Class 11 Physics

Chapter 13 - Kinetic Theory

Some solved examples related to:

- Root mean square velocity of molecules of a gas

Detailed notes can be seen here

Solved example 13.12

A flask contains argon and chlorine in the ratio of 2:1 by mass. The temperature of the mixture is 27 °C. Obtain the ratio of (i) average kinetic energy per molecule, and (ii) root mean square speed v_{rms} of the molecules of the two gases. Atomic mass of argon = 39.9 u; Molecular mass of chlorine = 70.9 u

Solution:

Part (i):

1. We have the equation for the kinetic energy possessed by one molecule:

$\frac{1}{2}m\overline{v^2} = \frac{3}{2}K_BT$

2. We can apply this equation to individual gases:

• Kinetic energy possessed by one molecule of Argon = $\frac{1}{2}m\overline{v_{Ar}^2} = \frac{3}{2}K_BT$

• Kinetic energy possessed by one molecule of Chlorine = $\frac{1}{2}m\overline{v_{Cl}^2} = \frac{3}{2}K_BT$

3. We find that, both energies are the same

• So the required ratio will be **1:1**

Part (ii):

1. We have Eq.13.7: $v_{rms} = \left(\frac{3K_BT}{m}\right)^{\frac{1}{2}}$

2. We can apply this to individual gases:

• v_{rms} of one molecule of Ar = $\left(\frac{3K_BT}{m_{Ar}}\right)^{\frac{1}{2}}$

• v_{rms} of one molecule of Cl = $\left(\frac{3K_BT}{m_{Cl}}\right)^{\frac{1}{2}}$

3. So the required ratio is: $\left(\frac{3K_BT}{m_{Ar}}\right)^{\frac{1}{2}} : \left(\frac{3K_BT}{m_{Cl}}\right)^{\frac{1}{2}}$

$\Rightarrow$ The required ratio is: $\left(\frac{1}{m_{Ar}}\right)^{\frac{1}{2}} : \left(\frac{1}{m_{Cl}}\right)^{\frac{1}{2}}$

4. One mole of Ar atoms has a mass of 39.9 grams

♦ So mass of one atom of Ar = $m_{Ar} = \left(\frac{39.9\times10^{-3}}{6.02\times10^{23}}\right) = 6.63 \times 10^{-26}$ kg

5. One mole of Cl molecules has a mass of 70.9 grams

♦ So mass of one molecule of Cl = $m_{Cl} = \left(\frac{70.9\times10^{-3}}{6.02\times10^{23}}\right) = 1.18 \times 10^{-25}$ kg

6. Substituting these masses in (3), we get:

Required ratio is: **1.33 : 1**

7. Note:

• From the result in (5), we can infer that, Ar atom will be 1.33 times faster than the Cl_2 molecule. This is because, Ar atom is lighter than Cl_2 molecule

• The ratio 2:1 did not come in our calculations. This is because the composition of the mixture will not affect the speeds.

Solved example 13.13

Uranium has two isotopes of masses 235 and 238 units. If both are present in Uranium hexafluoride gas which would have the larger average speed ? If the atomic mass of fluorine is 19 units, estimate the percentage difference in speeds at any temperature.

Solution:

Part (i):

1. In the mixture, all molecules will be that of Uranium hexafluoride

    ♦ Some of those molecules will contain U235

    ♦ The others will contain U238

• Those molecules containing U235 will be lighter

2. In the previous example, we saw that, the lighter molecule will have greater average speed

• So the molecules containing U235 will have larger average speed

Part (ii):

1. In the previous example we saw that, the ratio between the average velocities is: $\left(\frac{1}{m_A}\right)^{\frac{1}{2}} : \left(\frac{1}{m_B}\right)^{\frac{1}{2}}$

♦Where m_A and m_B are the mass of one molecule of gas A and gas B

2. In our present case,

• m_A is the mass of one molecule of Uranium hexafluoride containing U235

• m_B is the mass of one molecule of Uranium hexafluoride containing U238 (Remember that, 'hexa' indicates six. So there will be six fluorine atoms in Uranium hexafluoride)

• So we get:

♦ $m_A = (235 + 6\times19) = 349$ units

♦ $m_B = (238+6\times19) = 352$ units

3. Substituting the above masses in (1), we get:

$$\overline{v_A} : \overline{v_B} = \left(\frac{1}{349}\right)^{\frac{1}{2}} : \left(\frac{1}{352}\right)^{\frac{1}{2}} = 1.0044$$

4. Thus we get: $\frac{\overline{v_A}}{\overline{v_B}} = 1.0044$

• So percentage difference = $\left(\frac{\overline{v_A} - \overline{v_B}}{\overline{v_B}}\right) \times 100$

$$= \left(\frac{\overline{v_A}}{\overline{v_B}} - 1\right) \times 100 = 0.44\%$$
