Stage 1

Prove by induction that $11^n - 6$ is divisible by 5 for every positive integer n.

$$11^n - 6 \text{ is divisible by } 5.$$

Base Case: When $n = 1$ we have $11^1 - 6 = 5$ which is divisible by 5. So $P(1)$ is correct.

Induction hypothesis: Assume that $P(k)$ is correct for some positive integer k. That means $11^k - 6$ is divisible by 5 and hence $11^k - 6 = 5m$ for some integer m. So $11^k = 5m + 6$.

Stage 2

Induction step: We will now show that $P(k+1)$ is correct. Always keep in mind what we are aiming for and what we know to be true. In this case we want to show that $11^{k+1} - 6$ can be expressed as a multiple of 5, so we will start with the formula $11^{k+1} - 6$ and we will rearrange it into something involving multiples of 5. At some point we will also want to use the assumption that $11^k = 5m + 6$.

$$\begin{aligned} 11^{k+1} - 6 &= (11 \times 11^k) - 6 && \text{by the laws of powers} \\ &= 11(5m + 6) - 6 && \text{by the induction hypothesis} \\ &= 11(5m) + 66 - 6 && \text{by expanding the bracket} \\ &= 5(11m) + 60 && \\ &= 5(11m + 12) && \text{since both parts of the formula have a common factor of 5.} \end{aligned}$$

As $11m + 12$ is an integer we have that $11^{k+1} - 6$ is divisible by 5, so $P(k+1)$ is correct. Hence by mathematical induction $P(n)$ is correct for all positive integers n.

Stage 3

Validate the proof.