-3 A-APR.C.5 A-REI.A.2 A-REI.B.3 A-REI.D.11 A-SSE.B.3 **Functions** F-IF.A.1-3 F-IF.B.4-6 F-IF.C.7-9 F-LE.A.1-4 F-LE.B.5 F-BF.1-2 F-BF.B.3-4 **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8	**Algebra** A-REI.D.11 A-SSE.B.4 **Functions** F.BF.B.3 F-BF.B.5 F-IF.C.7-8 F-LF.A.1-4 **Number and Quantity** N-CN.A.1-3 N-CN.C.7-9 N-RN.A.1-2 **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1	**Functions** F-TF.A.1-4 F-TF.B.5-7 F-TF.C.8-9 **Geometry** G-C.A.2 G-C.A.4 G-C.B.5 **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1

	Technology 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.1.12.PB.1 9.2.12.CAP.6 9.4.12.CT.1 9.4.12.CT.2 9.4.12.CT.3	8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.1.12.PB.1 9.2.12.CAP.6 9.4.12.CT.1 9.4.12.CT.2 9.4.12.CT.3	**Career Readiness, Life Literacies, and Key Skills** 9.1.12.PB.1 9.2.12.CAP.6 9.4.12.CT.1 9.4.12.CT.2 9.4.12.CT.3
ENDURING UNDERSTANDINGS: *(Students will understand that . . .*	• Algebraic and graphing techniques can be used to explain and analyze the general properties and behaviors of functions or relations. • Functions can be used to model real-world phenomena and solve problems of varying quantities. • The properties of functions and function operations are used to model and analyze real-world applications and quantitative relationships. • Roots of polynomials can be used to solve real-world application problems. • Polynomials can be divided using steps similar to the long division steps used to divide whole numbers. • The characteristics of polynomial functions and their representations are useful in solving real-world problems. • A rational function is a ratio of polynomial functions. If a rational function is in simplified form and the polynomial in the denominator is not constant, the graph of the	• Logarithmic functions are the inverse of exponential functions. • Exponential and logarithmic functions model real-world phenomena. • Inverse functions allow us to solve equations algebraically. • Radical expressions can be written in different ways to allow simplification of the expression. • Exponential functions can be used to model growth and decay in real world situations. • Compound Interest can be used in financial situations such as banking and investments. • There are several types of compounding used by financial industries and banking to accumulate or charge interest. • Logarithms can be utilized to solve exponential equations; and conversely, exponents can be used to solve logarithmic equations • The exponential function y=bx is one-to-one, so its inverse x=b/y is a	• An angle can be thought of as the rotation of a ray about its endpoint. • Knowledge of the unit circle is vital to understanding the trigonometric functions and the trigonometric ratios. • Three ways to represent an angle measure are degrees, radians and DMS which are all used commonly in higher math. • Discover how to use the Unit Circle to obtain the exact values for sine, cosine and tangent of special angles in all quadrants. • Understand reference angles and how they are used to solve applied problems. • Utilize, analyze and be able to compute amplitude and period for sine, cosine and tangent. • Prove that trigonometric identities and equations allow for better comprehension and mastery of trigonometric functions.

	rational function features asymptotic behavior. ● The properties of the real number system, commutative, associative and distributive properties also apply to complex numbers.	function. To express "y as a function of x" for the inverse, write y=logbx.	● We can derive the graphs of other trigonometric functions from the graphs of sine, cosine and tangent. ● Trigonometric functions are extended to the set of real numbers when we relate a real number to a point on the unit circle.
ESSENTIAL QUESTIONS: *(What provocative questions will foster inquiry, understanding, and transfer of learning?)*	● How can functions be interpreted and what are some real-life manifestations? ● How can we use the properties of various types of functions to determine domain and range of any function without modeling with a graph? ● What is a local maximum and minimum when looking at a modeled polynomial function? What is a global maximum and minimum? ● What are the three notation types when describing an interval such as a domain or range? ● How can we use the ideas of complex numbers to evaluate or simplify equations and expressions and how can they be used in application problems? ● How can we determine the real and rational roots for any given polynomial equation through the idea and use of factors? ● Why do factors play a key role in determining the zeros of a given polynomial function?	● How do exponential functions model real-world problems and their solutions? ● How do logarithmic functions model real-world problems and their solutions? ● How can we use properties of logarithms to expand or compress a logarithmic expression? ● How can we use the idea of inverses to solve an exponential or logarithmic equation? ● How do we use characteristics of logarithms to graph the functions on the coordinate plane? ● How do we use characteristics of exponential functions to graph them on the coordinate plane? ● How can we evaluate a logarithm using the change of base formula? ● How can we use compound interest to solve financial application problems? ● What does it mean to compound annually, semi-annually, continuously… etc? How does that change the compound interest formula? ● How can we use exponential growth and decay to determine the effects of climate change on populations, ecosystems, and the atmosphere?	● How can we solve applied problems using the fact that trigonometric ratios stay constant in similar triangles? ● How can we derive the conversions from radians to degrees and from degrees to radians and how does DMS measurement fit in? ● How do you find values of inverse trigonometric functions, applying appropriate domain and range restrictions? ● How do we graph the six trigonometric functions and identify characteristics/transformations such as period, amplitude, phase shift, and asymptotes? ● How can we use basic trigonometric definitions and identities to prove trigonometric equations and expressions? ● Can you apply the Pythagorean and Reciprocal Identities to verify identities and solve equations? ● Why are certain values undefined for certain functions? ● How are the six basic trig functions related to each other?

STAGE 2: ASSESSMENT EVIDENCE

What evidence will be collected to determine whether or not the understandings have been developed, the knowledge and skills attained, and the State Standards met? [Anchor the work in performance tasks that involve application, supplemented as needed by prompted work, quizzes, observations, etc.]

PERFORMANCE TASKS:
(Through what authentic performance tasks will students demonstrate the desired understandings?)
(By what criteria will performances of understanding be judged?)
	• Group Activities
	• Projects
	• District and State Standardized Testing
	• Computer-Technology Based Projects
	• Independent assignments
	• Written Responses on real-world problems and non-routine problems.
	• Get together with your classmates in groups of two or three. Sketch a graph of a function, but do not show it to the other members of your group. Using the language of functions, describe your function as completely as you can. Exchange descriptions with the others in your group and see if you can reproduce each other's graphs.
		• Group Activities
		• Projects
		• District and State Standardized Testing
		• Computer-Technology Based Projects
		• Independent assignments
		• Written Responses on real-world problems and non-routine problems.
		• In genealogy you are considered generation 0. Your two parents are generation 1. Your four grandparents are generation 2. Your great grandpar-ents are generation 3 and your 2nd great grandparents are gen-eration 4.
		  • (a) How many 4th great grandparents do you have? What gen-eration are they?
		  • (b) Use an exponential function to model the number of nth
		  • great grandparents you have.
		  • (c) Use the model you found in (b) and find the number of 6th
		  • great grandparents you have.
		  • (d) How many 25th great grandparents do you have?
		  • (e) Discuss how many years it takes to span 25 generations at
		  • an average of 30 years between generations. The world's population in 1250 is thought to have been about 400 million.
		• To how much of the world's population in 1250 might you be related?
			• Group Activities
			• Projects
			• District and State Standardized Testing
			• Computer-Technology Based Projects
			• Independent assignments
			• Written Responses on real-world problems and non-routine problems.
			• Group Activity A musical note, like that produced by a tun-ing fork, is a pressure wave. Typically, its frequency is mea-sured in hertz (cycles per second), and its sound pressure in pascals. Table 4.5 gives frequency (in Hz) of several musical notes. The sound-pressure data (in Pa) in Table 4.6 show local deviations in atmospheric pressure, measured using a CBL and a microphone for a specific turning fork.

OTHER EVIDENCE: *(Through what other evidence (e.g., quizzes, tests, academic prompts, observations, homework, journals, benchmark assessments, etc.) will students demonstrate achievement of the desired results?)* *(How will students self-assess their learning?)*	• Classwork • Homework • Quizzes / Tests • Journals • Projects • Group Discussions • Class Discussions • Teacher Observations • Presentations • Activities • Peer and self- evaluations • Rubrics • Error analysis • Web-based assessments • Benchmark assessments • LinkIt!	• Classwork • Homework • Quizzes / Tests • Journals • Projects • Group Discussions • Class Discussions • Teacher Observations • Presentations • Activities • Peer and self- evaluations • Rubrics • Error analysis • Web-based assessments • Benchmark assessments • LinkIt!	• Classwork • Homework • Quizzes / Tests • Journals • Projects • Group Discussions • Class Discussions • Teacher Observations • Presentations • Activities • Peer and self- evaluations • Rubrics • Error analysis • Web-based assessments • Benchmark assessments • LinkIt!
RESOURCES:	• Contemporary PreCalculus (Digital Textbook) • Student workbook • Assessment resources • Graphing Calculators • Supporting videos • Chromebooks • Socrative • Youtube.com • Tiered Worksheet • Relevant websites such as Khan Academy, IXL.com, and Kuta Software • Canvas	• Contemporary PreCalculus (Digital Textbook) • Student workbook • Assessment resources • Graphing Calculators • Supporting videos • Chromebooks • Socrative • Youtube.com • Tiered Worksheet • Relevant websites such as Khan Academy, IXL.com, and Kuta Software • Canvas	• Contemporary PreCalculus (Digital Textbook) • Student workbook • Assessment resources • Graphing Calculators • Supporting videos • Chromebooks • Socrative • Youtube.com • Tiered Worksheet • Relevant websites such as Khan Academy, IXL.com, and Kuta Software • Canvas
STAGE 3: LEARNING PLAN ***What learning experiences and instruction will enable students to achieve the desired results? Utilize the WHERETO* acronym to consider key design elements.***			
SKILLS AND TOPICS: *(What specific activities will students do and what skills will students know as a result of the unit?)*	• Identify the graph of a function and obtain information from the graph of a function. • Determine continuity, increasing-decreasing behavior, local minima and maxima,	• Evaluate exponential expressions and solve exponential equations through the use of properties and inverses. • Identify and graph exponential, common and natural logarithmic	• Convert between radians and degrees and find arc length, angular and linear speed, and area of a sector of a circle. • Define and evaluate the six trigonometric functions in terms of

	symmetry, asymptotes and end behavior of a function both graphically and algebraically. ● Find the average rate of change of a function as well as the instantaneous rate of change when solving real world application problems. ● Identify and graph polynomial functions, predict their end behavior and find their real zeros algebraically and graphically. ● Find the domain and asymptotes of rational functions, and analyze and construct graphs of rational functions. ● Apply the Remainder Theorem and Factor Theorem to solve application problems. ● Determine the complex zeros of polynomial equations, and determine the polynomial with the specified zeros.	functions on the coordinate plane using characteristics identifiable from the function. ● Use exponential growth and decay to model real life problems such as financial applications and banking. ● Convert equations between logarithmic form and exponential form to simplify an expression or solve for a variable. ● Evaluate common and natural logarithms with and without the use of a calculator. ● Apply the properties of logarithms to evaluate expressions by compressing or expanding an expression. ● Solve exponential and logarithmic equations algebraically and graphically. ● Use exponential and logarithmic equations to solve real life problems	the lengths of the sides of a right triangle, the rotation of a ray in standard position, and a point on a unit circle both exactly and approximately. ● Determine the range, domain, and period of trigonometric functions as well as graph the function and transformations. ● Find an exact and approximate value of an inverse sine, cosine or tangent function and use them to solve equations checking for domain. ● Know the definitions of the inverse secant, cosecant and cotangent functions and use the calculator to evaluate $\sec^{-1}x$, $\csc^{-1}x$, $\cot^{-1}x$. ● Use algebra, Reciprocal Trigonometric Identities, Quotient Identities, Pythagorean Identities, Co-Function Identities and Odd-Even Identities to simplify trigonometric expressions and solve trigonometric equations. ● Apply the Sum and Difference Formulas, Double-angle Formulas, and Half-angle Formulas to solve.
CROSS-CURRICULAR/ DIFFERENTIATION: *(What cross-curricular (e.g., writing, literacy, math, science, history, Career Readiness, Life Literacies, and Key Skills, technology) learning activities are included in this unit that will help achieve the desired results?) (What type of differentiated instruction will be used for Special Education, ELL, At Risk, and Gifted and Talented students?)*	**Cross-Curricular Connections:** **English Language Arts** **(RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)** ● Current event problems Exploratory Activities Project-Based Learning	**Cross-Curricular Connections:** **English Language Arts** **(RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)** ● Current event problems Exploratory Activities Project-Based Learning	**Cross-Curricular Connections:** **English Language Arts** **(RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)** ● Current event problems Exploratory Activities Project-Based Learning

	Differentiation: General: ● Using task cards ● Homogeneous and heterogeneous grouping when completing exploration or reinforcement activities. ● Creating graphic organizers ● Flipped classroom ● Open-ended collaboration projects and assignments with rubrics created by the students. Special Education-- Students with IEP/504 Plan: ● Provide modifications and accommodations as listed in the student's IEP/504 Plan ● Position students near a helping peer or have quick access to a teacher. ● Provide teacher notes and homework answer keys so students can check work and use teacher notes to study. ● Use collaborative grouping strategies such as small groups where homogeneous grouping will be used so the teacher can provide additional help. ● Use online resources ● NJDOE resources ● Students will receive modifications such as numbered coordinate plans and numbered lines as well as formulas and graphic organizers for all notes, worksheets and assessments. English Language Learners:	**Science** **(HS-ESS3-6)** Lesson Performance Task: ● Use computational representation to illustrate the relationships between Earth systems and climate change. **Differentiation:** General: ● Using task cards ● Homogeneous and heterogeneous grouping when completing exploration or reinforcement activities. ● Creating graphic organizers ● Flipped classroom ● Open-ended collaboration projects and assignments with rubrics created by the students. Special Education-- Students with IEP/504 Plan: ● Provide modifications and accommodations as listed in the student's IEP/504 Plan. ● Position students near a helping peer or have quick access to a teacher. ● Students will receive modifications such as a graphic organizer with logarithmic properties and identities, the compound interest formulas and equations as well as the properties and identities of radical functions and rational exponents. ● Provide teacher notes and homework answer keys so students can check work and use teacher notes to study. ● Context clues will be provided for word problems involving compound interest where key points will be highlighted.	**Differentiation:** General: ● Using task cards ● Homogeneous and heterogeneous grouping when completing exploration or reinforcement activities. ● Creating graphic organizers ● Flipped classroom ● Open-ended collaboration projects and assignments with rubrics created by the students. Special Education-- Students with IEP/504 Plan: ● Provide modifications and accommodations as listed in the student's IEP/504 Plan. ● Position students near a helping peer or have quick access to a teacher. ● Students will receive modifications such as a graphic organizer with trigonometric equations and identities, and the first quadrant of a unit circle. ● Provide teacher notes and homework answer keys so students can check work and use teacher notes to study. ● Use collaborative grouping strategies such as small groups where homogeneous grouping will be used so the teacher can provide additional help. ● Use online resources ● NJDOE resources ● Context clues will be provided for word problems involving trigonometry and solving trigonometric equations. Word

	• Provide word banks for translated vocabulary for words such as "domain, range, maximum, minimum, difference quotient, inverse, composite, rational, long division,…etc" • Place the student next to the same-language speaker, if possible. • Provide text to speech for math problems as well as allow the use of translation dictionaries or software. • Provide graphic organizers including formulas for completing the square, creating a polynomial with complex roots, the quadratic formula, as well as the steps for determining roots of polynomials. • Providing context clues for all word problems including when to find the vertex and use the vertex formulas and when to complete the square to determine vertex and concavity. • Modification plan provided with the guidance department with the language program at the school. • NJDOE resources • Adapt a strategy adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm Gifted and Talented Students: • Processes should be modified: higher-order thinking skills, open-ended thinking, and discovery to derive methods for finding all of the roots of a given polynomial function, determining domain of functions by the use of graphing and applying the idea of domain and	• When solving properties of logarithms, only 1-2 properties will be assessed for any given problem. • Use collaborative grouping strategies such as small groups where homogeneous grouping will be used so the teacher can provide additional help. • Use online resources • NJDOE resources English Language Learners: • Provide word banks for translated vocabulary for words such as "logarithm, exponential, inverse, radicals, compound interest, rate…etc" • Place the student next to the same-language speaker, if possible. • Provide text to speech for math problems as well as allow the use of translation dictionaries or software. • Provide graphic organizers including formulas for computing compound interest, interest rates, logarithmic transformations for graphing, logarithmic properties and logarithmic definitions. • Provided context clues for solving word problems involving compound interest such as highlighting the important words and vocabulary. • Modification plan provided with the guidance department with the language program at the school. • NJDOE resources • Adapt a strategy adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm	problems will have a corresponding picture for reference when solving real world problems. English Language Learners: • Place the student next to the same-language speaker, if possible. • Provide text to speech for math problems as well as allow the use of translation dictionaries or software. • Provide word banks for translated vocabulary for words such as "sine, cosine, tangent, secant, cosecant, cotangent, radian, unit circle, measure, inverse, theta, …etc" • Provide graphic organizers including formulas for the identities of trigonometric functions, Unit Circle's first quadrant, and the formulas for area of a sector, arc length, angular speed, and vector speed. • Modification plan provided with the guidance department with the language program at the school. • NJDOE resources • Adapt a strategy adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm Gifted and Talented Students: • Processes should be modified: higher-order thinking skills, open-ended thinking, and discovery for students to derive identities and formulas used to solve equations. • Utilize exploratory connections to higher-grade concepts when proving trigonometric identities and equations- including

	range to solve real world application problems. ● Utilize project-based learning for greater depth of knowledge when graphing rational functions on the coordinate plane or solving for zeros of functions. ● Utilize exploratory connections to higher-grade concepts when solving for complex solutions, graphing rational functions with oblique asymptotes and finding inverse for any type of function. ● Contents should be modified: abstraction, complexity, variety, and organization. ● Derivations for all formulas will be completed as a higher order task for students to master content material. ● Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations ● Learning environments should be modified: student-centered learning to discover topics, independence, openness, and complexity, groups varied. ● Use of web based resources such as http://www.tenmarks.com www.khanacademy.orggeogebra.org ● NJDOE resources At-Risk Students: ● Tiered interventions following RtI framework as well as provided an Rtl Intervention Bank ● Use additional practice and textbook RTI resources for students to practice graphing functions, finding domain and range.	Gifted and Talented Students: ● Processes should be modified: higher-order thinking skills, open-ended thinking, and discovery for students to derive methods for graphing exponential functions and logarithmic functions on the coordinate plane as well as the properties used to solve and simplify expressions and equations. ● Utilize project-based learning for greater depth of knowledge and show mastery in the content. ● Utilize exploratory connections to higher-grade concepts by solving word problems with compound interest relating to financial real world situations. ● Contents should be modified: abstraction, complexity, variety, and organization. ● Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations. ● Learning environments should be modified: student-centered learning, independence, openness, and complexity, groups varied. ● Use of web based resources such as http://www.tenmarks.com www.khanacademy.orggeogebra.org ● NJDOE resources At-Risk Students: ● Tiered interventions following RtI framework as well as provided an Rtl Intervention Bank ● Use additional practice and textbook RTI resources for students to practice graphing logarithms, utilizing the logarithmic and radical	addition/subtracting, half and double angle equations and identities. ● Contents should be modified: abstraction, complexity, variety, and organization in higher order thinking questions and examples. ● Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations. ● Learning environments should be modified: student-centered learning for discovery topics, independence, openness, and complexity, groups varied to promote student centered learning. ● Use of web based resources such as http://www.tenmarks.com www.khanacademy.orggeogebra.org ● NJDOE resources At-Risk Students: ● Tiered interventions following RtI framework as well as provided an Rtl Intervention Bank ● Use additional practice and textbook RTI resources for students to practice graphing trigonometric functions on the coordinate plane. ● Use additional practice and textbook RTI resources for students to practice simplifying trigonometric expressions and solving trigonometric equations using inverses. ● Provide small group intervention to work on skills and concepts during group work such as homogeneous grouping. ● NJDOE resources

	• Provide small group intervention to work on skills and concepts during group work such as homogeneous grouping. • NJDOE resources • Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org	properties to solve problems and simplifying expressions. • Provide small group intervention to work on skills and concepts during group work such as homogeneous grouping. • NJDOE resources • Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org	• Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org

***WHERETO**

W = Help the students know **WHERE** the unit is going and **WHAT** is expected. Help the teacher know **WHERE** the students are coming from (prior knowledge, interests).

H = **HOOK** all students and **HOLD** their interest.

E = **EQUIP** students, help them **EXPERIENCE** the key ideas and **EXPLORE** the issue.

R = Provide opportunities to **RETHINK** and **REVISE** their understanding and work.

E = Allow students to **EVALUATE** their work and its implications.

T = **TAILORED** to the different needs, interests, and abilities of learners.

O = **ORGANIZE** to maximize initial and sustained engagement as well as effective learning.

UNIT #: *Unit Title* Number of Days	**Unit 4:** *Applications of Trigonometry* 25 days	**Unit 5:** *Conics and Polar Graphing* 25 days	**Unit 6:** *Systems of Equations and Matrices* 25 days
STAGE 1: DESIRED RESULTS ***What will students understand as a result of the unit? What are the BIG ideas?***			
ESTABLISHED GOALS: *(NJSLS-Mathematics)*	**Geometry** G-SRT.B.4-5 G-SRT.C.6-8 G-SRT.D.9-11 **Number and Quantity** N-VM.A.1(+)	**Algebra** A-SSE.B.3.b **Functions** F-TF.B.7 **Geometry**	**Algebra** A-SSE.B.4 A-CED.A.2-3 A-REI.C.5-9 A-REI.D.10-12 **Functions**

	N-VM.A.2(+) N-VM.A.3(+) N-VM.B.4.a(+) N-VM.B.4.b(+) N-VM.B.4.c(+) N-VM.B.5.a(+) N-VM.B.5.b(+) N-VM.C.6(+) **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.1.12.PB.1 9.2.12.CAP.6 9.4.12.CT.1 9.4.12.CT.2 9.4.12.CT.3	G-C.A.4 G-GPE.A.1 G-GPE.A.3 **Number and Quantity** N-CN.B.4 N-CN.B.6 **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.1.12.PB.1 9.2.12.CAP.6 9.4.12.CT.1 9.4.12.CT.2 9.4.12.CT.3	F-IF.A.3 F-BF.A.2 F-LE.A.2 **Number and Quantity** N-VM.C.7(+) N-VM.C.8(+) N-VM.C.9(+) N-VM.C.10(+) N-VM.C.11(+) N-VM.C.12(+) **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.1.12.PB.1 9.2.12.CAP.6 9.4.12.CT.1 9.4.12.CT.2 9.4.12.CT.3
ENDURING UNDERSTANDINGS: *(Students will understand that . . .)*	• DMS and DDS are units of measure that play a key role in real-world application problems	• The angle or direction you cut through a cone determines the type of conic you get.	• Patterns provide insights to potential relationships.

	• Special right triangles are useful tools used to solve real-world application problems by means of finding angles and lengths. • Law of Sines represent • Explain how to solve a variety of problems using the Law of Sines. • Explain how to solve acute and obtuse triangles and to determine the area of a triangle in terms of the measures of the sides and angles using the Law of Cosines. • Vectors can be utilized to display the magnitude and direction of an object in a physical application. • Vectors are used to describe quantities that have both magnitude and direction. • The algebraic representations of vectors make them applicable to many situations.	• Conic sections reflect real-world phenomena. • Certain key points and axis are vital to graphing and using the conic sections to real world problems. • Functions and relations can be represented using vectors, parametric equations, and polar coordinates. • Polar form makes it possible to find solutions to equations that couldn't be found in rectangular form. • There is a relationship between the polar form of a number, rectangular form of a number, and complex numbers. • Polar coordinates are an alternative way to specify location on the plane. • Equations of conic sections can be reorganized and analyzed to produce adequate graphs.	• A variety of representations of linear systems of equations, including matrices, are used to model and solve real-world problems. • Matrices are used to store and operate with data to solve real world situations. • Linear and non-linear systems of equations are used to predict outcomes in many real world scientific & engineering applications. • A variety of representations of linear systems of equations, including matrices, are used to model and solve real-world problems. • Matrices can be added, subtracted and multiplied because systems are closed under the operations. • Matrices allow us to represent multidimensional problems. • Sequences and series can solve a variety of real life problems, including compound interest.
ESSENTIAL QUESTIONS: *(What provocative questions will foster inquiry, understanding, and transfer of learning?)*	• In solving missing sides and angles of triangles, when are trig ratios, Law of Sines or Law of Cosines appropriate? • How do we differentiate between using Law of Sines or Law of Cosines? • What are the two methods used for finding the area of any given triangle and how do we differentiate between each method? • How does triangle trigonometry relate to real world applications? • When will I use vectors to model physical quantities?	• Where do conic sections appear in the real world? • How does the intersection of a plane and a cone create the different conic sections? • How do we identify the characteristics of conic functions such as the major and minor axis, center, vertices, foci… etc? • When is a polar system more useful than rectangular? • What algebraic operations are easier in polar form? • Why is the polar system necessary for finding all solutions for an equation?	• What are some of the different relationships between patterns? • Do all series have a finite solution? • How can sequences and series be used to model compound interest and annuity accruals? • What methods can be used to solve systems of equations? • How are systems of linear equations useful? • How do you evaluate determinants of 2x2 and 3x3 matrices? • How do you use Cramer's rule to solve systems of linear equations?

	• How are vectors and the concept of slope related? • How are speed and velocity related? • How do parametric equations and vectors solve real-world problems?	• What sorts of real-world contexts can be represented using a polar coordinate system? • What is the relationship between the ordered pairs in the Cartesian coordinate system versus a polar system?	• How do you find and use inverse matrices? • How do we perform operations of matrices and why are we allowed to complete operations for systems?
STAGE 2: ASSESSMENT EVIDENCE ***What evidence will be collected to determine whether or not the understandings have been developed, the knowledge and skills attained, and the State Standards met? [Anchor the work in performance tasks that involve application, supplemented as needed by prompted work, quizzes, observations, etc.]***			
PERFORMANCE TASKS: *(Through what authentic performance tasks will students demonstrate the desired understandings?)* *(By what criteria will performances of understanding be judged?)*	• Group Activities • Projects • District and State Standardized Testing • Computer-Technology Based Projects • Independent assignments • Written Responses on real-world problems and non-routine problems.	• Group Activities • Projects • District and State Standardized Testing • Computer-Technology Based Projects • Independent assignments • Written Responses on real-world problems and non-routine problems. • Group Activity: Designing a Suspension Bridge - The main cables of a suspension bridge uniformly distribute the weight of the bridge when in the form of a parabola. The main cables of a particular bridge are attached to towers that are 600 ft apart. The cables are attached to the towers at a height of 110 ft above the roadway and are 10 ft above the roadway at their lowest points. If vertical support cables are at 50-ft intervals along the level roadway, what are the lengths of these vertical cables? • Students find and maximize the productive value of existing and	• Group Activities • Projects • District and State Standardized Testing • Computer-Technology Based Projects • Independent assignments • Written Responses on real-world problems and non-routine problems. • Group Activity: Describe all possibilities for the number of solutions to a system of two equations in two variables if the graphs of the two equations are (a) a line and a circle, and (b) a circle and a parabola.

		new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.	
OTHER EVIDENCE: *(Through what other evidence (e.g. quizzes, tests, academic prompts, observations, homework, journals) will students demonstrate achievement of the desired results?)* *(How will students self-assess their learning?)*	● Classwork ● Homework ● Quizzes / Tests ● Journals ● Projects ● Group Discussions ● Class Discussions ● Teacher Observations ● Presentations ● Activities ● Peer and self- evaluations ● Rubrics ● Error analysis ● Web-based assessments ● Benchmark assessments ● LinkIt!	● Classwork ● Homework ● Quizzes / Tests ● Journals ● Projects ● Group Discussions ● Class Discussions ● Teacher Observations ● Presentations ● Activities ● Peer and self- evaluations ● Rubrics ● Error analysis ● Web-based assessments ● Benchmark assessments ● LinkIt!	● Classwork ● Homework ● Quizzes / Tests ● Journals ● Projects ● Group Discussions ● Class Discussions ● Teacher Observations ● Presentations ● Activities ● Peer and self- evaluations ● Rubrics ● Error analysis ● Web-based assessments ● Benchmark assessments ● LinkIt!
RESOURCES:	● Contemporary PreCalculus (Digital Textbook) ● Student workbook ● Assessment resources ● Graphing Calculators ● Supporting videos ● Chromebooks ● Socrative ● Youtube.com ● Tiered Worksheet ● Relevant websites such as Khan Academy, IXL.com, and Kuta Software	● Contemporary PreCalculus (Digital Textbook) ● Student workbook ● Assessment resources ● Graphing Calculators ● Supporting videos ● Chromebooks ● Socrative ● Youtube.com ● Tiered Worksheet ● Relevant websites such as Khan Academy, IXL.com, and Kuta Software	● Contemporary PreCalculus (Digital Textbook) ● Student workbook ● Assessment resources ● Graphing Calculators ● Supporting videos ● Chromebooks ● Socrative ● Youtube.com ● Tiered Worksheet ● Relevant websites such as Khan Academy, IXL.com, and Kuta Software

	• Canvas	• Canvas	• Canvas
STAGE 3: LEARNING PLAN ***What learning experiences and instruction will enable students to achieve the desired results? Utilize the WHERETO* acronym to consider key design elements.***			
SKILLS AND TOPICS: *(What specific activities will students do and what skills will students know as a result of the unit?)*	• Solve all types of triangles using right triangle properties, Law of Sines and Law of Cosines. • Find the area of any triangle using the formula for any given non-right triangle. • To solve real-world problems using right and non -right triangle trigonometry as well as area of triangles. • To find the area of a given triangle using Heron's formula. • Calculate and apply the dot product of vectors. • Calculate the magnitude / norm of a vector.	• Identify a conic from its equation. • State the domain, range, center, vertex and foci of each conic as it applies and sketch the related graph. • Analyze real world situations that are simulated by conic sections, such as roller coasters, planetary motion, satellite sigh, headlights and construction models. • Graph polar points and equations on the polar grid in both degrees and radians. • Utilize the polar distance formula to solve real world problems relating to topics such as sea navigation and aviation coordinates. • Identify and model classical curves (limacon, cardioid, rose, lemniscate, circle, … etc) • Convert between polar and rectangular coordinates and equations. • Formulate the polar equation of a conic given a verbal description.	• Justify whether a sequence is arithmetic, geometric or neither. • Write a formula for the nth term of a given sequence. • Use summation notation to represent a series. • Calculate the nth term of a given sequence as well as the sum of a series, if it exists. • Calculate the sum of an infinite sequence to determine that the infinite sequence does not have a sum. • Represent a system of two or more linear equations in matrix form. • Solve systems of two or more linear equations using a variety of methods. • Determine whether a square matrix has a multiplicative inverse and determine the inverse of any square matrix, if it exists. • Evaluate for the determinant of a 2x2 and 3x3 matrix and solve a system using Cramer's Rule. • Add, subtract, and multiply any given set of matrices.
CROSS-CURRICULAR / DIFFERENTIATION: *(What cross-curricular (e.g. writing, literacy, math, science, history, Career Readiness, Life Literacies, and Key Skills Technology) learning activities are included in this unit that will help achieve the desired results?)*	**Cross-Curricular Connections:** **English Language Arts (RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies (6.3.12.A.2 , 6.3.12.D.2)**	**Cross-Curricular Connections:** **English Language Arts (RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies (6.3.12.A.2 , 6.3.12.D.2)**	**Cross-Curricular Connections:** **English Language Arts (RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies (6.3.12.A.2 , 6.3.12.D.2)**

(What type of differentiated instruction will be used for Special Education, ELL, At Risk, and Gifted and Talented students?)	● Current event problems Exploratory Activities Project-Based Learning **Differentiation:** <u>General:</u> ● Using task cards ● Homogeneous and heterogeneous grouping when completing exploration or reinforcement activities. ● Creating graphic organizers ● Flipped classroom ● Open-ended collaboration projects and assignments with rubrics created by the students. <u>Special Education--</u> <u>Students with IEP/504 Plan</u>: ● Provide modifications and accommodations as listed in the student's IEP/504 Plan. ● Position students near a helping peer or have quick access to a teacher. ● Students will receive modifications such as a graphic organizer with the equations for Law of Sines, Law of Cosines, area of a non-right triangle, as well as Heron's formula. ● Provide teacher notes and homework answer keys so students can check work and use teacher notes to study. ● Context clues will be provided for word problems involving triangles. Word problems will have a corresponding picture for reference when solving real world problems.	● Current event problems Exploratory Activities Project-Based Learning **Differentiation:** <u>General:</u> ● Using task cards ● Homogeneous and heterogeneous grouping when completing exploration or reinforcement activities. ● Creating graphic organizers ● Flipped classroom ● Open-ended collaboration projects and assignments with rubrics created by the students. <u>Special Education--</u> <u>Students with IEP/504 Plan</u>: ● Provide modifications and accommodations as listed in the student's IEP/504 Plan. ● Position students near a helping peer or have quick access to a teacher. ● Students will receive modifications such as numbered polar grids, provided blank tables to fill out when graphing and conversion formulas provided on a graphic organizer for all notes, worksheets and assessments. ● Provide teacher notes and homework answer keys so students can check work and use teacher notes to study. ● Use collaborative grouping strategies such as small groups where homogeneous grouping will	● Current event problems Exploratory Activities Project-Based Learning **Differentiation:** <u>General:</u> ● Using task cards ● Homogeneous and heterogeneous grouping when completing exploration or reinforcement activities. ● Creating graphic organizers ● Flipped classroom ● Open-ended collaboration projects and assignments with rubrics created by the students. <u>Special Education--</u> <u>Students with IEP/504 Plan</u>: ● Provide modifications and accommodations as listed in the student's IEP/504 Plan. ● Position students near a helping peer or have quick access to a teacher. ● Students will receive modifications such as a graphic organizer with the discriminant formulas, Cramer's Rule, Explicit and Recursive formulas, and the sum definitions that can be used on all assessments and notes worksheets. ● Provide teacher notes and homework answer keys so students can check work and use teacher notes to study. ● Use collaborative grouping strategies such as small groups where homogeneous grouping will be used so the teacher can provide additional help. ● Use online resources

	● Use collaborative grouping strategies such as small groups where homogeneous grouping will be used so the teacher can provide additional help. ● Use online resources ● NJDOE resources <u>English Language Learners</u>: ● Place the student next to the same-language speaker, if possible. ● Provide text to speech for math problems as well as allow the use of translation dictionaries or software. ● Provide word banks for translated vocabulary for words such as "sine, cosine, tangent, secant, cosecant, cotangent, radian, Heron's formula, Law of Sines, Law of Cosines, vector, magnitude, dot product,…etc" ● Provide graphic organizers including formulas for Law of Cosines, Law of Sines, the area formulas and vector operations. ● Providing context clues for all word problems including solving triangles and finding the area. Key terms and items will be identified. ● Modification plan provided with the guidance department with the language program at the school. ● NJDOE resources ● Adapt a strategy adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm <u>Gifted and Talented Students</u>:	be used so the teacher can provide additional help. ● Use online resources ● NJDOE resources <u>English Language Learners</u>: ● Provide word banks for translated vocabulary for words such as "conic, ellipse, circle, hyperbola, foci, vertex, asymptote, axes…etc" ● Place the student next to the same-language speaker, if possible. ● Provide text to speech for math problems as well as allow the use of translation dictionaries or software. ● Provide graphic organizers including standard form of a circle, ellipse and parabola as well as the distance and midpoint formulas. ● Providing context clues for all word problems including when to find the center, vertices, co-vertices, foci… etc" ● Provide the polar distance formula on a graphic organizer as well as notes, charts and tables to be filled out. ● Modification plan provided with the guidance department with the language program at the school. ● NJDOE resources ● Adapt a strategy adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm <u>Gifted and Talented Students</u>: ● Processes should be modified: higher-order thinking skills,	● NJDOE resources <u>English Language Learners</u>: ● Place the student next to the same-language speaker, if possible. ● Provide text to speech for math problems as well as allow the use of translation dictionaries or software. ● Provide word banks for translated vocabulary for words such as "determinant, augmented matrix, system of equations, Cramer's rule, inverse, Gauss-Jordan, and Gaussian. ● Provide graphic organizers including formulas for determinants, Cramer's rule, inverse, and multiplying matrices to be used on all notes worksheets and assessments. ● Providing context clues for all word problems including how to create a system of equations. ● Modification plan provided with the guidance department with the language program at the school. ● NJDOE resources ● Adapt a strategy adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm <u>Gifted and Talented Students</u>: ● Processes should be modified: higher-order thinking skills, open-ended thinking, and discovery to derive solving systems using an augmented matrix and row operations to simplify and create new equations. ● Utilize project-based learning for greater depth of knowledge when

	• Processes should be modified: higher-order thinking skills, open-ended thinking, and discovery for students to derive identities and formulas used to solve equations. • Utilize exploratory connections to higher-grade concepts when solving real world application problems involving triangle trigonometry as well as the applications of vectors to physics. • Contents should be modified: abstraction, complexity, variety, and organization in higher order thinking questions and examples. • Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations. • Learning environments should be modified: student-centered learning for discovery topics, independence, openness, and complexity, groups varied to promote student centered learning. • Use of web based resources such as http://www.tenmarks.com www.khanacademy.orggeogebra.org • NJDOE resources At-Risk Students: • Tiered interventions following RtI framework as well as provided an Rtl Intervention Bank • Use additional practice and textbook RTI resources for students to practice graphing trigonometric functions on the coordinate plane. • Use additional practice and textbook RTI resources for students to practice simplifying	open-ended thinking, and discovery to derive the equations for conic sections and their graphs, as well as conversions for polar to rectangular points and equations. • Utilize project-based learning for greater depth of knowledge when graphing polar functions on the coordinate plane. • Utilize exploratory connections to higher-grade concepts when solving and graphing conic sections and converting between polar and rectangular functions relating to the conic sections. • Contents should be modified: abstraction, complexity, variety, and organization. • Derivations for all formulas such as the polar distance formula will be completed as a higher order task for students to master content material. • Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations • Learning environments should be modified: student-centered learning for discover topics, independence, openness, and complexity, groups varied. • Use of web based resources such as http://www.tenmarks.com www.khanacademy.orggeogebra.org • NJDOE resources • Have students explore using scalar multiplication and addition of matrices to solve a real-world problem.	evaluating summations and creating sequence functions. • Utilize exploratory connections to higher-grade concepts when solving matrices using real world situations and when using determinants and inverses. • Contents should be modified: abstraction, complexity, variety, and organization. • Derivations for all formulas will be completed as a higher order task for students to master content material. • Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations • Learning environments should be modified: student-centered learning for discover topics, independence, openness, and complexity, groups varied. • Use of web based resources such as http://www.tenmarks.com, www.khanacademy.orggeogebra.org • NJDOE resources At-Risk Students: • Use additional practice and textbook RTI resources for students to practice multiplying matrices and solving systems of any size using row echelon form. • Use additional practice and textbook RTI resources for students to practice creating sequence equations and evaluating summations. • Topics will be broken down and assessed in smaller sections such as an assessment for solving matrices then an assessment for properties and operations with matrices.

	trigonometric expressions and solving trigonometric equations using inverses. ● Provide small group intervention to work on skills and concepts during group work such as homogeneous grouping. ● NJDOE resources ● Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org	At-Risk Students: ● Students may struggle with setting up an equation of a parabola before completing the square. Have students identify the quadratic expression and explain how to complete the square. ● Tiered interventions following RtI framework as well as provided an Rtl Intervention Bank ● Use additional practice and textbook RTI resources for students to practice converting from polar to rectangular and graphing the conic sections. ● Topics will be broken down and assessed in smaller sections such as an assessment for graphing conic sections then an assessment for writing equations and identifying characteristics. ● Provide small group intervention to work on skills and concepts during group work such as homogeneous grouping. ● NJDOE resources ● Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org	● Provide small group intervention to work on skills and concepts during group work such as homogeneous grouping. ● NJDOE resources ● Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org ● Students may struggle to understand the notation associated with matrices. Have students use the given notation to practice writing a matrix.

***WHERETO**

W = Help the students know **WHERE** the unit is going and **WHAT** is expected. Help the teacher know **WHERE** the students are coming from (prior knowledge, interests).

H = **HOOK** all students and **HOLD** their interest.

E = **EQUIP** students, help them **EXPERIENCE** the key ideas and **EXPLORE** the issue.

R = Provide opportunities to **RETHINK** and **REVISE** their understanding and work.

E = Allow students to **EVALUATE** their work and its implications.

T = **TAILORED** to the different needs, interests, and abilities of learners.

O = **ORGANIZE** to maximize initial and sustained engagement as well as effective learning.

UNIT #: *Unit Title* Number of Days	**Unit 7:** *Limits, Continuity, and Derivatives* 30 days		
STAGE 1: DESIRED RESULTS ***What will students understand as a result of the unit? What are the BIG ideas?***			
ESTABLISHED GOALS: *(NJSLS-Mathematics)*	**Algebra** A-REI.A.2 A-CED.A.4 A-APR.C.5 **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.1.12.PB.1 9.2.12.CAP.6 9.4.12.CT.1 9.4.12.CT.2 9.4.12.CT.3		

ENDURING UNDERSTANDINGS: *(Students will understand that . . .)*	• The concept of limits can be applied to the asymptotic behavior of functions and average rate of change of a function. • The limit of a function is the value approached by f(x) as *x* approaches a given value or infinity. • We can use the limits and the difference quotient to solve for the instantaneous rate of change of any given function. • To apply the Continuity Test to determine if a function is continuous at a given point. • To differentiate between the three types of discontinuities and accurately identify them from a graph and algebraic function. • The Intermediate Value theorem can be used to prove intersections exist on a given interval.		
ESSENTIAL QUESTIONS: *(What provocative questions will foster inquiry, understanding, and transfer of learning?)*	• What is a limit and how is it used to approximate a function's value? • What are the conditions under which a limit does not exist? • What does it mean for a function to be considered continuous? • How can we differentiate between the three types of discontinuity and how can we determine them graphically / analytically? • How do we evaluate infinite limits and limits at infinity? • What is the Intermediate Value Theorem and how is it used to prove that appointments within an interval exist?		

STAGE 2: ASSESSMENT EVIDENCE ***What evidence will be collected to determine whether or not the understandings have been developed, the knowledge and skills attained, and the State Standards met? [Anchor the work in performance tasks that involve application, supplemented as needed by prompted work, quizzes, observations, etc.]***			
PERFORMANCE TASKS: *(Through what authentic performance tasks will students demonstrate the desired understandings?)* *(By what criteria will performances of understanding be judged?)*	• Group Activities • Projects • District and State Standardized Testing • Computer-Technology Based Projects • Independent assignments • Written Responses on real-world problems and non-routine problems. • Provide students with a table of data points. (a) Draw a scatter plot of the data that is provided. (b) Find a logistic regression model for the data. Find the limit of that model as time approaches infinity. (c) What can you conclude about the limit of the rabbit population growth in the county? (d) Provide a reasonable explanation for the population growth limit		
OTHER EVIDENCE: *(Through what other evidence (e.g. quizzes, tests, academic prompts, observations, homework, journals) will students demonstrate achievement of the desired results?)* *(How will students self-assess their learning?)*	• Classwork • Homework • Quizzes / Tests • Journals • Projects • Group Discussions • Class Discussions • Teacher Observations • Presentations		

	• Activities • Peer and self- evaluations • Rubrics • Error analysis • Web-based assessments • Benchmark assessments • LinkIt!		
RESOURCES:	• Contemporary PreCalculus (Digital Textbook) • Student workbook • Assessment resources • Graphing Calculators • Supporting videos • Chromebooks • Socrative • Youtube.com • Tiered Worksheet • Relevant websites such as Khan Academy, IXL.com, and Kuta Software • Canvas		
STAGE 3: LEARNING PLAN ***What learning experiences and instruction will enable students to achieve the desired results? Utilize the WHERETO* acronym to consider key design elements.***			
SKILLS AND TOPICS: *(What specific activities will students do and what skills will students know as a result of the unit?)*	• Calculate instantaneous velocities and acceleration using limits. • Use the properties of limits to evaluate one-sided limits, two sided limits and limits involving infinity. • Evaluate a limit numerically, analytically and graphically from the left and right sides. • Apply the Continuity Test to determine if a function is continuous at specific values and intervals. • Understand and apply the Intermediate Value Theorem.		

	● Evaluate infinite limits and limits at infinity. ● Evaluate the limit-definition of a derivative when given any function.		
CROSS-CURRICULAR / DIFFERENTIATION: *(What cross-curricular (e.g. writing, literacy, math, science, history, Career Readiness, Life Literacies, and Key Skills, technology) learning activities are included in this unit that will help achieve the desired results?)* *(What type of differentiated instruction will be used for Special Education, ELL, At Risk, and Gifted and Talented students?)*	**<u>Cross-Curricular Connections</u>:** **English Language Arts** **(RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)** ● Current event problems Exploratory Activities Project-Based Learning **Differentiation:** <u>General:</u> ● Using task cards ● Homogeneous and heterogeneous grouping when completing exploration or reinforcement activities. ● Creating graphic organizers ● Flipped classroom ● Open-ended collaboration projects and assignments with rubrics created by the students. <u>Special Education--</u> <u>Students with IEP/504 Plan</u>: ● Provide modifications and accommodations as listed in the student's IEP/504 Plan. ● Position students near a helping peer or have quick access to a teacher.		

	● Students will receive modifications such as a graphic organizer stating basic definitions such as how to evaluate limits, how to evaluate the cases of limits at infinity, and the 3 types of discontinuities. ● Provide teacher notes and homework answer keys so students can check work and use teacher notes to study. ● Use collaborative grouping strategies such as small groups where homogeneous grouping will be used so the teacher can provide additional help. ● Use online resources ● NJDOE resources <u>English Language Learners</u>: ● Place student next to same-language speaker, if possible ● Provide text to speech for math problems as well as allow the use of translation dictionaries or software. ● Provide word banks for translated vocabulary for words such as "limit, continuity, infinite, discontinuity, removable, intermediate value theorem, and asymptotes. ● Provide graphic organizers including the three types of discontinuities as well as the Continuity Test parameters. ● Providing context clues for all word problems indicating critical information. ● Modification plan provided with the guidance department with the language program at the school. ● NJDOE resources		

	● Adapt a strategy adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm Gifted and Talented Students: ● Tiered interventions following RtI framework as well as provided an Rtl Intervention Bank ● Use additional practice and textbook RTI resources for students to practice evaluating basic limits numerically, graphically and analytically. ● Topics will be broken down and assessed in smaller sections such as an assessment for evaluating limits 3 different ways and another assessment for continuity and discontinuity. ● Word problems for continuity will have key words and phrases highlighted for students to apply the Intermediate Value Theorem. ● Provide small group intervention to work on skills and concepts during group work such as homogeneous grouping. ● NJDOE resources ● Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org At-Risk Students: ● Processes should be modified: higher-order thinking skills, open-ended thinking, and discovery for limits numerically as well as graphically and analytically. ● Utilize project-based learning for greater depth of knowledge when		

	graphing limits both as x approaches c and when x approaches infinity. ● Utilize exploratory connections to higher-grade concepts when determining continuity/ discontinuity in functions. ● Contents should be modified: abstraction, complexity, variety, and organization. ● Derivations for all formulas such as intermediate value theorem and limits at infinity will be completed as a higher order task for students to master content material. ● Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations ● Learning environments should be modified: student-centered learning to discover topics, independence, openness, and complexity, groups varied. ● Use of web based resources such as http://www.tenmarks.com www.khanacademy.orggeogebra.org ● NJDOE resources		

***WHERETO**

W = Help the students know **WHERE** the unit is going and **WHAT** is expected. Help the teacher know **WHERE** the students are coming from (prior knowledge, interests).

H = **HOOK** all students and **HOLD** their interest.

E = **EQUIP** students, help them **EXPERIENCE** the key ideas and **EXPLORE** the issue.

R = Provide opportunities to **RETHINK** and **REVISE** their understanding and work.

E = Allow students to **EVALUATE** their work and its implications.

T = **TAILORED** to the different needs, interests, and abilities of learners.

O = **ORGANIZE** to maximize initial and sustained engagement as well as effective learning.