Mathematics Curriculum Guide

Unit 5: Fractions

Fourth Grade

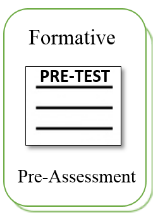
Curriculum Components

Class Discussion

Class discussion is a forum for students to discuss their work on an investigative task or problem. The teacher will facilitate the discussion by asking assessing and advancing questions that deepen student understanding and advance learning. The curriculum guide includes questions that may be used; however, questions should not be limited to this list as the discussion may prompt the teacher to use additional question prompts. A class discussion is utilized when anticipated student solution strategies and entry points are not as varied as tasks warranting a math congress.

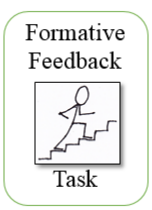
Content Standard(s)

The daily focus content standards are listed at the beginning of each day. Lessons or tasks may include other standard(s); if they are not listed, they are not the primary focus for the day.



Formative Assessment

Formative assessment tools and techniques provide the teacher with data on student understanding of concepts. These are presented at various points throughout the unit, and multiple options are often presented. These options include research-based mathematical formative assessment techniques journal prompts, and/or specific exit tickets linked to guided instruction/mini-lesson. Necessary modifications to instruction should be made based on this data.

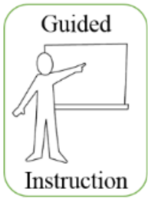


Formative Feedback Task

A formative feedback task is used solely for the purpose of gathering individual student data, providing feedback, and ultimately advancing each student's learning. It should never be graded. The task is a problem students will complete at a strategic point in the unit. The student will demonstrate current understanding of a big mathematical idea, concept, and/or strategy recently taught. This task should be completed with no assistance from the teacher or peers. The teacher will provide the student oral and written feedback through assessing and advancing questions. The ultimate goal of this feedback is to advance the individual learning of each student. Data from these tasks should be used for forming groups, as well as invitational group and/or intervention instructional decisions.

Formative Pre-Assessment

The pre-assessment should be administered, and analyzed, prior to beginning the unit, and results should guide instructional decisions. Students will complete the pre-assessment on paper. The data will provide evidence of: what students already know, understand, and are able to do, individual strengths and weaknesses, learning differences among students, and students in need of support or enrichment. The data should guide the teacher in the following decisions: Time allotment for various learning targets, guidance with whole group instruction, differentiated instruction, formation of groups, small group instructional focus, and identification of misconceptions.

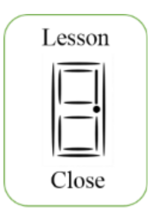


Guided Instruction/Mini-Lesson
A guided instruction/mini-lesson typically follows investigative tasks or problems. It serves to further develop conceptual and procedural knowledge. Segments from identified Eureka Math lessons will be utilized during the guided instruction/mini-lesson in the curriculum guide. The scripted Eureka lesson components serve as a guide for teachers (the script should not be read). Adjustments should be made based on student needs and allocated time. A guided instruction/mini-lesson is followed by the problem set.

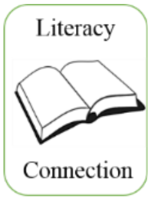
Homework
Homework from the Eureka Math student workbook will follow the guided instruction/mini-lesson. If homework in the guide does not coincide with a designated homework day, it may be assigned on a subsequent day. However, it should not be assigned before it appears in the guide. It is suggested that math homework be given on Monday, Tuesday, and Thursday of each week. The teacher may choose to select specific problems or ask that it be completed in its entirety, depending on the length of the assignment.

Investigative Task
New concepts are introduced through rigorous, investigative problems or tasks where students have opportunities to collaborate and construct conceptual knowledge. These tasks have multiple entry points and a variety of solution strategies, allowing students at all levels to build on individual background knowledge. While students are collaborating to complete these tasks, the teacher will confer with students asking assessing and advancing questions and facilitating productive struggle. Investigative tasks are followed by a math congress or classroom discussion.

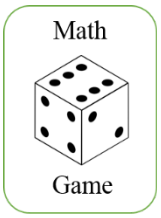
Learning Targets
The learning targets for each day are listed. The teacher will engage students in discussing these targets. This typically occurs at the beginning of the lesson; however, a teacher may decide to discuss a learning target at a different point in the lesson for strategic reasons. At the end of math instruction, the teacher should briefly revisit these with the class to assess achievement of each target. There is no time allotment in the curriculum guide for engaging in learning targets as it is considered part of the lesson introduction.

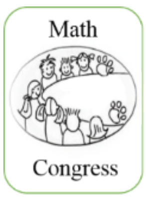


Lesson Close
To close the lesson, teachers will engage students in a brief assessment of daily learning target achievement, and what they will do in the subsequent math lesson. There is no time allotment in the curriculum guide for closing the lesson as it is considered part of the lesson wrap-up.



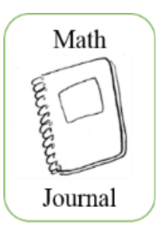
Literacy Connection
Specific books related to content are recommended through the Literacy Connection. They are introduced at strategic points during the unit. Books may be read on or after they appear in the guide. There is no time allotment in the guide as the teacher will determine if and when the books will be utilized (as a read-aloud, during Math Workshop, etc.).



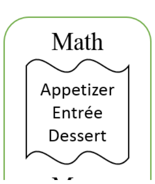


Math Congress
Math congress is an instructional, discourse strategy where students are provided a forum to discuss and listen actively to one another's solution strategies and thinking. It follows an investigative task where multiple entry points and solution strategies are evident. Students will justify their own thinking and reflect on others' thinking. It provides students with a positive and safe setting to share and challenge ideas. Students should be reassured that their ideas are valued and contribute to the learning of the entire class. It is recommended that students gather in a central classroom location (carpet area) for math congress. The teacher role in math congress is to scaffold ideas through pre-selected student work, facilitate discussion by posing assessing and advancing questions, and guide students to formulate mathematical connections. Student conversations about the work should be the focus. It is suggested the teacher utilize a combination of the following instructional practices to assess understanding and reinforce ideas and strategies communicated by students: use of hand signals to represent agreement/disagreement, turn-and-talk to reinforce understanding, and student restatement of others' ideas.

Math Games
Math games provide an engaging way to facilitate strategic mathematical thinking as students apply various strategies to solving problems. Games enhance computational fluency and serve as an inviting method for practicing skills and deepening understanding.



Math Journal Entry
Math journal entries provide students with a format for recording math terms, definitions, and examples. These entries are presented at various points throughout the lessons, once the associated ideas have been investigated and reinforced through the guided instruction/mini-lesson. There is no time allotment in the curriculum guide for the math journal entries, as there is flexibility on when this could be completed (during math workshop, upon completion of problem set, etc.). (The math journal will also be utilized for journal prompts and other formative assessments.)



Math Menu
A math menu of differentiated tasks provides students with opportunities to apply concepts learned in a variety of contexts. New tasks will be introduced each day.

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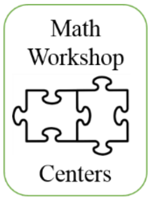
Students are expected to complete a minimum of four total tasks upon conclusion of math menu, one being a required task for all students.



Math Workshop

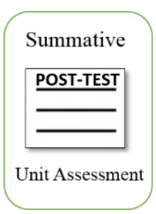
During Math Workshop students rotate to five different centers and complete designated independent tasks. One center will be a teacher-led, differentiated invitational group. Formation of groups for math workshop should be based on data from formative feedback tasks and other formative assessments. In this center, the teacher may choose to reteach a concept from a formative feedback task or formative assessment, expand on a guided instruction/mini-lesson previously taught, expand on a problem set using debrief questions, review homework, reteach a concept, conduct a cumulative review, or provide other tasks specific to student needs.

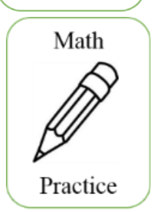
Number Talk

Number Talks are a daily, routine class discussion based on a structured strand of computational problems. It is tool to help students develop number sense and computational fluency. In the curriculum guide, a number talk is included for most days, and is designed to support standards in the unit. The focus strategies are listed in the heading, as well as the primary anticipated student strategy for each problem in the strand. The time allotment for a number talk is 15 minutes. A number talk should occur a minimum of three days per week. (If necessary, the teacher may choose to utilize this time for another lesson component up to two times per week.)

Performance Task

A Performance Task is a summative assessment completed either at the end of a major segment of a unit or at the end of the unit. These tasks should be completed independently, with no assistance from the teacher or peers. The tasks provide the student with an opportunity to apply the learning outcomes in another context. These tasks may be graded.





Problem Set

A problem set contained in the Eureka Math student workbook will follow the guided instruction/mini-lesson. Students should do their best to complete the designated problems within ten minutes. The teacher may specify certain problems to begin with, so that critical problems are completed by all students. Subsequent math workshop time may be utilized to finish problem sets (after center work is completed), or a number talk may be omitted for the week and used as additional problem set time (provided a minimum of three are completed during the week).

Summative Unit Assessment

The on-line summative unit assessment should be completed at the end of the unit. The objective is to evaluate student learning outcomes compared to standards and benchmarks. In addition, this data will be reviewed at an aggregate level to reflect on overall unit performance and instruction.

Unit 5 – Fractions

OVERVIEW

In this unit, students will explore fraction equivalence and broaden this understanding to mixed numbers. They will apply this understanding to compare fractions and mixed numbers using benchmark fractions and various models. Students will develop a conceptual understanding of decomposing fractions. They will build on their knowledge of whole number operations to construct an understanding of adding and subtraction fractions with like denominators as well as multiplying fractions by a whole number. Students will deepen their understanding of equivalent fractions justifying their thinking with models. They will develop a conceptual understanding of multiplying or dividing the numerator and denominator by the same number, which will result in an equivalent fraction as a result of multiplying or dividing by one. Students will extend their knowledge of fractions by investigating fractions greater than or equal to one, including decomposing, utilizing the distributive property to multiply by a mixed number by a whole number, and add and subtraction mixed numbers with like denominators. They will apply place value concepts and number sense to efficiently solve real-world, multi-step problems.

CCRS Content Standards:

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Represent and interpret data.

4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Unit 5 – Fractions

Vertical Alignment

3rd Grade

3.NF.1
Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.
3.NF.2
Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
3.NF.3
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate

4th Grade

4.OA.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
4.NF.1
Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
4.NF.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
4.NF.3
Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an

5th Grade

5.OA.3
Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane
5.NF.1
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
5.NF.2
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally, and assess the reasonableness of answers.
5.NF.5
Interpret multiplication as scaling (resizing), by:
a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case), explaining why multiplying a given number by a fraction less than 1 results in a

4/4 and 1 at the same point of a number line diagram. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

3.MD.4
Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. 3.G.2
Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

4.MD.4
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

product smaller than the given number, and relating the principle of fraction equivalence a/b = (n x a)/(n x b) t o the effect of multiplying by 1.

5.NF.7
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general by reasoning about the relationship between multiplication and division. However, division of a fraction by a fraction is not a requirement at this grade.)
a. Interpret division of a unit fraction by a nonzero whole number, and compute such quotients.
b. Interpret division of a whole number by a unit fraction, and compute such quotients.
c. Solve real-world problems involving division of unit fractions by nonzero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

5.NF.4a
Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
a. Interpret the product (a/b) x q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x q ÷ b.

5.MD.2
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.

Unit Vocabulary

- Benchmark – Benchmark is a standard or familiar reference point by which something is measured.
- Common denominator – A common denominator is when two or more fractions have the same denominator.
- Denominator – A denominator names the fractional unit.
- Fraction greater than one – A fraction greater than one is a fraction with a numerator that is greater than the denominator.
- Line plot – A line plot is a display of data on a number line, using an x or another mark to show frequency.
- Mixed number – A mixed number is a number made up of a whole number and a fraction.
- Numerator – A numerator indicates the number of fractional units.

Unit 5 – Fractions

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31	Number Talk (15 min) Math Workshop (75 min): Eureka Math-Lesson 22 Sprint, 4.NF.2 Task(s),Eureka Math-Lesson 24 & 25 Problem Sets, 4.MD.4 Task(s), & Invitational Group	4.NF.2 4.NF.3 4.NF.4 4.MD.4	
32	Number Talk (15 min) Guided Instruction/Mini-Lesson: Eureka Math – Lesson 30 & 31 (20 min) Problem Set: Eureka Math – Lesson 30 & 31 (10 min) Formative Assessment: Eureka Math – Lesson 30 & 31 (5 min) Math Menu: Dividing Up the Land & Fractions Cookies Bakery (40 min) Homework: Eureka Math – Lesson 30 & 31	4.NF.3	
33	Number Talk (15 min) Guided Instruction/Mini-Lesson: Eureka Math – Lesson 32 & 33 (20 min) Problem Set: Eureka Math – Lesson 32 & 33 (10 min) Formative Assessment: Eureka Math – Lesson 32 & 33 (5 min) Math Menu: Jumping Rope & Fractions in a Box (40 min) Homework: Eureka Math – Lesson 32 & 33	4.NF.3	
Repeated Addition of Fractions as Multiplication			
34	Number Talk (15 min) Guided Instruction/Mini-Lesson: Eureka Math – Lesson 35 & 36 (20 min) Problem Set: Eureka Math – Lesson 35 & 36 (10 min) Formative Assessment: Eureka Math – Lesson 35 & 36 (5 min) Math Menu: Trading Blocks & Jellybean Fractions (40 min) Homework: Eureka Math – Lesson 35 & 36	4.NF.4	
35	Number Talk (15 min) Guided Instruction/Mini-Lesson: Eureka Math – Lesson 37 & 38 (20 min)	4.NF.4	

Unit 5 – Fractions

	Problem Set: Eureka Math – Lesson 37 & 38 (10 min) Formative Assessment: Eureka Math – Lesson 37 & 38 (5 min) Math Menu: Pizza Dough & Insect Collection (40 min) Homework: Eureka Math – Lesson 37 & 38		
37	Class Discussion – Required Menu Problem: Dividing Up the Land (30 min) Performance Task: Chocolate Bars (30 min) Math Menu: Wrap-up (30 min)	4.NF.3 4.NF.4	
38	Summative Unit Assessment		

Unit 5 – Fractions

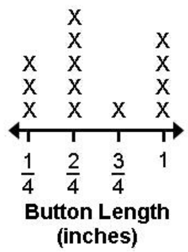
PRIOR TO BEGINNING UNIT

Formative PRE-TEST Pre-Assessment

Unit 5 – Fractions – Pre-Assessment (15 min)
Students will complete the pre-assessment on paper. The pre-assessment should be administered, and analyzed, prior to beginning the unit (approximately one week), and results should guide instructional decisions. The data will provide evidence of: what students already know, understand, and are able to do, strengths and weaknesses of individual students, learning differences among students, students in need of support, and students who may need enrichment. The data should guide the teacher in the following decisions: Time allotment for various learning targets, guidance with whole group instruction, differentiated instruction, formation of groups, invitational group instructional focus, intervention plans, and identification of misconceptions.

Pre-Assessment Questions	Standards	Source
1. Place the fraction ¾ on the number line. 0 1 What is an equivalent fraction for ¾? Place the equivalent fraction on the number line. 0 1	4.NF.1	(SanGiovanni, 2015)
2. Which is greater: 4/10 or 5/8? Explain how you know in the space below. Use pictures and/or words in explanation.	4.NF.2	(SanGiovanni, 2015)
3. Jo has a piece of tape that is 7/8 inch long. She cuts the tape into two pieces. One piece is 3/8 inch long. How long is the other piece of tape? Explain how you know.	4.NF.3	(SanGiovanni, 2015)
4. Courtney ran on a path that was $1\ ^{1}/_{5}$ of a mile in length. She ran the path three times. What is the total distance that Courtney ran? Draw a picture to represent your thinking.	4.NF.4	(SanGiovanni, 2015)
5. Jacqueline has a collection of colorful buttons represented by the line plot below.	4.MD.4	(Adapted from Scantron)

Unit 5 – Fractions

 What is the combined length of one of the buttons she has the most of and one of the buttons Jacqueline has the fewest of?		

<u>Answer Key</u>

1. 

   Answers will vary but should be equivalent to 3/4.
2. Five-eighths is greater than four-tenths.
3. The other piece is 4/8 inch long. Students may simplify the answer to ½ inch long, but this is not required.
4. Courtney ran $3\,{}^{3}/_{5}$ miles. Pictures may vary but should represent $3 \times 1\,{}^{1}/_{5} = 3\,{}^{3}/_{5}$.
5. The combined length is 5/4 inches or 1 ¼ inches.

Unit 5 – Fractions

DAY 1

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Fractions (15 min)

Adapted from NCTM, www.nctm.org/profdev)

1/3

Describe what it looks like	Where would it fall on a number line?
Is it closer to 0, ½, or 1?	Is it a unit fraction? If not, decompose into the sum of unit fractions.

Answer Key
Description: Answers will vary
Number Line: Divide space between 0 and 1 into 3 sections, and mark 1/3.
Estimate: It is between 0 and ½, so it rounds to ½.
Unit Fractions: It is a unit fraction.

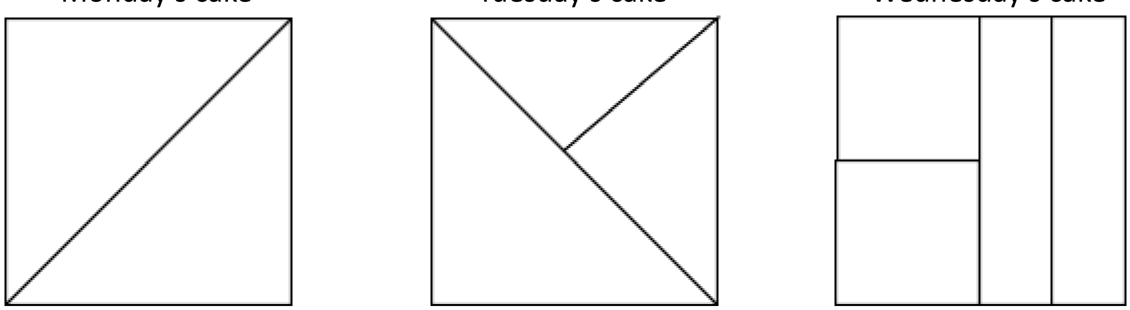
Learning Targets

- I can make sense of problems and persevere in solving them.

Weird Pieces of Cake (60 min)
(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)
The outcome of this task is to construct conceptual knowledge of equivalent fractions and decomposing fractions as a sum of unit fractions.

Part 1: A baker makes square cakes and decides to cut the pieces different each day of the week. If she wants to make eight dollars for the whole cake, how much money will each individual piece sell for?

Monday's cake Tuesday's cake Wednesday's cake



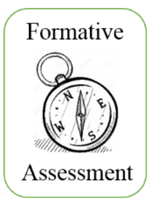
Part 2: While shopping on Wednesday, Martina says to the baker, "Buying two pieces of cake today will cost the same as one piece on Monday." Is Martina correct? Explain why or why not.

Answer Key
Part 1: On Monday, each piece sells for four dollars. On Tuesday, the large piece sells for four dollars, and the small pieces are two dollars each. On Wednesday, the slices are two dollars each.
Part 2: Martina is correct because Monday's slices are half of the whole cake, and Wednesday's slices are two fourths of the whole cake. One half equals two fourths.

Supporting the Investigative Task
As students work, the teacher will confer with them making notes about generalizations students are making.

- Decomposing each cake into the smallest size piece
- Cake slices can be cut differently but represent equivalent fractions of the cake (Wednesday's cake)
- Dividing the total cost of the cake into the fractional amounts of each slice
- The fractional amount of two slices can equal the fractional amount of another slice

<u>Differentiation</u>
- Support – If students are having difficulty, ensure they are breaking apart the whole into equal size pieces.
- Enrichment – The baker is considering how to cut the square cake for Thursday. She wants to slice the cake into seven slices. Four slices are each $^{1}/_{12}$ of the cake, two slices are each one fourth of the cake, and there is one remaining slice. What fraction of the cake is the remaining slice? Draw a model to prove your thinking.



Formative Assessment (15 min)
(<u>Mathematics Formative Assessment</u> by P. Keeley & C. Tobey)
This formative assessment may be administered during a gallery walk.
- <u>Strategy Harvest</u>
  "In this strategy, students complete a problem solving task and then circulate among their peers to find students who used a strategy different from theirs to solve the problem. Students record the other strategies and describe how the strategy is different from the one they used. During the process, students give feedback to each other on their strategy."

My Strategy	___________'s Strategy
___________'s Strategy	___________'s Strategy

Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Weird Pieces of Cake will be discussed in math congress, and they will continue decomposing fractions.

Unit 5 – Fractions

DAY 2

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

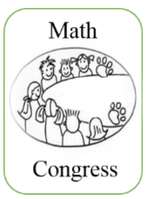
Focus: Fractions (15 min)
Adapted from NCTM, www.nctm.org/profdev)

Describe what it looks like	Where would it fall on a number line?
3/4	
Is it closer to 0, ½, or 1?	Is it a unit fraction? If not, decompose into the sum of unit fractions.

Answer Key
Description: Answers will vary
Number Line: Divide space between 0 and 1 into 4 sections, and mark ¾.
Estimate: It is between ½ and 1, so it rounds to 1.
Unit Fractions: ¼ + ¼ + ¼ = ¾

Learning Targets

- I can make sense of problems and persevere in solving them.
- I can explain equivalent fractions using visual models.
- I can decompose fractions into unit fractions.



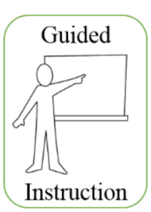
Math Congress – Weird Pieces of Cake (35 min)
Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

Example of Math Congress structure and sequence based on anticipated student strategies:

1. Decomposing each cake into the smallest size piece
2. Cake slices can be cut differently but represent equivalent fractions of the cake (Wednesday's cake)
3. Dividing the total cost of the cake into the fractional amounts of each slice
4. The fractional amount of two slices can equal the fractional amount of another slice

The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:

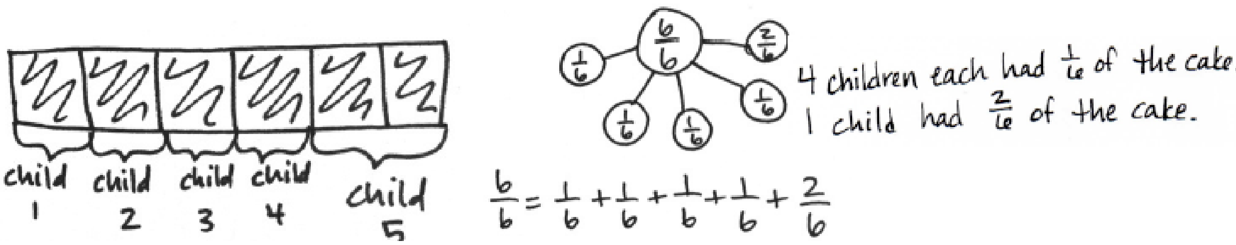
- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.



Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lessons 2 & 3 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

Application Problem (5 min)

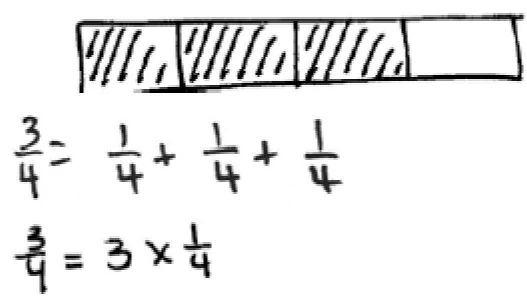
Mrs. Salcido cut a small birthday cake into 6 equal pieces for 6 children. One child was not hungry, so she gave the birthday boy the extra piece. Draw a tape diagram to show how much cake each of the five children received.



Note: This Application Problem is a review of the material presented in Lesson 1 and prepares students for the more advanced portion of this lesson objective that they encounter in today's lesson.

<u>Concept Development (20 min)</u>
Materials: (S) Personal white board

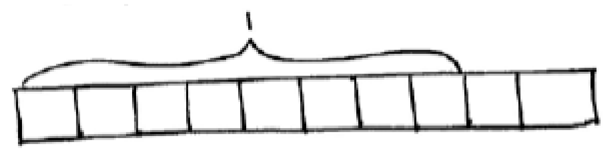
Problem 1: Express a non-unit fraction less than 1 as a whole number times a unit fraction using a tape diagram.
T: Look back at the tape diagram that we drew in the Application Problem. What fraction is represented by the shaded part?
S: 3/4.
T: Say 3/4 decomposed as the sum of unit fractions.
S: 3/4 = 1/4 + 1/4 + 1/4.
T: How many fourths are there in 3/4?
S: 3.
T: We know this because we count 1 fourth 3 times. Discuss with a partner. How might we express this using multiplication?



S: We have 3 fourths. That's 1/4 + 1/4 + 1/4 or three groups of 1 fourth. Could we multiply 3 × 1/4?
T: Yes! If we want to add the same fraction of a certain amount many times, instead of adding, we can multiply. Just like you multiplied 6 copies 4 times, we can multiply 1 fourth 3 times. What is 3 copies of 1/4?
S: It's 3/4. My tape diagram proves it!
Repeat with 2/3 and 7/8. Instruct students to draw a tape diagram to represent each fraction (as on the previous page), to shade the given number of parts. Then, direct students to write an addition number sentence and a multiplication number sentence.

Problem 2: Determine the non-unit fraction greater than 1 that is represented by a tape diagram, and then write the fraction as a whole number times a unit fraction.
T: (Project the tape diagram of 10/8 as shown below.) What fractional unit does the tape diagram show?

S: It shows tenths! →It shows eighths!
T: We first must identify 1. It’s bracketed here. (Point.) How many units is 1 partitioned into?
S: 8.
T: The bracketed portion of the tape diagram shows 8 fractional units. What is the total number of eighths?

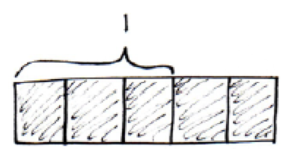


$$\frac{10}{8} = \frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$$

$$\frac{10}{8} = 10 \times \frac{1}{8}$$

S: 10.
T: What is the fraction?
S: 10 eighths.
T: Say this as an addition number sentence.
Use your fingers to keep track of the number of units as you say them.
S: 10/8 = 1/8 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8.
T: As a multiplication number sentence?
S: 10/8 = 10 × 1/8.
T: What are the advantages of multiplying fractions instead of adding?
S: It’s easier to write. →It’s faster. →It’s more efficient.

Problem 3: Express a non-unit fraction greater than 1 as a whole number times a unit fraction using a tape diagram.



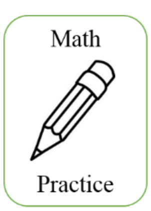
$$\frac{5}{3} = 5 \times \frac{1}{3}$$

$$\frac{5}{3} = (3 \times \frac{1}{3}) + (2 \times \frac{1}{3})$$

T: Let’s put parentheses around 8 eighths so that we can see 10 eighths can also be written to show 1 and 2 more eighths. (Write 10/8 = (8 × 1/8) + (2 × 1/8).
T: Discuss with your partner how to draw a tape diagram to show 5 thirds.
S: I can draw one unit at a time. The units are thirds, so I’ll draw five small rectangles together. →I know 5 thirds is greater than 1, so I’ll draw 1. That’s 3 thirds. So, then I can draw another 1. I’ll just shade 5 parts. →I will draw a long rectangle and break it into 5 equal parts. Each part represents 1 third. I’ll bracket 3 thirds to show 1.
T: How can we express 5/3 as a multiplication expression?
S: We have five thirds. That’s 5 × 1/3.
T: Is there another way we can express 5/3 using multiplication?
S: Can we express the 1 as 3 × 1/3 and then add 2 × 1/3?
T: Yes! We can use multiplication and addition to decompose fractions.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Offer an alternative to Problem 2 on the Problem Set for students working above grade level. Challenge students to compose a word problem of their own to match one or more of the tape diagrams they construct for Problem 2. Always offer challenges and extensions to learners as alternatives, rather than additional *busy* work.



Problem Set (10 min)
Eureka Math – Lessons 3 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the

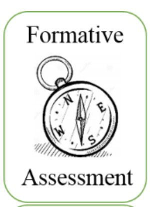
assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

You may choose to use any combination of the questions below to lead the discussion.

- In all of the problems, why do we need to label 1 on our tape diagrams? What would happen if we did not label 1?
- What is an advantage to representing the fractions using multiplication?
- What is similar in Problems 3(c), 3(d), and 3(e)? Which fractions are greater than 1? Which is less than 1?
- Are you surprised to see multiplication sentences with products less than 1? Why?
- In our lesson, when we expressed 5/3 as (3 × 1/3) + (2 × 1/3), what property were we using?
- How is multiplying fractions like multiplying whole numbers?



Formative Assessment (10 min)
- Eureka Math – Lessons 2 & 3 Exit Ticket (Module 5)

Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time they will continue investigating decomposing fractions.

Homework
Eureka Math – Lessons 2 & 3 (Module 5)

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Fractions (15 min)

Adapted from NCTM, www.nctm.org/profdev)

Describe what it looks like	Where would it fall on a number line?
6/8	
Is it closer to 0, ½, or 1?	Is it a unit fraction? If not, decompose into the sum of unit fractions.

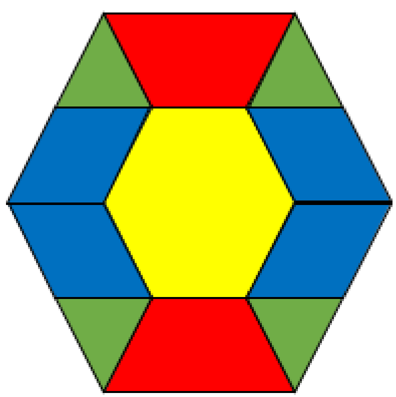
Answer Key
Description: Answers will vary
Number Line: Divide space between 0 and 1 into 8 sections, and mark $^{6}/_{8}$.
Estimate: It is between ½ and 1, so it rounds to 1.
Unit Fractions: $^{1}/_{8} + ^{1}/_{8} + ^{1}/_{8} + ^{1}/_{8} + ^{1}/_{8} + ^{1}/_{8} = ^{6}/_{8}$

Learning Targets

- I can make sense of problems and persevere in solving them.

Freedom Quilt (60 min)
(Written by teachers from Trussville City Schools)
The outcome of this task is to construct conceptual knowledge of decomposing fractions into a sum of unit fractions.

Students in Ms. Cook's class have been studying about the quilts, known as Freedom Quilts, which may have been used as a secret code for fugitive slaves during the days of the Underground Railroad. Some historians say that the quilts were used to alert fugitive slaves of food, the way north, and danger. Because quilts were very popular, they could be displayed on porches or fences without causing suspicion. On one of the quilts, each patch of the quilt had this design.



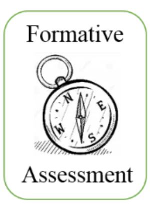
1. Determine what fraction of each patch is blue, red, yellow, and green.

2. Write an addition sentence for blue showing how each piece could be added together for a sum that is equal to the fraction. Repeat for red and green.

<u>Answer Key</u>
1. The patch is 8/24 or 4/12 blue, 6/24 or 2/8 red, 6/24 or 1/4 yellow, and 4/24 green. In simplest form the patch is 1/3 blue, 1/6 green, 1/4 yellow and 1/4 red, but students are <u>not</u> expected to determine the simplest form of these fractions.
2. Blue – 2/24 + 2/24 + 2/24 + 2/24 = 8/24 or 1/12 + 1/12 + 1/12 + 1/12 = 4/12
Red – 3/24 + 3/24 = 6/24 or 1/8 + 1/8 = 2/8
Green – 1/24 + 1/24 + 1/24 + 1/24 = 4/24

<u>Supporting the Investigative Task</u>
As students work, the teacher will confer with them making notes about generalizations students are making.
- Replacing blocks with common or like pieces (common denominator)
- An equation for each type of pattern block including a fractional value for each piece that equals the total fractional amount that pattern block represents of the quilt
- The simplest way to write the answer.

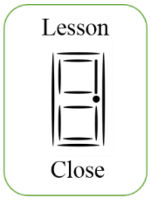
<u>Differentiation</u>
- Support – If students are having difficulty, have them explain whether or not the pieces should be the same size and ask them how to build the quilt with pieces that are the same size.
- Enrichment – Design a new Freedom Quilt patch using between ten to fifteen pieces of at least four different colors. Repeat the investigation questions using your Freedom Quilt patch.



Formative Assessment (15 min)
(<u>Mathematics Formative Assessment</u> by P. Keeley & C. Tobey)
This formative assessment may be administered during a gallery walk.
- <u>Strategy Harvest</u>

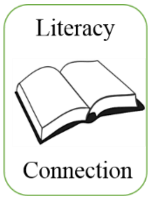
  "In this strategy, students complete a problem solving task and then circulate among their peers to find students who used a strategy different from theirs to solve the problem. Students record the other strategies and describe how the strategy is different from the one they used. During the process, students give feedback to each other on their strategy."

My Strategy	___________'s Strategy
___________'s Strategy	___________'s Strategy



Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Freedom Quilt will be discussed in math congress, and they will continue decomposing fractions.



Literacy Connection

These books may be utilized as a read-aloud to reinforce specific concepts at this point in the unit.

- <u>Sweet Clara and the Freedom Quilt</u> by Deborah Hopkinson
- <u>Follow the Drinking Gourd</u> by Jeanette Winter
- <u>The Secret to Freedom</u> by Marcia K. Vaughan
- <u>Under the Quilt of Night</u> Deborah Hopkinson

Unit 5 – Fractions

DAY 4

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Fractions (15 min)

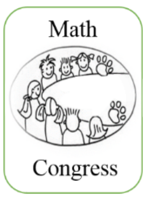
Adapted from NCTM, www.nctm.org/profdev)

Describe what it looks like	Where would it fall on a number line?
1 1/4	
Is it closer to 0, ½, or 1?	Is it a unit fraction? If not, decompose into the sum of unit fractions.

<u>Answer Key</u>
Description: Answers will vary
Number Line: Divide space between 0 and 1 into 4 sections, and mark an additional ¼ beyond 1.
Estimate: It is greater than 1, so it rounds to 1.
Unit Fractions: ${}^1/_4 + {}^1/_4 + {}^1/_4 + {}^1/_4 + {}^1/_4 = 1\frac{1}{4}$

Learning Targets

- I can make sense of problems and persevere in solving them.
- I can decompose fractions into the sum of unit fractions.



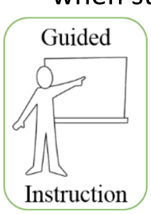
Math Congress – Freedom Quilt (35 min)
Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

Example of Math Congress structure and sequence based on anticipated student strategies:

1. Replacing blocks with common or like pieces (common denominator)
2. An equation for each type of pattern block including a fractional value for each piece that equals the total fractional amount that pattern block represents of the quilt
3. The simplest way to write the answer.

The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:

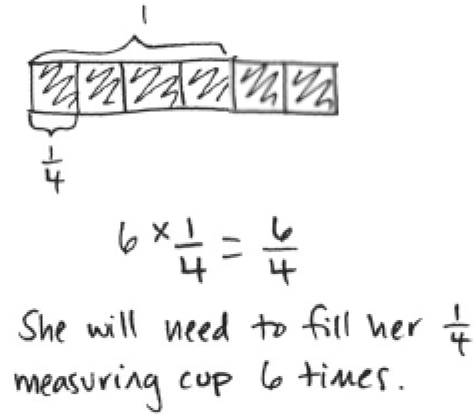
- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.



Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lesson 4 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Decompose fractions into sums of smaller unit fractions using tape diagrams.

Application Problem (5 min)

A recipe calls for 3/4 cup of milk. Saisha only has a 1/4-cup measuring cup. If she doubles the recipe, how many times will she need to fill the 1/4 cup with milk? Draw a tape diagram, and record as a multiplication sentence.



Note: This Application Problem reviews students' knowledge of fractions from Lesson 3 and prepares them for today's objective of decomposing unit fractions into sums of smaller unit fractions.

Concept Development (20 min)

Materials: (S) Personal white board

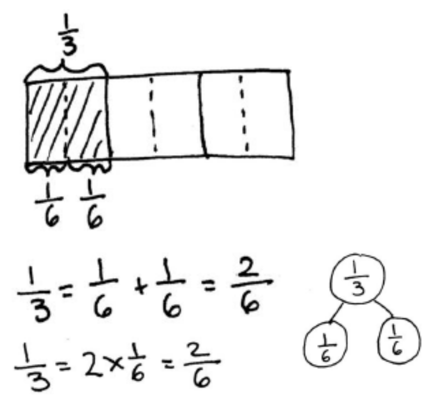
NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Cuisenaire rods can be used to model 1 whole (brown), 2 halves (pink), 4 fourths (red), and 8 eighths (white). If concrete Cuisenaire rods are unavailable or otherwise challenging, virtual rods can be found at the link below:

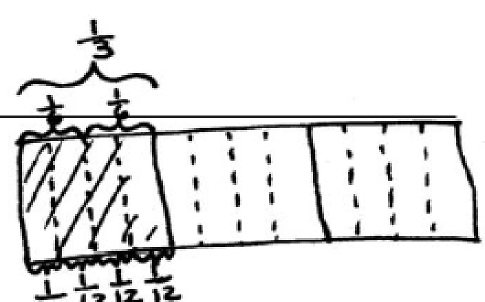
http://nrich.maths.org/4348.

Problem 1: Use tape diagrams to represent the decomposition of 1/3 as the sum of unit fractions.

T: Draw a tape diagram that represents 1, and shade 1 third. Decompose each of the thirds in half. How many parts are there now?
S: 6.
T: What fraction of 1 does each part represent?
S: 1 sixth.
T: How many sixths are shaded?
S: 2 sixths.
T: What can we say about 1 third and 2 sixths?
S: They are the same.
T: How can you tell?
S: They both take up the same amount of space.
T: Let's write that as a number sentence: 1/3 = 1/6 + 1/6 = 2/6.



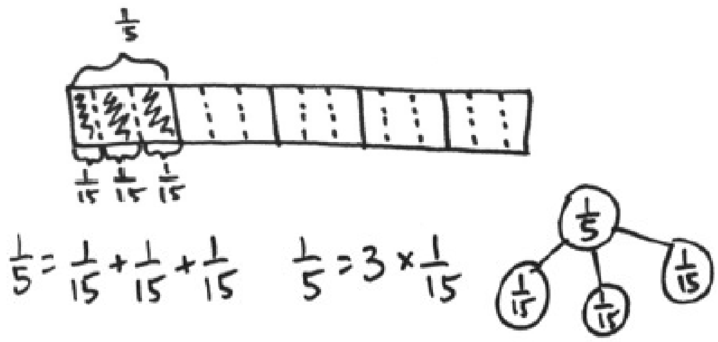
T: Now, decompose each sixth into 2 equal parts on your tape diagram. How many parts are in 1 now?
S: 12.
T: What fractional part of 1 does each piece represent?
S: 1 twelfth.
T: How many twelfths equal 1/6?
S: 2/12 equals 1/6.
T: Work with your partner to write a number sentence for how many twelfths equal 1/3.
S: (Write 1/3 = 1/6 + 1/6 = (1/12 + 1/12) + (1/12 + 1/12).
T: We can put parentheses around two groups of 1 twelfth to show that each combines to make 1/6.



$$\frac{1}{3}=\left(\frac{1}{12}+\frac{1}{12}\right)+\left(\frac{1}{12}+\frac{1}{12}\right)$$

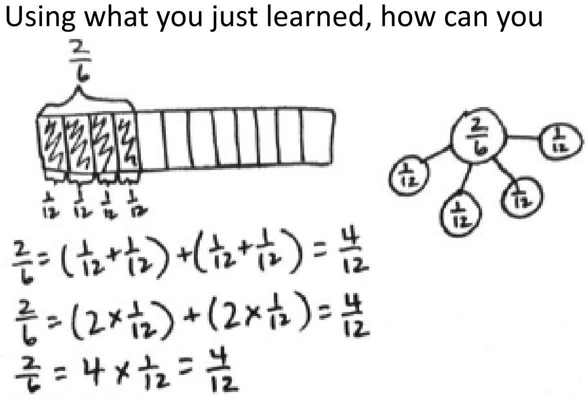
T: How can we represent this using multiplication?
S: (Write 1/3 = (2 × 1/12)+ (2 × 1/12) →1/3 = 4 × 1/12= 4/12.)

Problem 2: Use tape diagrams to represent the decomposition of 1/5 and 2/5 as the sum of smaller unit fractions.
T: Draw a tape diagram, and shade 1
5. Decompose each of the fifths into 3 equal parts. How many parts are there now?
S: There are 15 parts.
T: What fraction does each part represent?
S: 1/15.
T: Write an addition sentence to show how many fifteenths it takes to make 1 fifth.
S: (Write 1/5 = 1/15 + 1/15 + 1/15 = 3/15.)

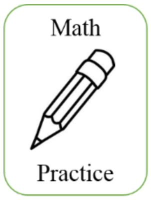


T: What can we say about one-fifth and three-fifteenths?
S: They are equal.
T: With your partner, write a number sentence that represents 2/5.
S: (Write 2/5 = 3/15 + 3/15 = 6/15. →2/5= (3 × 1/15) + (3 × 1/15) =6/15. →2/5 = 2 × 1/5 = 2 × 3/15 = 6/15.)

Problem 3: Draw a tape diagram, and use addition to show that 2/6 is the sum of 4 twelfths.
T: (Project 2/6 = 1/12 + 1/12 + 1/12 + 1/12 = 4/12.) Using what you just learned, how can you model to show that 2/6 is equal to 4/12?
S: We can draw a tape diagram and shade 2/6. Then, we can decompose it into twelfths.
T: How many twelfths are shaded?
S: 4.
T: We have seen that 1 third is equal to 2 sixths. We have seen 1 sixth is equal to 2 twelfths. So, how many twelfths equal 1 third?
S: 4 twelfths!
T: So, 2 thirds is how many twelfths? Explain to your partner how you know using your diagrams.



S: 1 third is 4 twelfths, so 2 thirds is 8 twelfths. →It's just double. →It's twice the area on the tape diagram. →It's the same as 4 sixths. 1 third is 2 sixths. 2 thirds is 4 sixths. 1 sixth is the same as 2 twelfths, so 4 times 2 is 8. 8 twelfths.

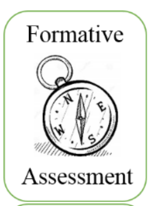


Problem Set (10 min)
Eureka Math – Lesson 4 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Decompose fractions into sums of smaller unit fractions using tape diagrams.

You may choose to use any combination of the questions below to lead the discussion.
- For Problem 1(a–d), what were some different ways that you decomposed the unit fraction?
- What is different about Problems 3(c) and 3(d)? Explain how fourths can be decomposed into both eighths and twelfths.
- For Problems 4, 5, and 6, explain the process you used to show equivalent fractions.
- Without using a tape diagram, what strategy would you use for decomposing a unit fraction?
- How did the Application Problem connect to today's lesson?



Formative Assessment (10 min)
- Eureka Math – Lesson 4 Exit Ticket (Module 5)

Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time they will continue exploring decomposing fractions.

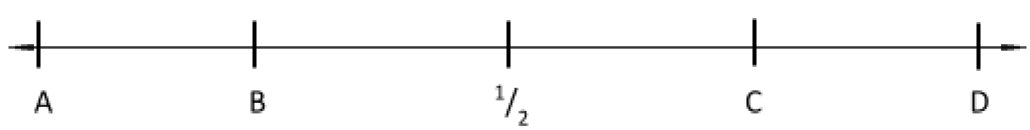
Homework
Eureka Math – Lesson 4 (Module 5)

Unit 5 – Fractions

DAY 5

Content Standard(s)
Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Fractions (15 min)

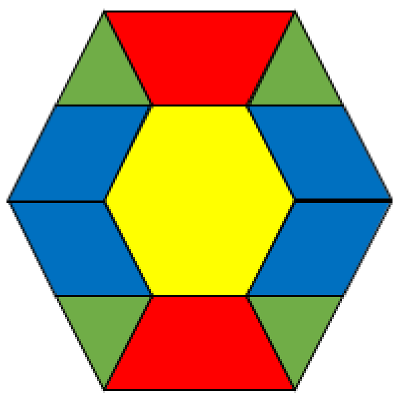


<u>Answer Key</u>
Point A: 0
Point B: ¼
Point C: ¾
Point D: 1

Learning Targets
- I can make sense of problems and persevere in solving them.

Freedom Quilt Part 2 (65 min)
(Written by teachers from Trussville City Schools)
The outcome of this task is to decompose fractions to construct conceptual knowledge of equivalent fractions.

The students in Mrs. Cook's class are very intrigued by the pattern on each patch of the Freedom Quilt they are studying. After determining what fraction of the patch is blue, red, yellow, and green, they began to notice other relationships.
1. Examine your fractions for yellow and red. Is it true that yellow and red combined are one half of the patch? Why or why not?
2. What is the simplest way to write each of your four fractions (blue, red, yellow, and green)?

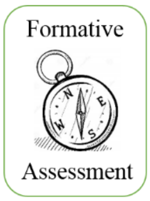


<u>Answer Key</u>
1. Yes, red and yellow combined are one half of the patch.
2. The simplest way to write the fraction for blue is 1/3 of the patch, red is ¼ of the patch, yellow is ¼ of the patch, and green is 1/6 of the patch.

<u>Supporting the Investigative Task</u>
As students work, the teacher will confer with them making notes about generalizations students are making.
- Rearranging the pattern block pieces to model red and yellow as one half of the patch
- Replacing blocks with common or like pieces
- Rearranging the pattern block pieces to model the simplest way to write the fraction for each type of pattern block
- A visual model of equivalence

<u>Differentiation</u>

- Support – If students are having difficulty, have them rearrange the pattern blocks grouping the yellow and red together to determine if they see a relationship.
- Enrichment – Write an equation adding these four fractions. Should your total equal one whole? Why or why not?



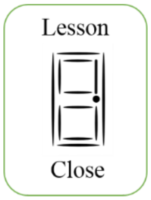
Formative Assessment Options (5 min):
(<u>Mathematics Formative Assessment</u> by P. Keeley & C. Tobey)

- <u>POMS (Point of Most Significance)</u>
  "POMS is the opposite of Muddiest Point. In this quick technique students are asked to identify the most significant learning or idea they gained from a lesson."
- <u>3-2-1</u>
  "3-2-1 provides a structured way for students to reflect on their learning. Students respond in writing to three reflective prompts, providing six responses (three for the first prompt, two for the second prompt, and one final response (to the last prompt) that describe what they learned from a lesson or instructional sequence."

3 new things I learned 1. 2. 3. **2** things I am still struggling with 1. 2. **1** thing that will help me tomorrow 1.

- <u>Look Back</u>
  "A Look Back is an account of what students learned over a given instructional period of time. Students recount specific examples of things they know now that they didn't know before and describe how they learned them."

What I Learned	How I Learned It



Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Freedom Quilt Part 2 will be discussed in math congress, and they will continue exploring fractions.

Unit 5 – Fractions

DAY 6

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Formative Feedback Task (15 min)

This task should be used solely for the purpose of gathering individual student data, providing feedback, and ultimately advancing each student's learning. It should never be graded. This task should be completed with no assistance from the teacher or peers. The teacher will provide the student oral and written feedback through assessing and advancing questions. The ultimate goal of this feedback is to advance the individual learning of each student.

Focus: Decomposing Fractions
(Adapted from Howard County Schools, https://jsangiovanni.wikispaces.hcpss.org/)

Name ____ Date ________________ #________

1. John, David, and Douglas painted their bedroom. John painted 2/8 of a wall in the bedroom. David and Douglas painted the rest of the wall, and they each painted the same amount. Use the box below to show how much of the wall David painted.

2. How much of the wall did Douglas and David paint together?

3. Write an equation to show much John, David and Douglas painted together.

Learning Targets

Learning Targets

- I can make sense of problems and persevere in solving them.
- I can decompose fractions to model equivalence.

Math Congress

Math Congress – Freedom Quilt Part 2 (35 min)

Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

Example of Math Congress structure and sequence based on anticipated student strategies:

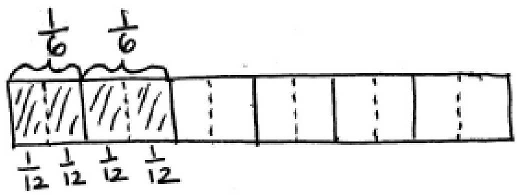
1. Rearranging the pattern block pieces to model red and yellow as one half of the patch
2. Replacing blocks with common or like pieces
3. Rearranging the pattern block pieces to model the simplest way to write the fraction for each type of pattern block
4. A visual model of equivalence

The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:

- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.

Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lessons 5 & 6 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Decompose fractions using area models to show equivalence.

<u>Application Problem (5 min)</u>
A loaf of bread was cut into 6 equal slices. Each of the 6 slices was cut in half to make thinner slices for sandwiches. Mr. Beach used 4 slices. His daughter said, “Wow! You used 2/6 of the loaf!” His son said, “No. He used 4/12.” Work with a partner to explain who was correct using a tape diagram.



$$\frac{2}{6} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{12}$$

$$\frac{2}{6} = \left(2 \times \frac{1}{12}\right) + \left(2 \times \frac{1}{12}\right)$$

or

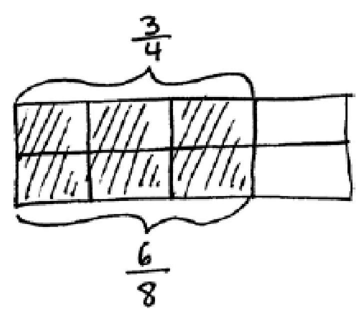
$$\frac{2}{6} = 4 \times \frac{1}{12}$$

Mr. Beach's son and daughter were both correct. $\frac{2}{6}$ represents the same amount as $\frac{4}{12}$.

Note: This Application Problem builds on Lesson 4’s objective of decomposing a fraction as the sum of smaller fractions. It also bridges to today’s lesson where students use the area model as another way to show both decomposition and equivalence.

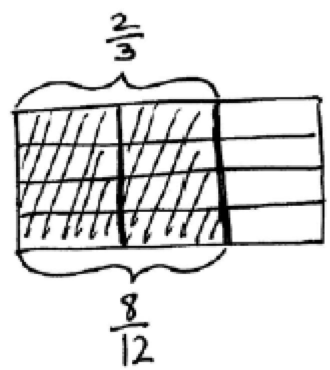
<u>Concept Development (20 min)</u>
Materials: (S) Personal white board

Problem 1: Use an area model to show that 3/4 = 6/8.
T: Draw an area model representing 1, and then shade 3/4.
T: Discuss with a partner how you can use this model to show the decomposition of 3 fourths into eighths.
S: We could draw a line so that each of the fourths is split into 2 equal parts. That would give us eighths. →Drawing a line will make each unit into 2 smaller units, which would be eighths.
T: How many eighths are shaded?
S: 6 eighths.



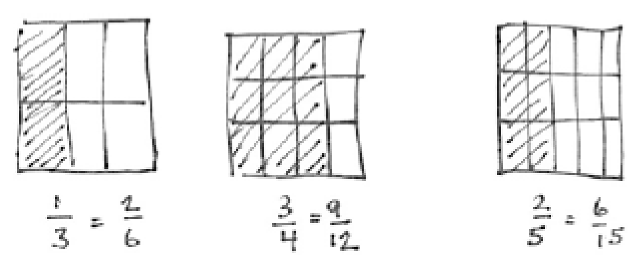
T: Work with a partner to write an addition and a multiplication sentence to describe the decomposition.
S: 3/4= (1/8 + 1/8) + (1/8 + 1/8) + (1/8 + 1/8) = 6/8. 3/4 = 3 × 2/8 = 6 × 1/8 = 6/8. →3/4 is equal to 6/8.
T: What do these addition and multiplication sentences tell you?
S: The shaded area didn't change. It's still the same amount. The number of pieces increased, but the size of the pieces got smaller. →Adding together all of the smaller units equals the total of the larger units shaded. →Multiplying also equals the total of the larger units shaded and is easier to write out!

Problem 2: Draw an area model to represent the equivalence of two fractions, and express the equivalence as the sum and product of unit fractions.



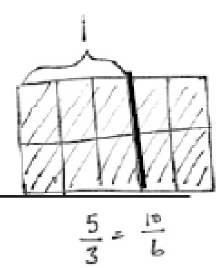
T: Let's draw an area model to show that 2/3 = 8/12. What fraction will you model first, and why? Discuss with a partner.
S: I will represent 2/3 first since thirds are the larger pieces. I can draw 1 divided into thirds and then shade 2 of them. Then, it's easy to split the thirds into parts to make twelfths. →We have to draw the larger units first and then decompose them into smaller ones, don't we?
T: Draw an area model representing 2 thirds.
T: How can we show that 2/3 = 8/12? Discuss.
S: We can split the thirds into parts until we have 12 of them. →Yes, but we need to make sure that they are equal parts. →We might have to erase our lines and then redraw to make them look equal. →We can draw three lines across the thirds. This will make 12 groups. →When I do that, the eight pieces are already shaded!
T: Express the equivalence as an addition sentence.
S: 2/3 = 1/3 + 1/3 = (1/12 + 1/12 + 1/12 + 1/12) + (1/12 + 1/12 + 1/12 + 1/12) = 8/12.
T: Express the equivalence as a multiplication sentence.
S: 2/3 = (8 × 1/12) = 8/12. →2/3 = (4 × 1/12) + (4 × 1/12) = 8/12.

Problem 3: Decompose to create equivalent fractions by drawing an area model and then dividing the area model into smaller parts.

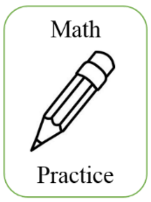


T: Let's use what we know to model equivalent fractions.

1. Draw an area model. The entire figure is 1.
2. Choose a fraction, and partition the whole using vertical lines.
3. Shade your fraction.
4. Switch papers with a partner. Write down the fraction that your partner has represented.
5. Draw one to three horizontal lines. What equivalent fraction have you modeled?



T: How could we model 5 thirds?
S: We can draw an area model and partition it into 5 parts. Each part is 1 third. We have to label 1 after 3 units.
T: Draw one horizontal line to model an equivalent fraction. How many units are in 1?
S: 6.
T: What fraction is represented?
S: 10/6.

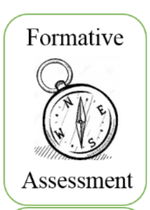


Problem Set (10 min)
Eureka Math – Lessons 6 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

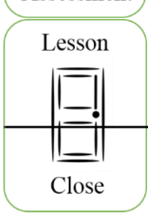
<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Decompose fractions using area models to show equivalence.

You may choose to use any combination of the questions below to lead the discussion.
- Look at Problems 1(c) and 2(b). Compare the two problems. How can 3/4 be equivalent to both fractions?
- Why do we use parentheses? What does it help show?
- In Problem 2 of the Concept Development, could you represent 8/12 first and then show the equivalence to 2/3? How would you show it?
- How can two different fractions represent the same portion of a whole?
- How did the Application Problem connect to today's lesson?



Formative Assessment (10 min)
- Eureka Math – Lessons 5 & 6 Exit Ticket (Module 5)



Closing the Lesson
- Revisit the learning targets with students.

- Inform students that next time they will continue their work on decomposing fractions by participating in a math workshop (where they will rotate to five different centers).

Homework
- Eureka Math – Lessons 5 & 6 (Module 5)

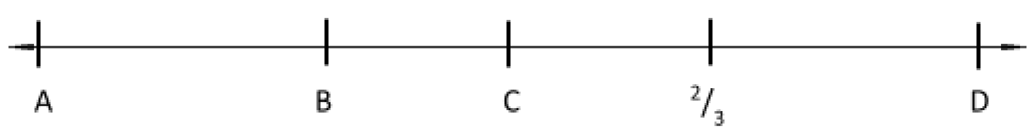
Unit 5 – Fractions

DAY 7

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Fractions (15 min)



<u>Answer Key</u>
Point A: 0
Point B: $^{1}/_{3}$
Point C: $^{1}/_{2}$
Point D: 1

Learning Targets

- I can make sense of problems and persevere in solving them.
- I can add and multiply unit fractions.
- I can decompose fractions into the sum of unit fractions.

Math Workshop – Centers (75 min)

Students will rotate through five different centers and complete designated tasks. Students will work independently, even though they will rotate as a group. Each rotation will last approximately 10-15 minutes. If a student finishes before the rotation time ends, he will complete any teacher specified unfinished problem set practice. The teacher will conduct an invitational group as one of the centers. This instruction should be differentiated based on the group needs. The teacher may choose to use debrief questions from Eureka lessons already completed and/or tasks related to learning targets. Differentiated instruction may include, but not be limited to debrief questions, tasks, problem set review, homework review, and cumulative review.

Center 1: Sprint (Multiplying Whole Numbers Times Fractions)

(Eureka Math, Lesson 6, Module 5)

A STORY OF UNITS — Lesson 6 Sprint 4•5

A

Number Correct: ______

Multiply Whole Numbers Times Fractions

1.	$\frac{1}{3} + \frac{1}{3} =$		23.	$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$	
2.	$2 \times \frac{1}{3} =$		24.	$4 \times \frac{1}{3} =$	
3.	$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$		25.	$\frac{5}{6} =$	$\_\_\_ \times \frac{1}{6}$
4.	$3 \times \frac{1}{4} =$		26.	$\frac{5}{6} =$	$5 \times \frac{}{}$
5.	$\frac{1}{5} + \frac{1}{5} =$		27.	$\frac{5}{8} =$	$5 \times \frac{}{}$
6.	$2 \times \frac{1}{5} =$		28.	$\frac{5}{8} =$	$\_\_\_ \times \frac{1}{8}$
7.	$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} =$		29.	$\frac{7}{8} =$	$7 \times \frac{}{}$
8.	$3 \times \frac{1}{5} =$		30.	$\frac{7}{10} =$	$7 \times \frac{}{}$
9.	$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} =$		31.	$\frac{7}{8} =$	$\_\_\_ \times \frac{1}{8}$
10.	$4 \times \frac{1}{5} =$		32.	$\frac{7}{10} =$	$\_\_\_ \times \frac{1}{10}$
11.	$\frac{1}{10} + \frac{1}{10} + \frac{1}{10} =$		33.	$\frac{6}{6} =$	$6 \times \frac{}{}$
12.	$3 \times \frac{1}{10} =$		34.	$1 =$	$6 \times \frac{}{}$
13.	$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} =$		35.	$\frac{8}{8} =$	$\_\_\_ \times \frac{1}{8}$
14.	$3 \times \frac{1}{8} =$		36.	$1 =$	$\_\_\_ \times \frac{1}{8}$
15.	$\frac{1}{2} + \frac{1}{2} =$		37.	$9 \times \frac{1}{10} =$	
16.	$2 \times \frac{1}{2} =$		38.	$7 \times \frac{1}{5} =$	
17.	$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$		39.	$1 =$	$3 \times \frac{}{}$
18.	$3 \times \frac{1}{3} =$		40.	$7 \times \frac{1}{12} =$	
19.	$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$		41.	$1 =$	$\_\_\_ \times \frac{1}{5}$
20.	$4 \times \frac{1}{4} =$		42.	$\frac{3}{5} =$	$\frac{1}{5} + \frac{1}{5} + \frac{}{}$
21.	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$		43.	$3 \times \frac{1}{4} =$	$\frac{}{} + \frac{1}{4} + \frac{1}{4}$
22.	$3 \times \frac{1}{2} =$		44.	$1 =$	$\frac{}{} + \frac{}{} + \frac{}{}$

A STORY OF UNITS — Lesson 6 Sprint 4•5

B

Number Correct: ______

Improvement: ______

Multiply Whole Numbers Times Fractions

1.	$\frac{1}{5} + \frac{1}{5} =$		23.	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$	
2.	$2 \times \frac{1}{5} =$		24.	$3 \times \frac{1}{2} =$	
3.	$\frac{1}{3} + \frac{1}{3} =$		25.	$\frac{5}{6} =$	$\_\_\_ \times \frac{1}{6}$
4.	$2 \times \frac{1}{3} =$		26.	$\frac{5}{6} =$	$5 \times \frac{}{}$
5.	$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$		27.	$\frac{5}{8} =$	$5 \times \frac{}{}$
6.	$3 \times \frac{1}{4} =$		28.	$\frac{5}{8} =$	$\_\_\_ \times \frac{1}{8}$
7.	$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} =$		29.	$\frac{7}{8} =$	$7 \times \frac{}{}$
8.	$3 \times \frac{1}{5} =$		30.	$\frac{7}{10} =$	$7 \times \frac{}{}$
9.	$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} =$		31.	$\frac{7}{8} =$	$\_\_\_ \times \frac{1}{8}$
10.	$4 \times \frac{1}{5} =$		32.	$\frac{7}{10} =$	$\_\_\_ \times \frac{1}{10}$
11.	$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} =$		33.	$\frac{8}{8} =$	$8 \times \frac{}{}$
12.	$3 \times \frac{1}{8} =$		34.	$1 =$	$8 \times \frac{}{}$
13.	$\frac{1}{10} + \frac{1}{10} + \frac{1}{10} =$		35.	$\frac{6}{6} =$	$\_\_\_ \times \frac{1}{6}$
14.	$3 \times \frac{1}{10} =$		36.	$1 =$	$\_\_\_ \times \frac{1}{6}$
15.	$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$		37.	$5 \times \frac{1}{12} =$	
16.	$3 \times \frac{1}{3} =$		38.	$6 \times \frac{1}{5} =$	
17.	$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$		39.	$1 =$	$4 \times \frac{}{}$
18.	$4 \times \frac{1}{4} =$		40.	$9 \times \frac{1}{10} =$	
19.	$\frac{1}{2} + \frac{1}{2} =$		41.	$1 =$	$\_\_\_ \times \frac{1}{3}$
20.	$2 \times \frac{1}{2} =$		42.	$\frac{3}{4} =$	$\frac{1}{4} + \frac{1}{4} + \frac{}{}$
21.	$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$		43.	$3 \times \frac{1}{5} =$	$\frac{}{} + \frac{1}{5} + \frac{1}{5}$
22.	$4 \times \frac{1}{3} =$		44.	$1 =$	$\frac{}{} + \frac{}{} + \frac{}{} + \frac{}{}$

Center 2: 4.NF.1 Task(s)

(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)

Name: ______________________________ Date: ____________ #: ___

Splitting to Make Equivalent Fractions

Task 1:
Jenna ate 1/3 of a cake and had 2/3 leftover for her friends. She split each of the remaining thirds into four pieces. How many pieces of cake did she have? What fraction of the whole was each piece?

Each of her friends ate the same amount of cake as Jenna. How many pieces would each friend get to eat 1/3 of the whole cake? Write or draw this fraction in two different ways.

Task 2:
Ronoldo ate ¼ of a pizza for dinner and had ¾ of the pizza leftover. He cut the leftover pizza into 6 equal slices for his friends. What fraction of the whole pizza was each piece?

Each of his friends ate the same amount of pizza as Ronoldo. How many pieces would each friend get in order to eat ¼ of the whole pizza? Represent (write or draw) the solution (fraction) in two different ways.

Task 3:

4/12 = 1/3 **2/8 = 1/4**

Look at the equivalent fractions from the story problems. What relationships do you notice between the numerators and denominators in each equation? What is happening to the numbers? We see the numbers being split. How can we see this idea happening in the models that you drew?

Can you think of additional examples that show the numerator and denominator of fractions being split by the same number? What is the result?

Center 3: 4.NF.3 Task(s)

(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)

Name: ________________________________ Date: ____________ #: ____

Candy Bucket

There are 12 pieces of candy in the bucket.
Maria and Sam each get 2 pieces of candy.
Tom gets 5 pieces of candy.
Vinny gets the rest of the candy.

What fraction does each student get? Write an equation to match this story.

Write a sentence to explain the strategy used to solve the problem.

Name: ________________________________ Date: ____________ #: ____

Boxing up Leftover Brownies

Amaria has brownies at her birthday party. At the end of the party there are the following brownies left over:
5 brownies with cream cheese frosting
4 plain chocolate brownies
3 chocolate brownies with nuts
7 brownies with caramel frosting

Part 1:
After the party the brownies are put into boxes. A box can hold 8 brownies. If each type of brownie were packed into their own box, what fraction of a box does each type of brownie take up? Draw pictures below to show your work.

Part 2:
Amaria and her Mom want to use fewer boxes and put different types of brownies into the same box. How many whole boxes do they fill? Will there be a box partially filled? If so what fraction of the box is partially filled? Draw pictures to show your work.

Part 3:
Write an equation to match the picture that you drew in Part 2.

Part 4:
Is there space for any more brownies? If so how many more brownies do you have room for? Write an equation that shows your work.

Center 4: 4.NF.4 Task(s)

(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)

Name: ________________________________ Date: ____________ #: ____

Serving Ice Cream

Katie uses 2/3 of a cup of ice cream for each ice cream sundae that she makes. For a party she makes 5 sundaes.

Part 1:
Write an addition equation to show this situation.

Show your answer with a number line or an area model.

Use numbers or words to explain how your model shows addition.

Part 2:
Write a multiplication equation to show this situation.

Show your answer with a number line or an area model.

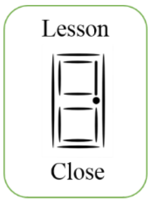
Use numbers or words to explain how your model shows multiplication.

Part 3
How are your addition and multiplication equations alike? Different? Would you use one over the other? Why or why not?

Part 4
If ice cream were sold in 1 cup containers how many containers does she need to buy for her party? Write a sentence explaining your reasoning.

Center 5: Invitational Group

The teacher will meet with each group during their Center 5 Rotation, where differentiated instruction should occur (providing each group with an experience based on need). The teacher may choose to reteach a concept from a formative feedback task or formative assessment, expand on a guided instruction/mini-lesson previously taught, expand on a problem set using debrief questions, review homework, reteach a concept, conduct a cumulative review, or provide other tasks specific to student needs.



Closing the Lesson

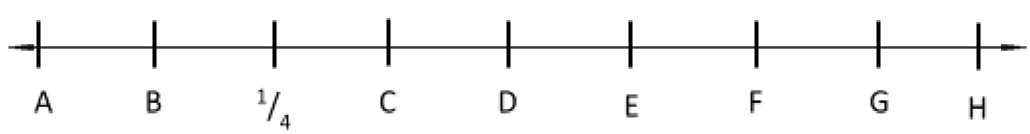
- Revisit the learning targets with students.
- Inform students that next time they will continue exploring fractions.

Unit 5 – Fractions

DAY 8

Content Standard(s)
Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Fractions (15 min)



<u>Answer Key</u>
Point A: 0
Point B: $^{1}/_{8}$
Point C: $^{3}/_{8}$
Point D: $^{1}/_{2}$

Point E: $^{5}/_{8}$
Point F: $^{6}/_{8}$
Point G: $^{7}/_{8}$
Point H: 1

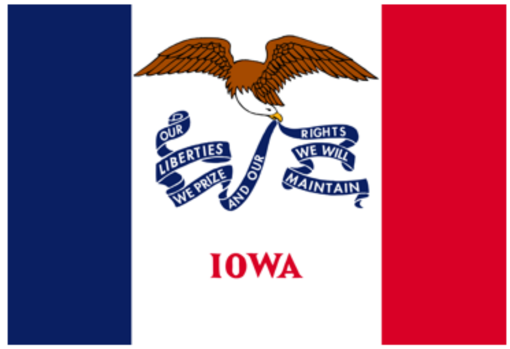
Learning Targets

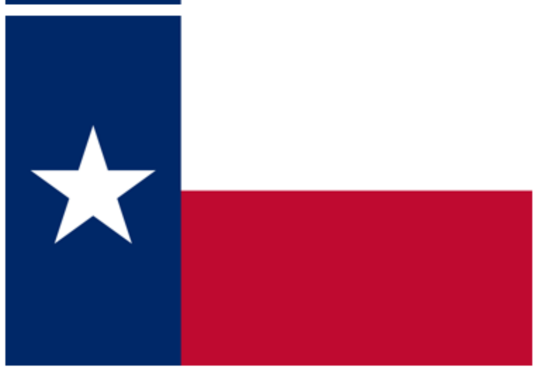
- I can make sense of problems and persevere in solving them.

Red Thread (75 min)
(NCTM Regional Conference, Houston, Texas, November 2014)
The outcome of this task is to construct conceptual knowledge of equivalent fractions using a type of area model.

Tell the following story: *Researchers are investigating what would happen if there was a shortage of red thread in the world. What items might be in short supply if there was a limited amount of red thread to produce them?* (Look around them room. Students may answer clothing, furniture, or other items.) *What about our flag? Did you know that thirty-four state flags use red thread in their design? Why is red such a popular color for flags? Most states choose to include the color red in their flag because it is a sign of power. Determine which flags will be in the most danger if there is a shortage of red thread.*

Students will use the given six flags while investigating.
1. Determine how much of each flag is red using the most accurate way you can.
2. Order the flags from the least amount of red to the greatest amount of red. Be sure to include detailed evidence and proof for each flag.





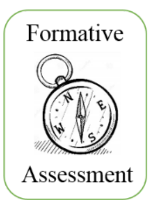
<u>Answer Key</u>
The focus of this investigation is students' development of a conceptual understanding of fractions rather than calculating an exact answer.
1. The Alabama flag is ~ $^{36}/_{100}$ red. The Arkansas flag is ½ red. The Iowa flag is ~1/4 red. The Maryland flag is 4/16 or ¼ red. The North Carolina flag is 3/8 red. The Texas flag is 1/3 red.

<u>Supporting the Investigative Task</u>
As students work, the teacher will confer with them making notes about generalizations students are making.

- Students may fold the flag to divide it into sections equal in size
- Relationship between the inverted red space and white space of the Maryland flag
- Use of measurement to determine fractional amounts
- Understanding of geometric attributes to determine fractional amounts (cutting the triangular corners of the Arkansas flag and rearranging them to create squares)
- Comparison strategies to order the flags from the least amount of red to the greatest amount of red

<u>Differentiation</u>

- Support – If students are having difficulty, encourage them to start with the Texas flag and encourage students to measure the large blue, white, and red sections of the flag.
- Enrichment – Use graph paper to design a blueprint for a new state flag that is three feet wide and six feet long. The only colors in the flag are red, blue, green, and black, and it has five times as much red as blue, twice as much red as green, and ten times as much red as black.



Formative Assessment (15 min)

- <u>Check Me Cards (During Investigative Task)</u>
  Check Me Cards are used by students to self-assess their work and obtain feedback during a task. Students place a clothespin next to the statement that best describes the status of their work. This allows the teacher to focus on the students in most need of help during the task. The teacher may also utilize the cards to provide feedback if a student(s) is not on task.

Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Red Thread will be discussed in math congress, and they will continue exploring equivalent fractions.

Unit 5 – Fractions

DAY 9

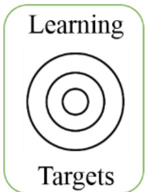
Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Fractions (15 min)

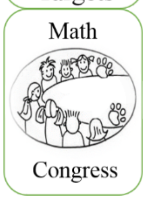
<u>Answer Key</u>
Point A: 0
Point B: $^{1}/_{4}$
Point C: $^{1}/_{2}$

Point D: $^{3}/_{4}$
Point D: 1 ¼



Learning Targets

- I can make sense of problems and persevere in solving them.
- I can use multiplication and an area model to explain equivalent fractions.



Math Congress – Red Thread (35 min)

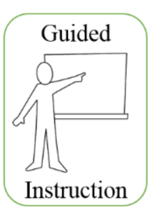
Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

Example of Math Congress structure and sequence based on anticipated student strategies:

1. Students may fold the flag to divide it into sections equal in size
2. Relationship between the inverted red space and white space of the Maryland flag
3. Use of measurement to determine fractional amounts
4. Understanding of geometric attributes to determine fractional amounts (cutting the triangular corners of the Arkansas flag and rearranging them to create squares)
5. Comparison strategies to order the flags from the least amount of red to the greatest amount of red

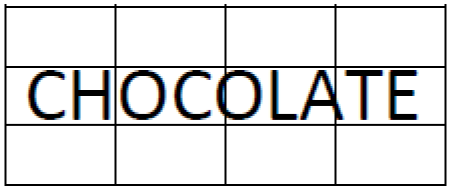
The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:

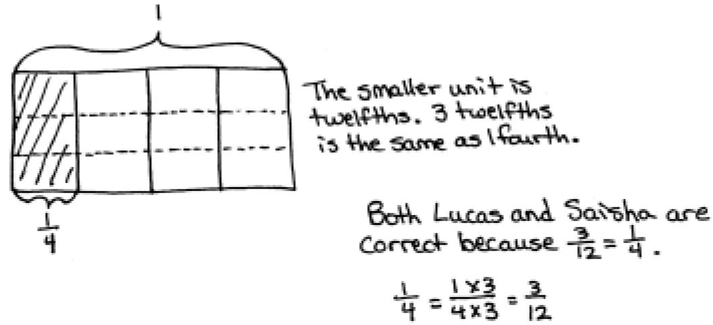
- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understandin.



Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lessons 7 & 8 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Use the area model and multiplication to show the equivalence of two fractions.

<u>Application Problem (5 min)</u>
Saisha gives some of her chocolate bar, pictured below, to her younger brother Lucas. He says, “Thanks for 3/12 of the bar.” Saisha responds, “No. I gave you 1/4 of the bar.” Explain why both Lucas and Saisha are correct.

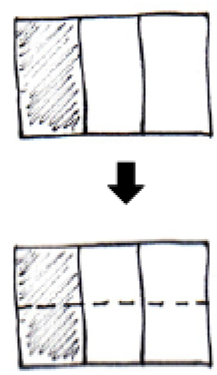




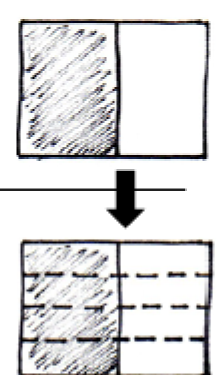
Note: This Application Problem reviews content from Lesson 7. This bridges into today's lesson, where students determine equivalent fractions of non-unit fractions. Revisit this problem in the Student Debrief by asking students to write the remaining portion as two equivalent fractions.

<u>Concept Development (20 min)</u>
Materials: (S) Personal white board

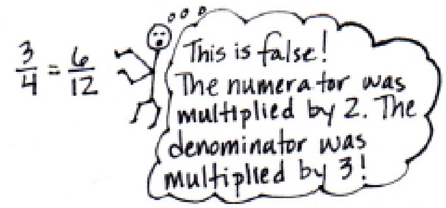
Problem 1: Determine that multiplying the numerator and denominator by n results in an equivalent fraction.
T: Draw an area model representing 1 whole partitioned into thirds. Shade and record 1/3 below the area model. Draw 1 horizontal line across the area model.
S: (Draw, partition, and shade an area model.)
T: What happened to the size of the fractional units?
S: The units got smaller. →The unit became half the size.
T: What happened to the number of units in the whole?
S: There were 3; now there are 6. →We doubled the total number of units.
T: What happened to the number of selected units when we drew the dotted line?
S: There was 1 unit selected, and now there are 2. →It doubled, too!



T: That's right. We can record the doubling of units with multiplication:
$\frac{1}{3}=\frac{1\times 2}{3\times 2}=\frac{2}{6}$.
S: Hey, I remember from third grade that 1/3 is the same as 2/6.
T: Yes. They are equivalent fractions.
T: Why didn't doubling the number of selected units make the fraction larger?
S: We didn't change the amount of the fraction, just the size of the units. →Yeah. So, the size of the units became half as big.
T: Draw an area model representing 1 partitioned with a vertical line into 2 halves.
T: Shade and record 1



2 below the area model. If we want to rewrite 1/2 using 4 times as many units, what should we do?



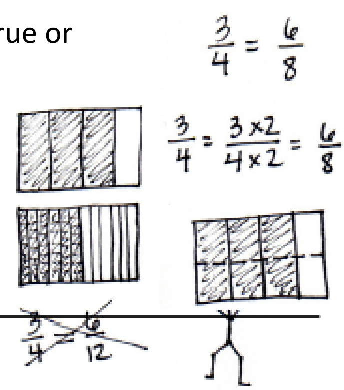
S: Draw horizontal dotted lines—three of them. →Then, we can write a number sentence using multiplication. →This time, it's 4 times as many, so we will multiply the top number and bottom number by 4.
T: Show me. (Allow time for students to partition the area model.) What happened to the size of the fractional unit?
S: The size of the fractional unit got smaller.
T: What happened to the number of units in the whole?
S: There are 4 times as many. →They quadrupled.
T: What happened to the number of selected units?
S: There was 1, and now there are 4. →The number of selected units quadrupled!
T: Has the size of the selected units changed?
S: There are more smaller-unit fractions instead of one bigger-unit fraction, but the area is still the same.
T: What can you conclude about 1/2 and 4/8?
S: They are equal!
T: Let's show that using multiplication:

$$\frac{1}{2}=\frac{1\times 4}{2\times 4}=\frac{4}{8}. \qquad \left(\frac{\textit{4 times as many selected units}}{\textit{4 times as many units in the whole}}\right)$$

T: When we quadrupled the number of units, the number of selected units quadrupled. When we doubled the number of units, the number of selected units doubled. What do you predict would happen to the shaded fraction if we tripled the units?
S: The number of units within the shaded fraction would triple, too.

Problem 2: Determine that two fractions are equivalent using an area model and a number sentence.

T: (Project 3/4 = 6/8.) If the whole is the same, is this statement true or false?
S: 3 times 2 is 6, and 4 times 2 is 8. Yes. It's true. →If we multiply both the numerator and denominator by 2, we get 6/8. →Doubling the selected units and the number of units in the whole has the same area as 3/4.
T: Represent the equivalence in a number sentence using multiplication, and draw an area model to show the equivalence.
S: (Do so as pictured to the right.)
T: (Project 3/4 = 6/12.) If the wholes are the same, is this statement true or false? How do you know? Discuss with your partner.
S: Three times 2 is 6, and 4 times 3 is 12. It's false. We didn't multiply by the same number. →This is false. I drew a model for 3/4 and then decomposed it into twelfths. There are 9 units shaded, not 6. →The numerator is being multiplied by 2, and the denominator is being multiplied by 3.



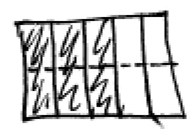
$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

They are not equivalent fractions.
T: With your partner, revise the right side of the equation to make a true number sentence.
S: We could change 6/12 to 9/12. →Or we could change 6/12 to 6/8 because both the numerator and denominator would be multiplied by 2.

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

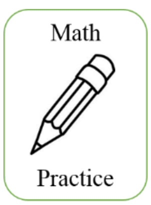
Problem 3: Write a number sentence using multiplication to show the equivalence of two fractions. Draw the corresponding area model.
T: Find an equivalent fraction without drawing an area model first. Write 3/5 on your personal white board. How have we found equivalent fractions?
S: We've doubled, tripled, or quadrupled the numerator and denominator. →We multiply the numerator and denominator by the same number.
T: Find an equivalent fraction to 3/5 using multiplication.
S: When I multiply the numerator and denominator by 2, I get 6/10.
T: Use an area model to confirm your number sentence.
S: (Do so, correcting any errors as necessary. Answers may vary.

$$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$



NOTES ON MULTIPLE MEANS OF ENGAGEMENT

Invite students working above grade level and others to test their discoveries about multiplying fractions by partitioning shapes other than rectangles, such as circles and hexagons. This work may best be supported by means of concrete or virtual manipulatives.



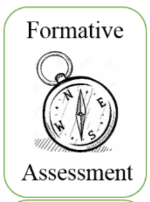
Problem Set (10 min)
Eureka Math – Lesson 8 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Use the area model and multiplication to show the equivalence of two fractions.

You may choose to use any combination of the questions below to lead the discussion.

- For Problem 3(a–d), how did you determine the number of horizontal lines to draw in each area model?
- For Problem 5(c), did you and your partner have the same answer? Explain why you might have different answers.
- Explain when someone might need to use equivalent fractions in daily life.
- How are we able to show equivalence without having to draw an area model?
- Think back to the Application Problem. What fraction of the bar did Saisha receive?



Formative Assessment (10 min)

- Eureka Math – Lessons 7 & 8 Exit Ticket (Module 5)

Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time they will continue exploring equivalent fractions.

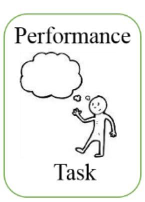
Homework

Eureka Math – Lessons 7 & 8 (Module 5)

Unit 5 – Fractions

DAY 10

Content Standard(s)
Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.



Performance Task – How Much Punch is Left (30 min)
This task will serve as an independent summative assessment of student learning. It should be completed with no assistance.

<u>How Much Punch is Left?</u>
Part 1
There are 2 gallons of punch left in the punch bowl. It gets divided between 8 students, with each getting a different amount.
Micah takes 1/12 of a gallon of punch.
Roberta takes three times as much Micah.

Steve takes twice as much as Roberta.
Yanni takes 2/12 of a gallon of punch less than Steve.
Amy takes 1/12 of a gallon of punch less than Yanni.
The remaining punch is divided between Tom, Jackie, and Henry.
Tom and Jackie had the same amount of punch.
Henry had less punch than both Tom and Jackie.
How much punch did each person take?
Draw a picture and write an equation to match this context.

Part 2
At the next party, the amount of punch doubled to 4 gallons. Each person took the same fraction of the punch. How much would each person get?

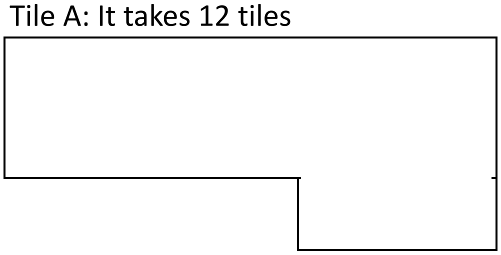
Answer Key
Micah took 1/12 of a gallon of punch, Robert took 3/12 or ¼ of a gallon of punch, Steve took 6/12 or ½ of a gallon of punch, Yanni took 4/12 or 1/3 of a gallon of punch, Amy took 3/12 or ¼ of a gallon of punch, Henry took 1/12 of a gallon of punch, and Tom and Jack each get 3/12 of a gallon of punch.

Learning Targets
- I can make sense of problems and persevere in solving them.

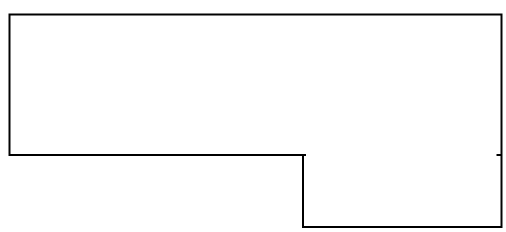
Covering the Patio (65 min)
(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)
The outcome of this task is to construct conceptual knowledge of using an area model to show the equivalence of two fractions.

Cover the patio below with the same kind of tile.
Part 1

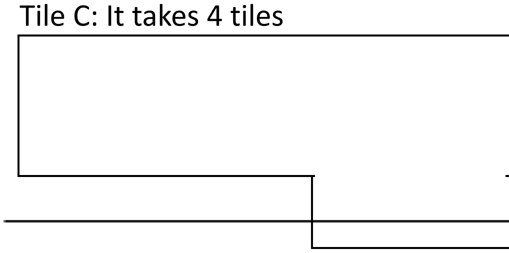
Tile A: It takes 12 tiles



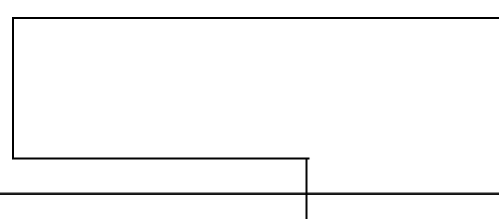
Tile B: It takes 6 tiles



Tile C: It takes 4 tiles



Tile D: It takes 8 tiles



Part 2

Cover half of the patio. Complete the table below for how many tiles it would take to cover half of the tile.

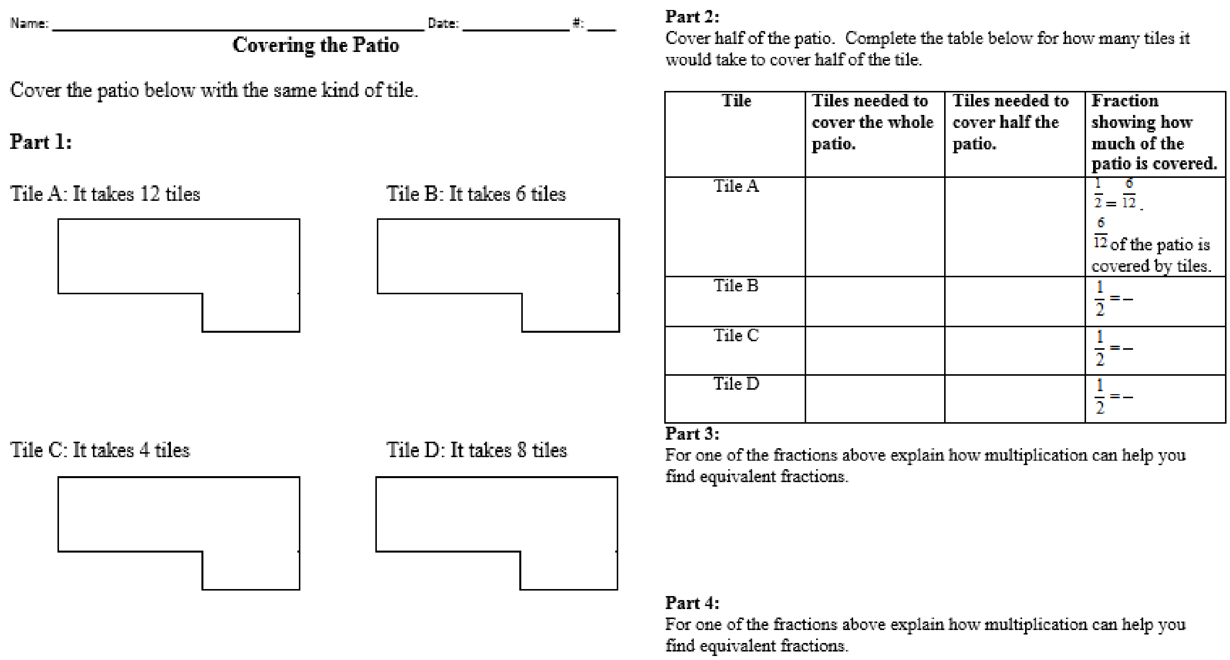
Tile	Tiles needed to cover the whole patio.	Tiles needed to cover half the patio.	Fraction showing how much of the patio is covered.
Tile A			$\frac{1}{2} = \frac{6}{12}$ $\frac{6}{12}$ of the patio is covered by tiles.
Tile B			
Tile C			
Tile D			

Part 3

For one of the fractions above explain how multiplication can help you find equivalent fractions.

Part 4

For one of the fractions above explain how multiplication can help you find equivalent fractions.


Name: ______________________ Date: __________ #: ___

Covering the Patio

Cover the patio below with the same kind of tile.

Part 1:

Tile A: It takes 12 tiles

Tile B: It takes 6 tiles

Tile C: It takes 4 tiles

Tile D: It takes 8 tiles

Part 2:

Cover half of the patio. Complete the table below for how many tiles it would take to cover half of the tile.

Tile	Tiles needed to cover the whole patio.	Tiles needed to cover half the patio.	Fraction showing how much of the patio is covered.
Tile A			$\frac{1}{2} = \frac{6}{12}$. $\frac{6}{12}$ of the patio is covered by tiles.
Tile B			$\frac{1}{2} = -$
Tile C			$\frac{1}{2} = -$
Tile D			$\frac{1}{2} = -$

Part 3:

For one of the fractions above explain how multiplication can help you find equivalent fractions.

Part 4:

For one of the fractions above explain how multiplication can help you find equivalent fractions.

<u>Answer Key</u>

Part 1 – Each region should be accurately partitioned into equal sections.

Part 2 –

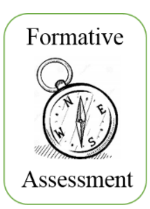
Tile	Tiles needed to cover the whole patio.	Tiles needed to cover half the patio.	Fraction showing how much of the patio is

			covered.
Tile A	12	6	½ = ${}^{6}/_{12}$
Tile B	6	3	½ = ${}^{3}/_{6}$
Tile C	4	2	½ = ${}^{2}/_{4}$
Tile D	8	4	½ = ${}^{4}/_{8}$

Part 3 – Answers may vary. Example: Multiplying both the numerator and the denominator by the same number will result in an equivalent fraction because you are multiplying the fraction by 1.
Part 4 – Answers may vary. Example: Dividing both the numerator and denominator by the same number will result in an equivalent fraction because you are dividing the fraction by 1.

Supporting the Investigative Task
As students work, the teacher will confer with them making notes about generalizations students are making.
- Dividing the patio into equal size sections
- A visual model of equivalence
- Relationship of multiplying by the same numerator and denominator to create an equivalent fraction is multiplying by 1
- Relationship of dividing by the same numerator and denominator to create an equivalent fraction is dividing by 1

Differentiation
- Support – If students are having difficulty, have them use square tiles to model the patios.
- Enrichment – Price Elementary School offers an Art Club and Theatre Club after school. The school as one thousand students and three fifths of them are girls. Out of the girls, five-twelfths of them signed up for an after-school program. Two-fifths of the girls that signed up for a program join the Theater Club. How many girls joined the Art Club.



Formative Assessment Options (5 min):
(Mathematics Formative Assessment by P. Keeley & C. Tobey)
- POMS (Point of Most Significance)
  "POMS is the opposite of Muddiest Point. In this quick technique students are asked to identify the most significant learning or idea they gained from a lesson."
- 3-2-1
  "3-2-1 provides a structured way for students to reflect on their learning. Students respond in writing to three reflective prompts, providing six responses (three for the first prompt, two for the second prompt, and one final response (to the last prompt) that describe what they learned from a lesson or instructional sequence."

3 new things I learned

1.
2.
3.

2 things I am still struggling with
1.
2.

1 thing that will help me tomorrow
1.

- <u>Look Back</u>
  "A Look Back is an account of what students learned over a given instructional period of time. Students recount specific examples of things they know now that they didn't know before and describe how they learned them."

What I Learned	How I Learned It

Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Covering the Patio will be discussed in math congress, and they will continue exploring equivalent fractions.

Unit 5 – Fractions

DAY 11

Content Standard(s)

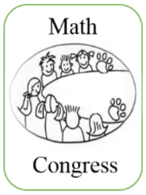
Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Comparing Fractions (15 min)

Compare the fractions.		Answer Key		
$^{2}/_{5}$	$^{3}/_{5}$	$^{2}/_{5}$	<	$^{3}/_{5}$
$^{2}/_{5}$	$^{2}/_{4}$	$^{2}/_{5}$	<	$^{2}/_{4}$
$^{2}/_{5}$	$^{2}/_{6}$	$^{2}/_{5}$	>	$^{2}/_{6}$
$^{2}/_{6}$	$^{1}/_{6}$	$^{2}/_{6}$	>	$^{1}/_{6}$
$^{6}/_{6}$	1	$^{6}/_{6}$	=	1

Learning Targets

- I can make sense of problems and persevere in solving them.
- I can use an area model and division to create equivalent fractions.



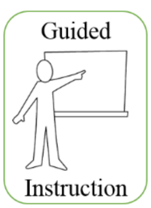
Math Congress – Cover the Patio (35 min)
Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

Example of Math Congress structure and sequence based on anticipated student strategies:

1. Dividing the patio into equal size sections
2. A visual model of equivalence
3. Relationship of multiplying by the same numerator and denominator to create an equivalent fraction is multiplying by 1
4. Relationship of dividing by the same numerator and denominator to create an equivalent fraction is dividing by 1

The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:

- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.



Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lessons 9 & 10 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Use the area model and division to show the equivalence of two fractions.

<u>Application Problem (5 min)</u>
Nuri spent 9/12 of his money on a book and the rest of his money on a pencil.

a. Express how much of his money he spent on the pencil in fourths.
b. Nuri started with $1. How much did he spend on the pencil?

Note: This Application Problem connects Topic A and Lesson 9 by finding the other fractional part of the whole and expressing equivalent fractions. Using what students know about money, ask why it is preferable to answer in

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

English language learners may confuse the terms *decompose* and *compose*.

- Demonstrate that the prefix *de* can be placed before some words to add an opposite meaning.
- Use gestures to clarify the meanings: *Decompose* is to take apart, and *compose* is to put together.
- Refresh students' memory of decomposition and composition in the context of the operations with whole numbers.

Nuri spent 25¢ on the pencil.

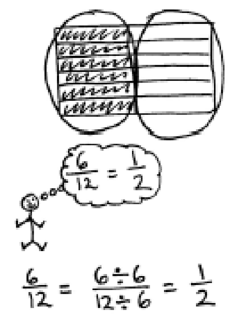
fourths rather than twelfths. Students connect fourths to quarters of a dollar. Revisit this problem in the Student Debrief to express how much money was sp

NOTES ON MULTIPLE MEANS OF EXPRESSION:

As the conceptual foundation for simplification is being set, the word *simplify* is initially avoided with students as they compose higher-value units. The process is rather referred to as *composition,* the opposite of decomposition, which relates directly to their drawing, work throughout the last two lessons, and work with whole numbers. When working numerically, the process is referred to at times as *renaming,* again in an effort to relate to whole number work.

<u>Concept Development (25 min)</u>
Materials: (S) Personal white board

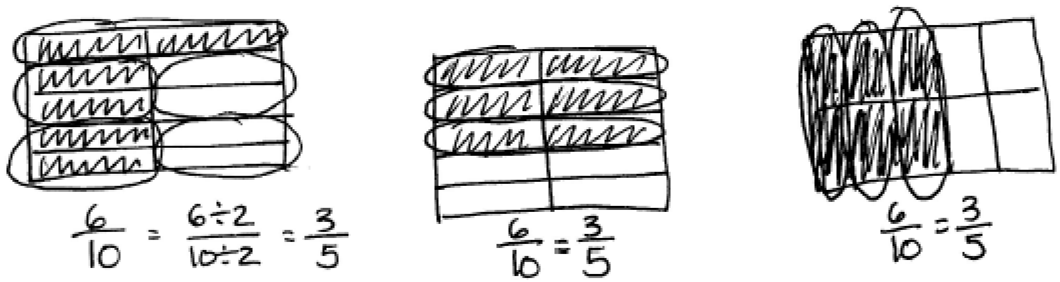
Problem 1: Simplify 6/12 by composing larger fractional units using division.
T: (Project area model showing 6/12.) What fraction does the area model represent?
S: 6/12.
T: Discuss with a partner. Do you see any fractions equivalent to 6/12?
S: Half of the area model is shaded. The model shows 1/2.
T: Which is the larger unit? Twelfths or halves?
S: Halves!
T: Circle the smaller units to make the larger units. Say the equivalent fractions.
S: 6/12 = 1/2.
T:
(Write $\frac{6 \div 6}{12 \div 6} = \frac{}{}$,



and point to the denominator.) Twelve units were in the whole, and we made groups of 6 units. Say a division sentence to record that.
S: 12 ÷ 6 = 2.
T: (Record the 2 in the denominator, and point to the numerator.) Six units were selected, and we made a group of 6 units. Say a division sentence to record that.
S: 6 ÷ 6 = 1.
T: (Record the 1 in the numerator.) We write
$\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$,
dividing both the numerator and denominator by 6 to find an equivalent fraction.
T: What happened to the size of the units and the total number of units?
S: The size of the units got larger. There are fewer units in the whole. →The units are 6 times as large, but the number of units is 6 times less. →The units got larger. The number of units got smaller.

Problem 2: Draw an area model of a number sentence that shows the simplification of a fraction.
T: (Project $\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$.)
T: Draw an area model to show how this number sentence is true.

S: The numerator and denominator are both being divided by 2. I will circle groups of 2. →I know 2 is a factor of 6 and 10, so I could make groups of 2. →There are 3 shaded groups of 2 and 5 total groups of 2. →That's 3/5.



Problem 3: Simplify a fraction by drawing to find different common factors, and relate it to division.

T: With your partner, draw an area model to represent 8/12. Rename 8/12 using larger fractional units. You may talk as you work. (Circulate and listen.)

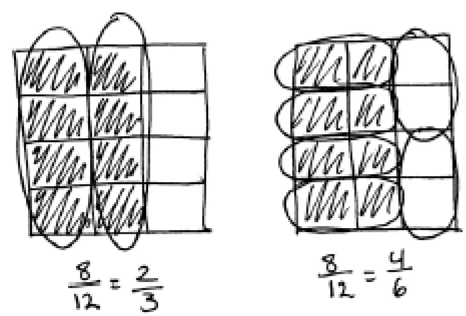
S: I can circle groups of 2 units. →2 is a factor of 8 and 12. →There are 6 groups of 2 units. →Four groups are shaded. That's 4/6.

T: What happens when I use 4 as a common factor instead of 2? Turn and talk.

S: Four is a factor of both 8 and 12. It works. →We can make larger units with groups of 4. →Thirds are larger than sixths. 8/12 = 2
3. →We have fewer units, but they're bigger.

T: Express the equivalent fractions as two division number sentences.

S: (Write $\frac{8}{12}=\frac{8 \div 4}{12 \div 4}=\frac{2}{3}$ and $\frac{8}{12}=\frac{8 \div 2}{12 \div 2}=\frac{4}{6}$.)



T: What can you conclude about 2/3 and 4/6?

S: They are both equivalent to 8/12.

T: What is true about dividing the numerator and denominator in 8/12 by 2 or 4?

S: Two and 4 are both factors of 8 and 12. →The larger the factor used, the larger the new fractional units will be.

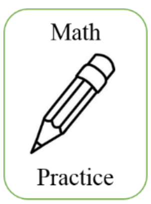
T: Interesting. Discuss what your classmate said. "The larger the factor, the larger the new fractional units."

S: When we divided by 2, we got sixths, and when we divided by 4, we got thirds. Thirds are larger. Four is larger than 2. A larger factor gave a larger unit. →When the factor is larger, it means we can make fewer units but larger ones.

Problem 4: Simplify a fraction using the largest possible common factor.

T: Discuss with your partner how to rename 8/12 with the largest units possible without using an area model.

S: Figure out the greatest number of units that can be placed in equal groups. →Divide the numerator and denominator by the same number, just like we've been doing. →Find a factor of both 8 and 12, and use it to divide the numerator and the denominator.
T: Express the equivalence using a division number sentence.
S: $\frac{8}{12} = \frac{8 \div 2}{12 \div 2} = \frac{4}{6}$.
Four and 6 are still both even, so that wasn't the largest factor.
→ $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$.
The only common factor 2 and 3 have is 1, so 4 must be the largest factor that 8 and 12 have in common.
T: How can we know we expressed an equivalent fraction with the largest units?
S: When we make equal groups, we need to see if we can make larger ones. →When we find the factors of the numerator and denominator, we have to pick the largest factor. Four is larger than 2, so dividing the numerator and denominator by 4 gets us the largest units. →When I found 4/6, I realized 2 and 4 are both even, so I divided the numerator and denominator again by 2. Two and 3 only have a common factor of 1, so I knew I made the largest unit possible. →Dividing by 2 twice is the same as dividing by 4. Just get it over with faster, and divide by 4.
T: It's not wrong to say that 8/12 = 4/6. It is true. It's just that, at times, it really is simpler to work with larger units because it means the denominator is a smaller number.



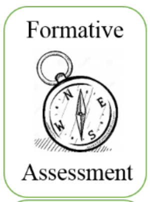
Problem Set (10 min)
Eureka Math – Lesson 10 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Use the area model and division to show the equivalence of two fractions.

You may choose to use any combination of the questions below to lead the discussion.
- In Problem 2(b), did you compose the same units as your partner? Are both of your answers correct? Why?
- In Problem 4(a–d), how is it helpful to know the common factors for the numerators and denominators?

- In Problem 4, you were asked to use the largest common factor to rename the fraction: 4/8 = 1/2. By doing so, you renamed 4/8 using larger units. How is renaming fractions useful?
- Do fractions always need to be renamed to the largest unit? Explain.
- Why is it important to choose a common factor to make larger units?
- How can you tell that a fraction is composed of the largest possible fractional units?
- When you are drawing an area model and circling equal groups, do all of the groups have to appear the same in shape? How do you know that they still show the same amount?
- Explain how knowing the factors of the numerator and the factors of the denominator can be helpful in identifying equivalent fractions of a larger unit size.



Formative Assessment (10 min)
- Eureka Math – Lesson 9 & 10 Exit Ticket (Module 5)

Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time they will continue exploring equivalent fractions.

Homework
Eureka Math – Lesson 9 & 10 (Module 5)

DAY 12

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Comparing Fractions (15 min)

Compare the fractions.		Answer Key
$^{1}/_{2}$	$^{1}/_{4}$	$^{1}/_{2}$ > $^{1}/_{4}$
$^{1}/_{2}$	$^{2}/_{4}$	$^{1}/_{2}$ = $^{2}/_{4}$
$^{1}/_{4}$	$^{1}/_{8}$	$^{1}/_{4}$ > $^{1}/_{8}$
$^{1}/_{4}$	$^{2}/_{8}$	$^{1}/_{4}$ = $^{2}/_{8}$
$^{2}/_{4}$	$^{2}/_{8}$	$^{2}/_{4}$ > $^{2}/_{8}$
$^{1}/_{3}$	$^{1}/_{6}$	$^{1}/_{3}$ > $^{1}/_{6}$
$^{1}/_{3}$	$^{2}/_{6}$	$^{1}/_{3}$ = $^{2}/_{6}$

Learning Targets

- I can make sense of problems and persevere in solving them.

Cat Walk (75 min)
(Math Forum-NCTM, Problem #673)
The outcome of this task is to construct conceptual knowledge of using a number line to explain fraction equivalence.

A dog takes three steps to walk the same distance for which a cat takes four steps. Suppose one step of the dog covers ½ foot. How many feet would the cat cover in taking 24 steps?

Answer Key
The cat would cover 9 feet.

Supporting the Investigative Task
As students work, the teacher will confer with them making notes about generalizations students are making.

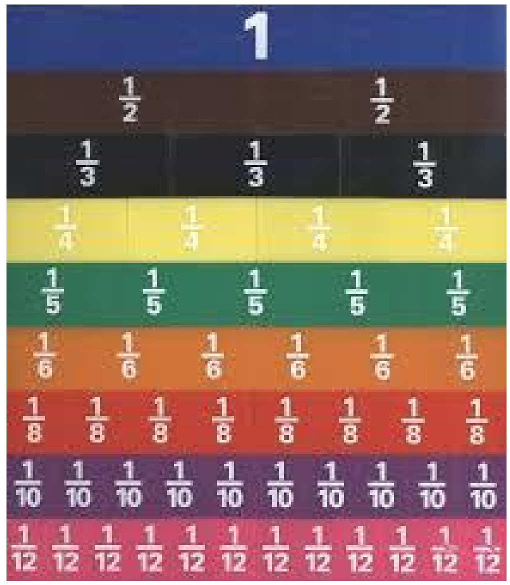
- Use of a double number line
- Ratio table to determine the number of dog steps
- Multiplying a whole number times a fraction

Differentiation

- Support – If students are having difficulty, have them act out the situation or model the steps of the dog and cat on a number line.
- Enrichment – Farmer Brown planted his spring vegetable garden. A section of his garden was made up of corn and squash plans, and 2 out of 3 of the plants were corn. He planted seventy-eight plants in all. How many were squash? How many were corn?

The Equivalent Fraction Game (30 min)
Partner students. Each student will need a set of the Fraction Bars that include one whole and various other fraction pieces such as halves, thirds, fourths, fifths, sixths, tenths, and twelfths like the ones pictures below. Each pair of students will also need two six-sided dice. Students take turn rolling the dice. Each player will take the digits on the dice to create a fraction using the larger digit as the denominator. That student can remove that fraction or an equivalent fraction from his or her game board of Fraction Bars. (It is recommend that students place these pieces in a bag, so the small pieces are not lost.) For example, if a student rolls one half, the student could take away one half, two-fourths, three-sixths, four-eighths, etc. If students are removing an equivalent fraction,

they must provide proof that the two fractions are equivalent. If a student cannot use a roll, he or she loses her turn, and it is the other students turn to roll the dice. Players take turns until one student has removed all of his or her Fraction Bars. The student who is able to remove all of the Fraction Bar pieces first is the winner. Different dice can be used, but a six-sided dice requires students to create an equivalent fraction to remove the tenths and twelfths Fraction Bars.



Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Cat Walk will be discussed in math congress, and they will continue exploring equivalent fractions.

DAY 13

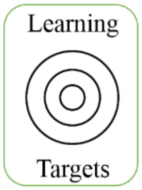
Content Standards

Content Standard(s)
Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

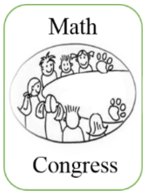
Number Talk

Focus: Comparing Fractions (15 min)

Compare the fractions.	Answer Key
$^{1}/_{2}$ $^{2}/_{3}$	$^{1}/_{2}$ < $^{2}/_{3}$
$^{2}/_{3}$ $^{3}/_{4}$	$^{2}/_{3}$ < $^{3}/_{4}$
$^{3}/_{4}$ $^{4}/_{5}$	$^{3}/_{4}$ < $^{4}/_{5}$
$^{5}/_{6}$ $^{4}/_{5}$	$^{5}/_{6}$ > $^{4}/_{5}$
$^{5}/_{6}$ $^{11}/_{12}$	$^{5}/_{6}$ < $^{11}/_{12}$
$^{7}/_{8}$ $^{3}/_{4}$	$^{7}/_{8}$ > $^{3}/_{4}$



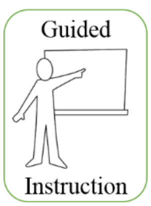
Learning Targets
- I can make sense of problems and persevere in solving them.
- I can use a number line to explain equivalent fractions.



Math Congress – Cat Walk (35 min)
Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

Example of Math Congress structure and sequence based on anticipated student strategies:
1. Use of a double number line
2. Ratio table to determine the number of dog steps
3. Multiplying a whole number times a fraction

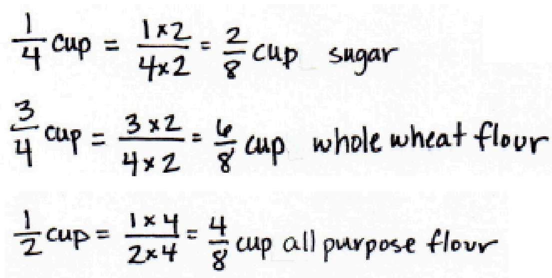
The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:
- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.



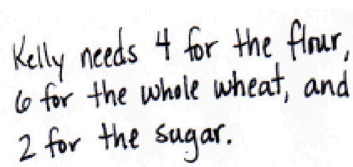
Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lesson 11 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Explain fraction equivalence using a tape diagram and the number line, and relate that to the use of multiplication and division.

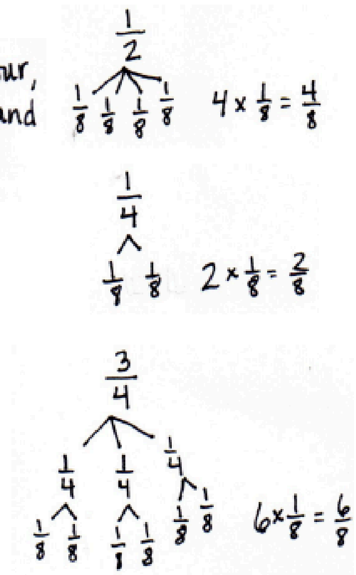
<u>Application Problem (5 min)</u>
Kelly was baking bread but could only find her 1/8-cup measuring cup. She needs 1/4 cup sugar, 3/4 cup whole wheat flour, and 1/2 cup all-purpose flour. How many 1/8 cups will she need for each ingredient?

Solution 1



Solution 2

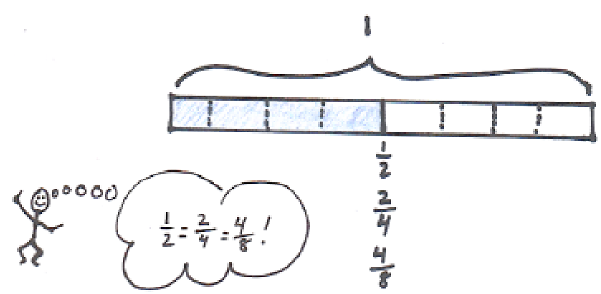




Note: This Application Problem places equivalent fractions into a context that may be familiar to students. Multiple solution strategies are possible. The first solution models the equivalency learned in Lessons 7 and 8. The second solution uses number bonds to find unit fractions, reviewing Topic A content.

<u>Concept Development (20 min)</u>
Materials: (S) Personal white board, ruler

Problem 1: Use a tape diagram and number line to find equivalent fractions for halves, fourths, and eighths.

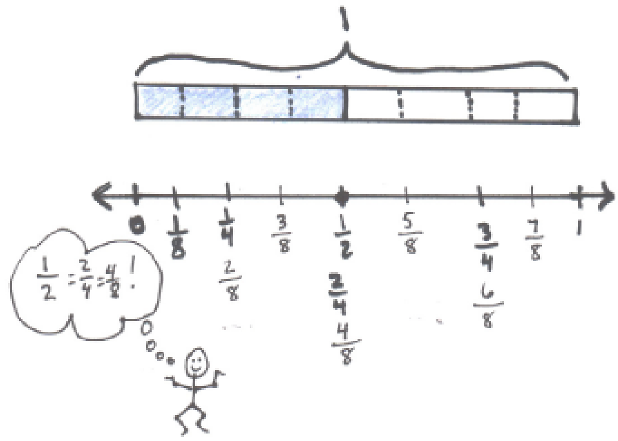


T: Draw a tape diagram to show 1 partitioned into halves.
S: (Draw a tape diagram.)
T: Shade 1/2. Now, decompose halves to make fourths. How many fourths are shaded?
S: 2 fourths.
T: On your personal white board, write what we did as a multiplication number sentence.
S: (Write $\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$.)
T: Decompose fourths to make eighths. How many eighths are shaded?
S: 4 eighths.
T: Write a multiplication number sentence to show that 2 fourths and 4 eighths are equal.
S: (Write $\frac{2}{4} = \frac{2 \times 2}{4 \times 2} = \frac{4}{8}$.)

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

To preserve the pace of the lesson, provide a tape diagram and number line Template (see Lesson 12) for some learners. Students may also choose to transform the tape diagram into a number line by erasing the top line, labeling points, and extending the endpoints.

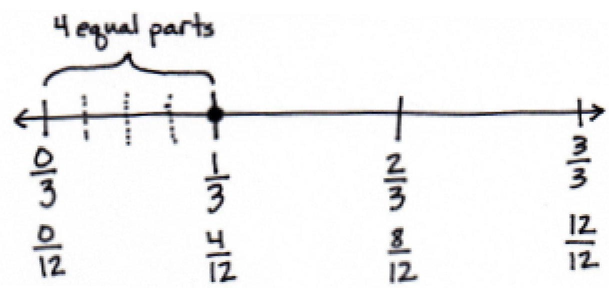
T: Now, use a ruler to draw a number line slightly longer than the tape diagram. Label points 0 and 1 so that they align with the ends of the tape diagram.
S: (Draw a number line.)
T: Label 1/2 on the number line. Decompose the number line into fourths. What is equivalent to 2/4 on the number line?
S: 1/2 = 2/4. We showed that on the tape diagram.
T: Decompose the number line into eighths.
S: (Label the eighths.)
T: What is 4/8 equal to on the number line?
S: 1/2 = 4/8. →2/4 = 4/8. →That also means 1/2 = 2/4 = 4/8.
T: Explain what happened on the number line as you decomposed the half.



S: When we decomposed the half into fourths, it was like sharing a licorice strip with four people instead of two. →We got 4 smaller parts instead of 2 larger parts. →There are 4 smaller segments in the whole instead of 2 larger segments. →We doubled the number of parts but made smaller parts, just like with the area model. →It made 2 lengths that were the same length as 1 half.

Problem 2: Use a number line, multiplication, and division to decompose and compose fractions.

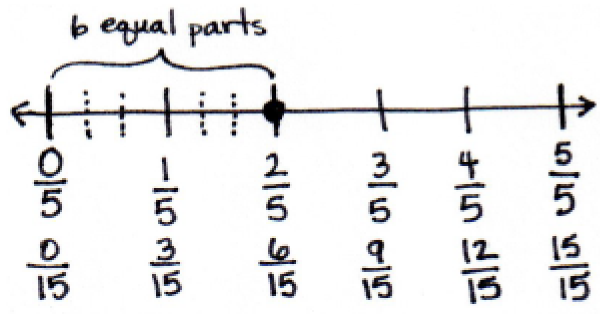
T: Partition a number line into thirds. Decompose 1 third into 4 equal parts.
T: Write a number sentence using multiplication to show what fraction is equivalent to 1 third on this number line.
S: (Write $\frac{1}{3}=\frac{1\times4}{3\times4}=\frac{4}{12}$.)



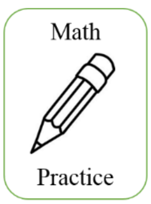
T: Explain to your partner why that is true.
S: It's just like the area model. We made more smaller units, but the lengths stayed the same instead of the area staying the same. →If we multiply a numerator and a denominator by the same number, we find an equivalent fraction. →1 third was decomposed into fourths, so we multiplied the number of units in the whole and the number of selected units by 4.
T: Write the equivalence as a number sentence using division.
S: (Write $\frac{4}{12}=\frac{4\div4}{12\div4}=\frac{1}{3}$.)

T: Explain to your partner why that is true.
S: We can join four smaller segments to make one longer one that is the same as 1 third. →We can group the twelfths together to make thirds. →Four copies of 1/12 equals 1/3. →Just like the area model, we are composing units to make a larger unit.

Problem 3: Decompose a non-unit fraction using a number line and division.
T: Draw a number line. Partition it into fifths, label it, and locate 2/5.
S: (Draw.)
T: Decompose 2
5 into 6 equal parts. First, discuss your strategy with your partner.
S: I will make each fifth into 6 parts. →No. We have to decompose 2 units, not 1 unit. Each unit will be decomposed into 3 equal parts. →Two units are becoming 6 units. We are multiplying the numerator and denominator by 3.



T: Write a number sentence to express the equivalent fractions.
S: (Write $\frac{2}{5}=\frac{2\times 3}{5\times 3}=\frac{6}{15}$.)



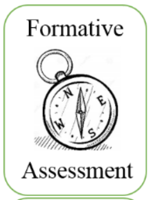
Problem Set (10 min)
Eureka Math – Lesson 11 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Explain fraction equivalence using a tape diagram and the number line, and relate that to the use of multiplication and division.

You may choose to use any combination of the questions below to lead the discussion.
- In Problem 1, compare the distance from 0 to each point on the number line you circled. What do you notice?
- In Problem 1, does the unshaded portion of the tape diagram represent the same length from the point to 1 on every number line? How do you know?
- Compare your number sentences in Problem 2. Could they be rewritten using division?
- In Problem 5, what new units were created when 2 fifths was decomposed into 4 equal parts?

- How is modeling with a number line similar to modeling with an area model? How is it different?
- In Grade 3, you found equivalent fractions by locating them on a number line. Do you now require a number line to find equivalent fractions? What other ways can you determine equivalent fractions?



Formative Assessment (10 min)

- Eureka Math – Lesson 11 Exit Ticket (Module 5)

Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time they will continue exploring fractions.

Homework

Eureka Math – Lesson 11 (Module 5)

Unit 5 – Fractions

DAY 14

Content Standard(s)
Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Focus: Estimation, Comparing Fractions (15 min)

Estimate if the fraction is closer to 0, ½ or 1.	<u>Answer Key</u>
$^{1}/_{5}$	0
$^{1}/_{4}$	½
$^{2}/_{4}$	½
$^{2}/_{5}$	½
$^{3}/_{5}$	½
$^{3}/_{4}$	1

Learning Targets

- I can make sense of problems and persevere in solving them.

Who Has More Gum? (75 min)
(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)
The outcome of this task is to construct conceptual knowledge of using benchmarks to compare fractions.

A group of friends buys a big long strip of gum and tear it into pieces. Sally has 2/3 of a foot of gum. Josey has 3/4 of a foot of gum. Mitch has 4/6 of a foot of gum. Gary has 3/6 of a foot of gum.

Part 1
Draw pictures and write an expression using the >, <, or = signs to show who has more gum

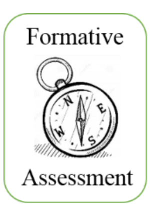
between:
- Gary or Sally?
- Mitch or Sally?
- Josey or Mitch?

Part 2
Taylor comes in and gets ½ of a foot of gum. Gary says, "We have the same amount." Is Gary correct? Why or why not?

Answer Key
Part 1 – Sally has more gum because 3/6 < 2/3. Mitch and Sally have the same amount of gum because 4/6 = 2/3. Mitch has more gum because ¾ < 4/6.
Part 2 – Gary is correct because 3/6 = ½.

Supporting the Investigative Task
As students work, the teacher will confer with them making notes about generalizations students are making.
- Use of a model to compare fractions
- Use of a number line to compare fractions
- Use of benchmarks to compare fractions
- Strategies to prove fractions are equivalent

Differentiation
- Support – If students are having difficulty, have them draw a model of each fraction and ensure the wholes are equivalent in size.
- Enrichment – Cruz and Erica were both getting ready for soccer. Cruz ran one lap around the school. Erica ran three laps around the playground. Erica said, "I ran more laps, so I ran farther." Cruz said, "Four laps around the school is 1 mile, but it takes 12 laps around the playground to go 1 mile. My laps are much longer, so I ran farther. Who is right? Draw a picture to help you explain your answer.



Formative Assessment Options (15 min)
(Mathematics Formative Assessment by P. Keeley & C. Tobey)
This formative assessment may be administered during a gallery walk.
- I Used to Think...But Now I Know...
  "I used to Think...But Now I Know... asks students to compare, orally or in writing, their ideas at the beginning of a lesson or instructional sequence to the ideas they have after completing the lesson(s)."
- Always, Sometime, or Never True
  "Always, Sometimes, or Never True involves a set of statements that students examine and decide if they are always true, sometimes true, or never true. This strategy is useful in revealing whether students overgeneralize or under

generalize a mathematical concept. In addition, they are asked to provide a justification for their answer."

The smaller the denominator, the bigger the fraction, so the fraction with the smallest denominator is always larger. ☐ Always ☐ Sometimes ☐ Never	Justify your answer.

Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Who Has More Gum will be discussed in math congress, and they will continue comparing fractions.

Unit 5 – Fractions

DAY 15

Content Standard(s)
Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Focus: Fractions (15 min)
Adapted from NCTM, www.nctm.org/profdev)

Name 3 equivalent fractions	Is it in simplest form? If not, what is the simplest form?
3/6	
Is it closer to 0, ½, or 1?	Name 3 fractions that are greater. Name 3 fractions that are less.

Learning Targets
- I can make sense of problems and persevere in solving them.
- I can use benchmarks to compare two fractions on a number line.

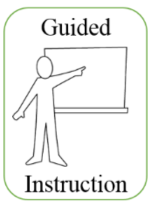
Math Congress – Who Has More Gum? (35 min)
Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

Example of Math Congress structure and sequence based on anticipated student strategies:

1. Use of a model to compare fractions
2. Use of a number line to compare fractions
3. Use of benchmarks to compare fractions
4. Strategies to prove fractions are equivalent

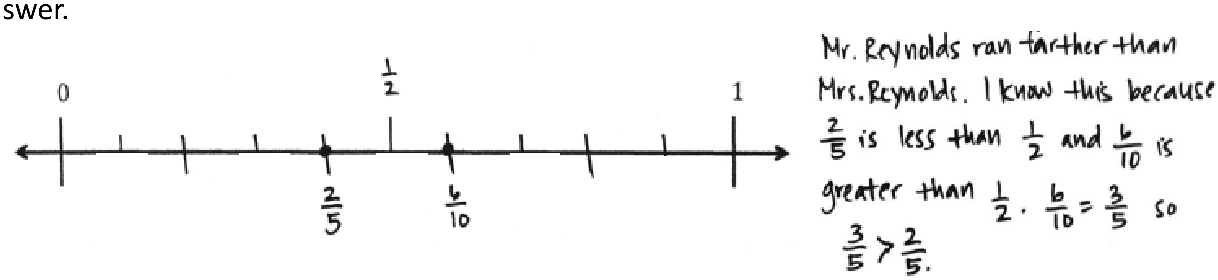
The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:

- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.



Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lesson 12-13 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Reason using benchmarks to compare two fractions on the number line.

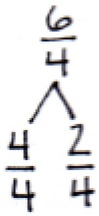
<u>Application Problem (5 min)</u>
Mr. and Mrs. Reynolds went for a run. Mr. Reynolds ran for 6/10 mile. Mrs. Reynolds ran for 2/5 mile. Who ran farther? Explain how you know. Use the benchmarks 0, 1/2, and 1 to explain your answer.



<u>Concept Development (20 min)</u>
Materials: (S) Personal white board, number line (Template)

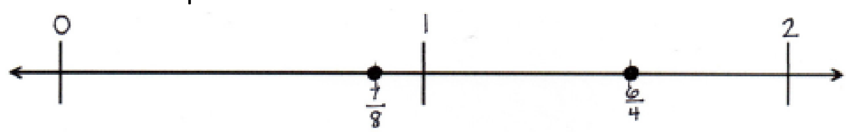
NOTES ON MULTIPLE MEANS OF ENGAGEMENT:
Some students may benefit from a review of how to change an improper fraction to a mixed number by drawing a number bond. Before the lesson, instruct students to draw a number bond for an improper fraction in which one addend has a value of 1 whole.

Problem 1: Reason to compare fractions between 1 and 2.
T: Compare 7/8 and 6/4 with your partner.
S: 7/8 is less than 1. 6/4 is greater than 1 because 1 is equal to 4/4.
T: Draw a number bond for 6/4 partitioning the whole and parts.
S: (Draw.)

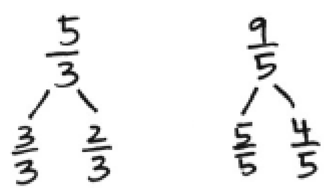


T: We can use the bond to help us locate 6/4 on the number line. Label a number line with endpoints 0 to 2, and locate 4/4.
S: (Put pencils on 4/4.)
T: 6/4 is 2/4 more. Imagine partitioning the line into fourths between 1 and 2. Where would you plot 6/4?
S: 6/4 is halfway between 1 and 2. →That's because 6/4 = 1 1/2. →6 fourths is 2 more fourths than 1. 2 fourths is the same as a half.
T: Plot 6/4 and 7/8. Write a statement to compare the two fractions.

S: $\frac{7}{8} < \frac{6}{4}$. → $\frac{6}{4} > \frac{7}{8}$.



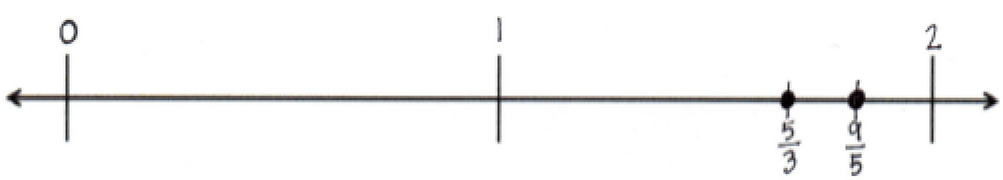
T: Next, compare 5/3 and 9/5. Discuss their relationship to 1.
S: Both are greater than 1 because 3/3 and 5/5 equal 1. →Neither is very close to 1, because 4/3 and 6/5 would be the fractions just a little bigger than 1.
T: Write a number bond to show 5/3 and 9/5 as a whole and some parts.
S: (Draw bonds.)
T: Use the number bond to write each fraction as 1 and some more fractional units.



S: $\frac{5}{3} = 1\frac{2}{3}$. → $\frac{9}{5} = 1\frac{4}{5}$.

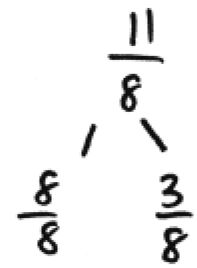
T: Label 0, 1, and 2 on another number line. We are plotting two points. One point is 2/3 greater than 1. The other is 4/5 greater than 1. Discuss with your partner how to plot these two points. Consider their placement in relation to 2.
S: 2/3 is 1 third less than 1. 4/5 is 1 fifth less than 1. Thirds are greater than fifths, so 2/3 is farther from 1 than 4/5. →1 2/3 is farther from 2 than 1 4/5. →The number bond lets me see that both fractions have 1 and some parts. The whole is the same, so I can compare just the parts and plot them between 1 and 2.
T: Plot the points. Compare 5/3 and 9/5. Write your statement using a comparison symbol.

S: (Write $\frac{5}{3} < \frac{9}{5}$ → $1\frac{2}{3} < 1\frac{4}{5}$.)



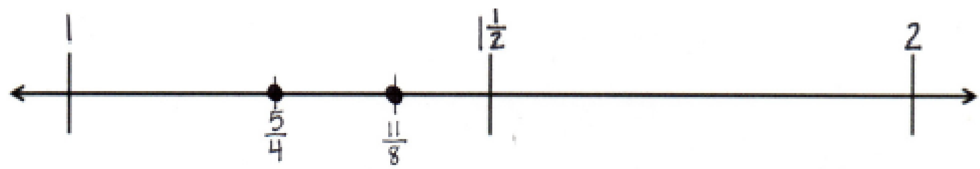
Continue the process with 7/4 and 9/5.

Problem 2: Reason about the size of fractions as compared to 1 1/2.



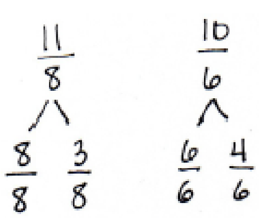
T: Is 11/8 less than 1 or greater than 1? Create a number bond to guide you in your thinking.
S: 11/8 is greater than 1 because 11/8 = 8/8 + 3/8. 8/8 is equal to 1, so 11/8 must be greater than 1.
T: Is 11/8 less than 1 1/2 or greater than 1 1/2?
S: 1 1/2 = 8/8 + 4/8, and 3/8 is less than 4/8, so 11/8 is less than 1 1/2. →1 1/2 is the same as 12/8. 11/8 is less than 12/8 , so 11/8 is less than 1 1/2.
T: Discuss with your partner if 5/4 is greater than or less than 1.
S: (Discuss.)
T: Plot 11/8 and 5/4 on another number line. You reasoned that both are between 1 and 2. Let's determine their placement using the benchmark 1 1/2. Label the number line with 1, 1 1/2, and 2. Talk it over with your partner before plotting.
S: 5/4 is the same as 1 1/4. That's halfway between 1 and 1 1/2. →There are 2 fourths in a half, so 5/4 is one unit away from 1 1/2 , and 11/8 is one unit away from 1 1/2. →Eighths are smaller than fourths, so 11/8 is closer to 1 1/2.
T: Compare 11/8 and 5/4. Write your statement using a comparison symbol.

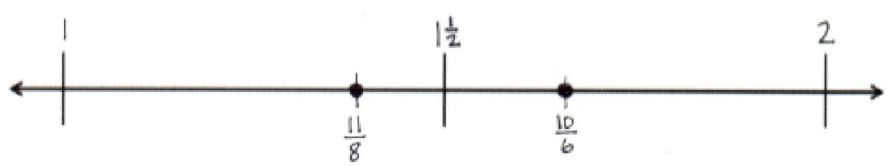
S: (Write $\frac{11}{8} > \frac{5}{4}$ or $1\frac{3}{8} > 1\frac{1}{4}$.)



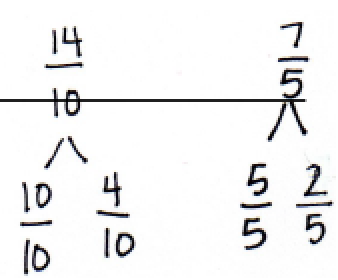
T: Compare 11/8 and 10/6 . Discuss with a partner using benchmarks to help explain.
S: Both fractions are greater than a whole but less than 2. →12/8 = 1 1/2. So, 11/8 is one unit less than 1 1/2. →9/6 = 1 1/2, so 10/6 is one unit more than 1 1/2. →I drew number bonds. Both numbers have a whole, so I just compared the parts. I thought of 3/8 and 4/6 compared to 1/2. I know 4/6 is more than 1/ , so I know

$1\frac{4}{6} > 1\frac{3}{8}$. → $\frac{11}{8} < \frac{10}{6}$.





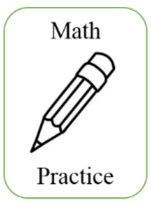
Problem 3: Reason using benchmarks to compare two fractions.
T: Which is greater: 14/10 or 7/5? Discuss with a partner. Use the benchmarks to help explain.
S: I used number bonds. Since both have 1 whole, I compared the parts: 4/10 and 2/5 are both less than 1 half. 4/10 is one unit away



from 1 half. But there are no fifths equal to 1 half. →4/10 is 4 units from zero. 2/5 is 2 units from zero. Fifths are half of tenths. I think they are equal! →I can make an equivalent fraction to compare.

$\frac{7}{5} = \frac{7 \times 2}{5 \times 2} = \frac{14}{10}$. $\frac{14}{10}$ is equal to $\frac{7}{5}$.

→14/10 = 7/5.
T: Compare 6/4 and 11/10.
S: 11/10 is 1/10 past 1. 6/4 = 1 1/2. →6/4 > 11/10.
T: Compare 10/8 and 8/4.
S: 1 2/8 is halfway between 1 and 1 1/2. →8/4 = 2. →10/8 < 8/4.

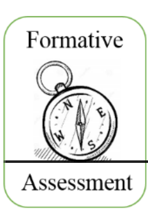


Problem Set (10 min)
Eureka Math – Lesson 13 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

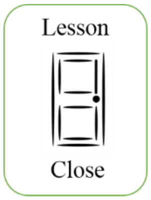
<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Reason using benchmarks to compare two fractions on the number line.

You may choose to use any combination of the questions below to lead the discussion.
- When were number bonds helpful in solving some of the problems on the Problem Set? Explain.
- Explain your thinking in comparing the fractions when you solved Problem 5(a–j). Were benchmarks always helpful?
- How did you solve Problem 5(h)?
- What other benchmarks could you use when comparing fractions? Why are benchmarks helpful?
- How did the Application Problem connect to today's lesson?



Formative Assessment (10 min)
- Eureka Math – Lesson 12 & 13 Exit Ticket (Module 5)



Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time they will continue comparing fractions.

Homework

Eureka Math – Lesson 12 & 13 (Module 5)

Unit 5 – Fractions

DAY 16

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Focus: Fractions (15 min)

Adapted from NCTM, www.nctm.org/profdev)

2/6

Name 3 equivalent fractions	Is it in simplest form? If not, what is the simplest form?
Is it closer to 0, ½, or 1?	Name 3 fractions that are greater. Name 3 fractions that are less.

Learning Targets

- I can make sense of problems and persevere in solving them.

Pattern Blocks (75 min)

(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)

The outcome of this task is to construct conceptual knowledge of comparing two fractions.

Task 1
Use pattern blocks.
- If a hexagon is one whole, which block represents ½? Which block represents 1/3? Which blocks represents 2/3?
- If a trapezoid if one whole, which block represents 1/3? Which block represents 2/3?
- If a blue rhombus is one whole, which block represents ½?
- If a blue rhombus is ½ of a whole, what would one whole look like?
- Find one half of a hexagon and one half of a blue rhombus. Why don't they make one whole altogether?
- If a green triangle is 1/3 of a whole, what would one whole look like? How many these wholes could you make with 3 hexagons?

Task 2
Use grid paper.
- Justin planted tomatoes in 1/3 of his 6' x 6' garden. Gina planted tomatoes in 1/3 of her 8' x 6' garden. How many square feet of the garden did each person use for tomatoes? If each person planted 1/3 of their garden with tomatoes, why did they use a different amount of square feet?
- Deon used a 9 x 9 grid to represent 1 whole and Shawn used a 12 x 12 grid to represent 1. Each boy shaded in squares to show 1/3 of the whole. How many squares did Deon shade? How many squares did Shawn shade? Why did they shade different numbers of squares if they each shaded in 1/3?

Answer Key
Task 1:
- The trapezoid is half of a hexagon, the blue rhombus is one-third of the hexagon, and two blue rhombuses are two-thirds of the hexagon.
- The green triangle is one-third of the trapezoid and the blue rhombus is two-thirds of the trapezoid.
- The green triangle is half of the blue rhombus.
- Since there is not a particular shape, student answers may vary. Example: The whole would look like a trapezoid with an extra triangle.
- They do not make one whole together because the wholes were different sizes.
- The whole would look like a red trapezoid. You could make six of these wholes.

Task 2:
- Justin used twelve square feet, and Gina used sixteen square feet. They used a different amount of square feet because their gardens (the whole) were different sizes.
- Deon shaded in twenty-seven squares, and Shawn shaded in forty-eight squares. They shaded in a different number of squares because their wholes were different sizes.

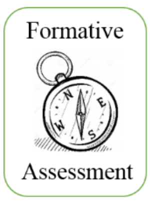
Supporting the Investigative Task

As students work, the teacher will confer with them making notes about generalizations students are making.

- The size of the whole matters
- Replacing blocks with common or like pieces
- Multiplication strategies to determine the number of total squares
- Splitting a set of objects into thirds

Differentiation

- Support – If students are having difficulty, ensure they can explain what halves and thirds mean at a conceptual level and have them use pattern blocks to build each situation.
- Enrichment – Mrs. Johnson and Mrs. Black each gave half of their students a pencil. Mrs. Johnson handed out five more pencils than Mrs. Black. How many students are in each class? Is this the only solution? Why or why not? Provide evidence to support your thinking.



Formative Assessment Options (15 min)
(Mathematics Formative Assessment by P. Keeley & C. Tobey)

- Is it Fair?

  "Is it Fair? asks students to examine a context in which several mathematical statements are made in response to a problem. Students examine the proposed solution to decide if it is fair (for example dividing up a quantity so everyone gets an equal portion). If they decide it is unfair, they explain what makes it unfair and how the situation can be made fair. Alternatively, if it is fair, students explain why it is fair. Common misconceptions or mistakes made in mathematics are situated in the context."

  Is It Fair? Why or Why Not?

  Mackenzie ordered two large pizzas for her family to have for dinner. Her father cut the first pizza into eight slices, and her mother cut the second pizza into four slices.

  - Mackenzie ate two slices of the first pizza
  - Her brother ate one slice of the second pizza

- Create the Problem

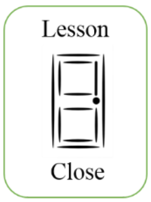
  "Create the Problem is a reverse problem-solving FACT. Instead of performing the computation, students are given the solution and are asked to figure out what the real-world problem might be."

  Create the Problem

${}^{3}/_{4} < {}^{7}/_{8}$

Create a problem that may have been solved with this expression.

Explain how your story matches the equation.



Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Pattern Blocks will be discussed in math congress, and they will continue comparing fractions.

DAY 17

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

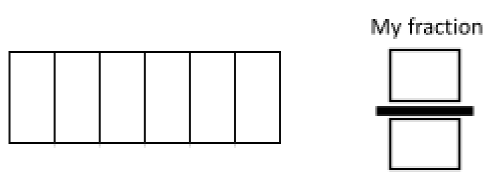
Formative Feedback Task (15 min)

This task should be used solely for the purpose of gathering individual student data, providing feedback, and ultimately advancing each student's learning. It should never be graded. This task should be completed with no assistance from the teacher or peers. The teacher will provide the student oral and written feedback through assessing and advancing questions. The ultimate goal of this feedback is to advance the individual learning of each student.

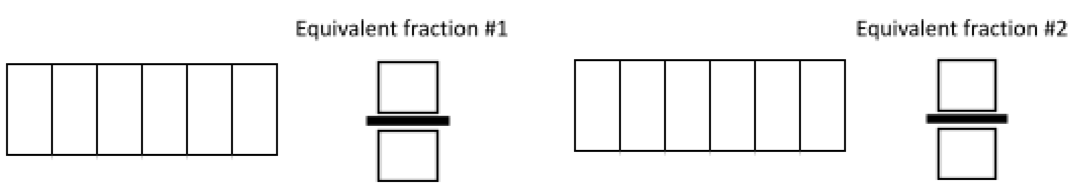
Focus: Equivalent Fractions
(Adapted from Howard County Schools, https://jsangiovanni.wikispaces.hcpss.org/)

Name ____ Date _________________ #_________

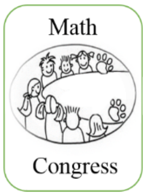
1. The rectangle is divided into equal pieces. Shade some of the parts. What fraction did you create?



2. Use the figures below to create two equivalent fractions to the fraction you created above. Then write the name of the fraction shown in the model.



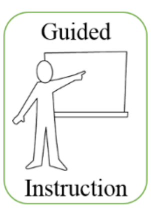
Learning Targets
- I can make sense of problems and persevere in solving them.
- I can compare two fractions.



Math Congress – Pattern Blocks (35 min)
Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

Example of Math Congress structure and sequence based on anticipated student strategies:
1. The size of the whole matters
2. Replacing blocks with common or like pieces
3. Multiplication strategies to determine the number of total squares
4. Splitting a set of objects into thirds

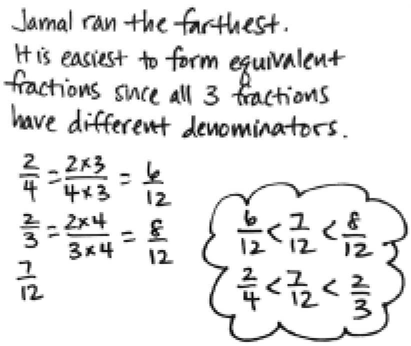
The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:
- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.

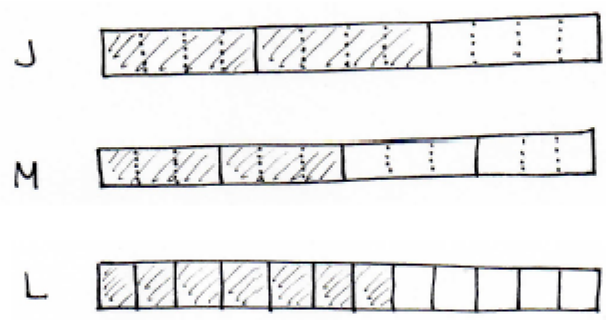


Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lesson 14 & 15 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Find common units or number of units to compare two fractions.

<u>Application Problem (5 min)</u>
Jamal ran 2/3 mile. Ming ran 2/4 mile. Laina ran 7/12 mile. Who ran the farthest? What do you think is the easiest way to determine the answer to this question? Talk with a partner about your ideas.

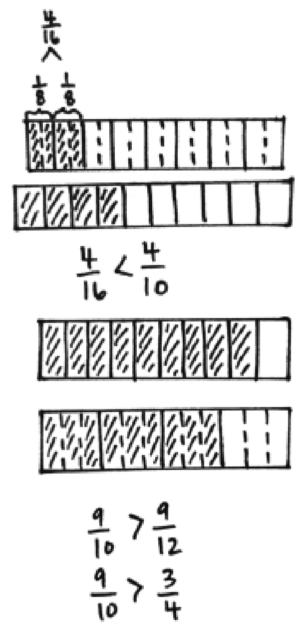
Unit 5 – Fractions





Note: This Application Problem reviews skills learned in Topic B to compare fractions and anticipates finding common units in this lesson. Be ready for conversations centered around comparing the fractions in other ways. Such conversations might include area models, tape diagrams, and finding equivalent fractions.

<u>Concept Development (20 min)</u>
Materials: (S) Personal white board



Problem 1: Compare fractions with related numerators.
T: (Display 2/8 and 4/10.) Draw a tape diagram to show each.
T: Partition the eighths in half. What fraction is now shown?
S: 4/16. The numerators are the same! →The number of shaded units is the same.
T: Compare 4/16 and 4/10.
S: 4/16 is less than 4/10 since sixteenths are smaller units than tenths. I can compare the size of the units because the numerators are the same.
T: Compare 2/8 and 4/10.
S: 2
8 is less than 4/10.
T: (Display 9/10 and 3/4.) Discuss a strategy for comparing these two fractions with your partner.
S: Let's make a common numerator of 9.
$\frac{3}{4}=\frac{3\times 3}{4\times 3}=\frac{9}{12}$. → $\frac{9}{10}$ is greater than 9/12. →9/10 is greater than 3/4.
→9/10 + 1/10 = 1, and 3/4 + 1/4 = 1. 1 tenth is less than 1 fourth, so 9 tenths is greater.

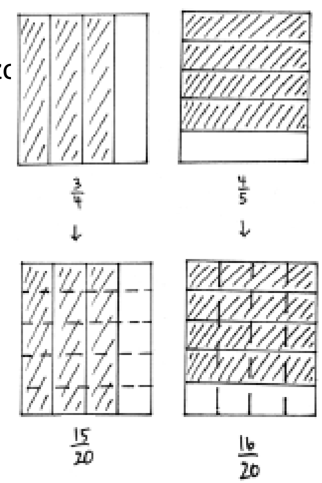
Problem 2: Compare two fractions with unrelated denominators using area models.

T: (Display 3/4 and 4/5.) We have compared fractions by using benchmarks to help us reason. Another way to compare fractions is to find like units.
T: Draw two almost-square rectangles that are the same size. Each model is 1. Partition the left area model into fourths by drawing vertical lines. (Model.)
S: (Draw two almost-square rectangles, partitioning the left area model into fourths.)
T: Shade 3/4 of the left area model. Partition the right area model into fifths by drawing horizontal lines. Shade 4/5. (Demonstrate.)

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

When comparing fractions, we seek to make common units. This can be modeled by representing $\frac{3}{4}$ vertically, while representing $\frac{4}{5}$ horizontally. Then, each model is decomposed to make twentieths. Both models then show common units of the same size and shape, even if the whole units are not drawn perfectly square.

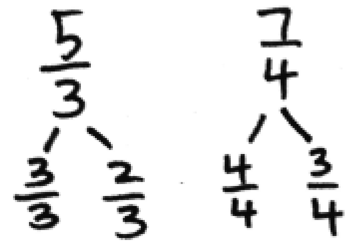
S: (Shade 3/4 of the left area model. Partition the right area model into fifths by drawing horizontal lines.
Shade 4/5.)
T: Do we have like denominators?
S: No.
T: Partition each fourth into 5 equal pieces. (Demonstrate drawing horizo
T: How many units are in the whole now?
S: 20.
T: What is the value of one of the new units?
S: 1 twentieth.
T: How many twentieths are shaded?
S: 15.
T: Now, let's decompose 4
5. Partition each fifth into 4 equal pieces. (Model the decomposition drawing vertical lines.) How many twentieths are the same as 4/5?
S: 16/20 is the same as 4/5.
T: Now that we have common units, can you compare the fractions?
S: Yes! 15/20 is less than 16/20, so 3/4 is less than 4/5.
T: How did we decompose 4/5 and 3/4 to compare?
S: We made common units so that we would be able to compare the fractions. First, we drew area models to show each fraction. We partitioned one using vertical lines and the other using horizontal lines. Then, we partitioned each model again to create like units. Once we had like units, it was easy to compare the fractions. We compared 15/20 and 16/20. Then, we knew that 3/4 < 4/5.
Repeat with 2/3 and 3/5, drawing thirds vertically and fifths horizontally. Then, partition the thirds into fifths and the fifths into thirds.



$$\frac{15}{20} < \frac{16}{20} \quad \text{so} \quad \frac{3}{4} < \frac{4}{5}$$

Problem 3: Compare two fractions greater than one with unrelated denominators using number bonds and area models.

T: (Display 5/3 and 7/4.) These fractions are greater than 1. Draw number bonds to show how 5/3 and 7/4 can be expressed as the sum of a whole number and a fraction.
S: 5/3 = 3/3 + 2/3 and 7/4 = 4/4 + 3/4.
T: Since the wholes are the same, we can just compare 2/3 and 3/4. Draw area models once again to help.
S: 2/3 is less than 3/4. →Since 2/3 is less than 3/4, 1 2/3 is less than 1 3/4. →5/3 is less than 7/4.



Repeat with 6/4 and 7/5.

Problem 4: Compare two fractions with unrelated denominators without an area model.

T: We modeled common units to compare 4/5 and 3/4. What was the common unit?
S: Twentieths!
T: Use multiplication to show that 4/5 is the same as 16/20.

S: $\frac{3}{4}=\frac{3\times 5}{4\times 5}=\frac{15}{20}$.

T: Use multiplication to show that 3/4 is the same as 15/20.

S: $\frac{3}{4}=\frac{3\times 5}{4\times 5}=\frac{15}{20}$.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

For students who struggle to represent fractions precisely, provide a template of equally sized rectangles that can be partitioned as area models. This helps them to compare fractions more easily.

T: We decomposed by multiplying by the denominator of the other fraction.
T: Let's compare 3/5 and 8/12 by multiplying the denominators. We could use area models, but that would be a lot of little boxes!

T: (Write $\frac{3}{5}=\frac{3\times 12}{5\times 12}=\frac{}{60}$.)

How many sixtieths are the same as 3 fifths? Write your answer as a multiplication sentence.

S: $\frac{3}{5}=\frac{3\times 12}{5\times 12}=\frac{36}{60}$.

T: (Write $\frac{8}{12}=\frac{8\times 5}{12\times 5}=\frac{}{60}$.)

How many sixtieths are the same as 8 twelfths? Write your answer as a multiplication sentence.

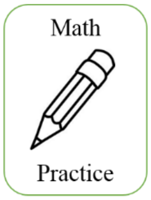
S: $\frac{8}{12}=\frac{8\times 5}{12\times 5}=\frac{40}{60}$.

T: Compare 3/5 and 8/12.
S: 36/60/< 40/60, so 3/5 < 8/12.
T: Write 9/5 and 10/8 . Express each as an equivalent fraction using multiplication.

S: $\frac{9}{5}=\frac{9\times 8}{5\times 8}=\frac{72}{40}$.

$\frac{10}{8}=\frac{10\times 5}{8\times 5}=\frac{50}{40}$.

T: 72/40 > 50/40. That means 9/5 > 10/8 .

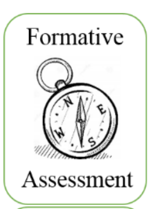


Problem Set (10 min)
Eureka Math – Lesson 15 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Find common units or number of units to compare two fractions.

You may choose to use any combination of the questions below to lead the discussion.
- In Problem 2, did you need to use multiplication for every part? Why or why not? When is multiplication not needed, even with different denominators?
- In Problem 2(b), did everyone use forty-eighths? Did anyone use twenty-fourths?
- In Problem 3, how did you compare the fractions? Why?
- Do we always need to multiply the denominators to make like units?
- If fractions are hard to compare, we can always get like units by multiplying denominators—a method that always works. Why is it sometimes not the best way to compare fractions?
- What new or significant math vocabulary did we use today to communicate precisely?
- How did the Application Problem connect to today's lesson?



Formative Assessment (10 min)
- Eureka Math – Lessons 14 & 15 Exit Ticket (Module 5)

Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time they will continue their work on equivalent fractions and comparing fractions by participating in a math workshop (where they will rotate to five different centers).

Homework

Eureka Math – Lessons 14 & 15 (Module 5)

Unit 5 – Fractions

DAY 18

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

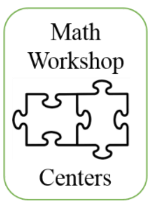
Unit 5 – Fractions

Focus: Adding and Subtracting (15 min)

	Answer Key	*Strategies that may be used*
$^{2}/_{8} + {}^{6}/_{8} = n$	n =	Helper Problem
$^{8}/_{8} = n + {}^{2}/_{8}$	n =	$^{8}/_{8} - {}^{2}/_{8}$
$^{7}/_{8} - n = {}^{2}/_{8}$	n =	$^{7}/_{8} - {}^{2}/_{8}$
$n = {}^{2}/_{8} + {}^{5}/_{8}$	n =	$({}^{2}/_{8} + {}^{6}/_{8}) - {}^{1}/_{8}$
$^{3}/_{8} + {}^{5}/_{8} = n$	n =	$({}^{2}/_{8} + {}^{5}/_{8}) + {}^{1}/_{8}$
$^{5}/_{8} = {}^{3}/_{8} + n$	n =	$^{5}/_{8} - {}^{3}/_{8}$

Learning Targets

- I can make sense of problems and persevere in solving them.
- I can compare two fractions.
- I can use models to explain equivalent fractions.



Math Workshop – Centers (75 min)

Students will rotate through five different centers and complete designated tasks. Students will work independently, even though they will rotate as a group. Each rotation will last approximately 10-15 minutes. If a student finishes before the rotation time ends, he will complete any teacher specified unfinished problem set practice. The teacher will conduct an invitational group as one of the centers. This instruction should be differentiated based on the group needs. The teacher may choose to use debrief questions from Eureka lessons already completed and/or tasks related to learning targets. Differentiated instruction may include, but not be limited to debrief questions, tasks, problem set review, homework review, and cumulative review.

Center 1: 4.NF.1 Task(s)

(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)

Name: ______________________ Date: __________ #: ___

Fraction Rectangles

Task 1: Use color tiles to make a rectangle or square that is one half red and one half blue.

Task 2: Use color tiles to make a rectangle or square that is one third red. Find at least three different ways to represent one third. Using pictures, numbers, and/or words, prove that the three models that you made are all equal to one third.

Repeat the task with ¼ and 1/6 for additional practice.

Name: ______________________ Date: __________ #: ___

Equivalent Pizzas

There is two-thirds of a pizza left.

How many pieces of pizza are left if the original pizza had a total of 3 slices? Provide evidence to support your thinking.

How many pieces of pizza are left if the original pizza had a total of 6 slices? Provide evidence to support your thinking.

How many pieces of pizza are left if the original pizza had a total of 12 slices? Provide evidence to support your thinking.

Center 2: 4.NF.1 Task(s)

(Howard County Schools, https://jsangiovanni.wikispaces.hcpss.org/)

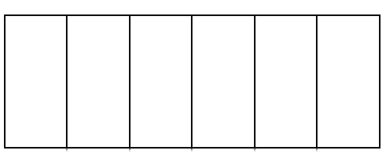
Name: ______________________ Date: __________ #: ___

4. NF. 1

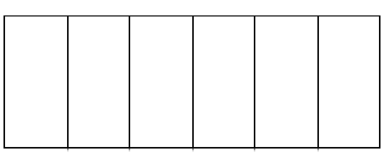
The rectangle is divided into some equal pieces. Shade some of the parts. What fraction did you create?

My fraction

Use the figures below and create two equivalent fractions to the fraction you created above. Then, write the name of the fraction shown in the model.



Equivalent fraction #1

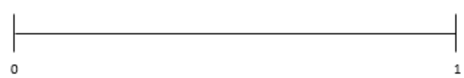


Equivalent fraction #2

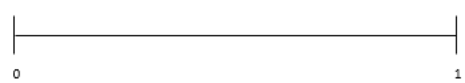
Name: ______________________ Date: __________ #: ___

4. NF. 1

1. Place the fraction $\frac{3}{4}$ on the number line.



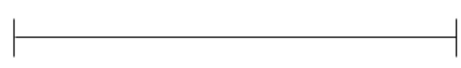
2. What is an equivalent fraction for $\frac{3}{4}$? ______________
3. Place your equivalent fraction on the number line.



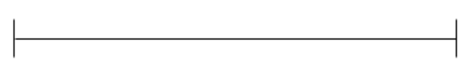
Name: ______________________ Date: __________ #: ___

4. NF. 1

1. Place the fraction $\frac{3}{4}$ on the number line.



2. What is an equivalent fraction for $\frac{3}{4}$? ______________
3. Place your equivalent fraction on the number line.



Center 3: 4.NF.2 Task(s)

(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)

Name: ______________________ Date: __________ #: ___

Who's On The Bus?

There are some children on the bus.

- 2/6 of the children are wearing tan pants.
- 6/10 of the children are wearing tennis shoes.
- 5/12 of the children are wearing a red shirt.
- 2/3 of the children are wearing a hat.

For each item of clothing, are more than half or less than half of the children wearing that item?

Write a sentence explaining how you know that you are correct.

Name: ______________________ Date: __________ #: ___

Which is Bigger?

If two hexagons have a value of 1 whole, think about the value of other pattern block. Draw each fraction below in terms of pattern blocks and determine which is bigger:

- 3/4 or 4/6
- 1/2 or 5/12
- 2/4 or 3/6
- 5/6 or 3/4

Write your own comparison question using fourths, sixths, or twelfths. Draw a picture to prove which is easier.

Pick one of the questions above and write a sentence explaining how you know that you are correct.

Center 4: 4.NF.2 Task(s)

(Howard County Schools, https://jsangiovanni.wikispaces.hcpss.org/)

Name: ______________________ Date: __________ #: ___
4.NF.2

Which is greater: $\frac{4}{10}$ or $\frac{5}{8}$? ________________

Explain how you know in the space below. Use pictures and/or words in explanation.

Name: ______________________ Date: __________ #: ___
4.NF.2

1. Circle all the inequalities that are true.

$\frac{1}{4} < \frac{2}{3}$	$\frac{1}{5} < \frac{1}{6}$	$\frac{7}{8} > \frac{3}{4}$
$\frac{4}{5} > \frac{1}{3}$	$\frac{2}{5} > \frac{3}{4}$	$\frac{6}{8} > \frac{3}{8}$

2. Pick one inequality that is false and rewrite it to make it true.

false ____________ true ____________

Name: ______________________ Date: __________ #: ___
4.NF.2

Which is greater: $\frac{4}{10}$ or $\frac{5}{8}$? ________________

Explain how you know in the space below. Use pictures and/or words in explanation

Name: ______________________ Date: __________ #: ___
4.NF.2

1. Circle all the inequalities that are true.

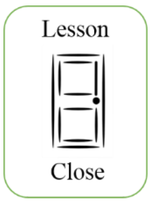
$\frac{1}{4} < \frac{2}{3}$	$\frac{1}{5} < \frac{1}{6}$	$\frac{7}{8} > \frac{3}{4}$
$\frac{4}{5} > \frac{1}{3}$	$\frac{2}{5} > \frac{3}{4}$	$\frac{6}{8} > \frac{3}{8}$

2. Pick one inequality that is false and rewrite it to make it true.

false ____________ true ____________

Center 5: Invitational Group

The teacher will meet with each group during their Center 5 Rotation, where differentiated instruction should occur (providing each group with an experience based on need). The teacher may choose to reteach a concept from a formative feedback task or formative assessment, expand on a guided instruction/mini-lesson previously taught, expand on a problem set using debrief questions, review homework, reteach a concept, conduct a cumulative review, or provide other tasks specific to student needs.



Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time they will continue exploring fractions.

Unit 5 – Fractions

DAY 19

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

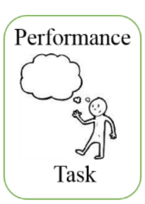
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.



Performance Task – Enough Soda (30 min)

This task will serve as an independent summative assessment of student learning. It should be completed with no assistance.

Enough Soda

Part 1

You need ¾ of a Liter of soda to make punch for a party. Which containers have enough soda in them to make punch? Provide evidence to support your thinking.

Container A – 2/4 of a Liter
Container B – 2/3 of a Liter
Container C – 5/6 of a Liter
Container D – 11/12 of a Liter
Container E – 7/12 of a Liter

Answer Key

Containers C and D have enough soda to make punch for a party.

Learning Targets

- I can make sense of problems and persevere in solving them.

Ed, Chip & Roy (75 min)

The outcome of this task is to construct conceptual knowledge of adding and subtracting fractions.

Ed, Chip and Roy make all the signs for the baseball team car wash. Ed made ¼ of them, and Chip made 3/8. What fraction of the signs did Roy make?

Answer Key

Roy made 3/8 of the signs.

Supporting the Investigative Task

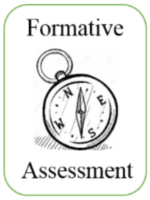
As students work, the teacher will confer with them making notes about generalizations students are making.

- The number of eighths equivalent to one whole (all of the signs)
- Strategies to create an equivalent fraction for Ed’s signs (1/4 = 2/8)
- Models to represent the situation
- Strategies for adding and subtracting fractions

Differentiation

- Support – If students are having difficulty, have them draw a model to represent all of the signs.

- Enrichment – Jerry made one gallon of sweetened tea and one half galloon of lemonade for a picnic. If he drank ¼ of each container, how many cups of tea did he drink? How many cups of lemonade? Remember that one gallon equals sixteen cups.



Formative Assessment
- <u>Check Me Cards (During Investigative Task)</u>
  Check Me Cards are used by students to self-assess their work and obtain feedback during a task. Students place a clothespin next to the statement that best describes the status of their work. This allows the teacher to focus on the students in most need of help during the task. The teacher may also utilize the cards to provide feedback if a student(s) is not on task.

Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Ed, Chip, & Roy will be discussed in math congress, and they will continue adding and subtracting fractions.

Unit 5 – Fractions

DAY 20

Content Standard(s)

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Focus: Subtracting a Fraction from One Whole (15 min)

	<u>Answer Key</u>	*Strategies that may be used*
$1 - {}^{1}/_{5} = n$	$n = {}^{4}/_{5}$	Helper Problem
$n = 1 - ¼$	$n = ¾$	$¾ + {}^{1}/_{4}$
$1 - {}^{1}/_{3} = n$	$n = {}^{2}/_{3}$	${}^{2}/_{3} + {}^{1}/_{3}$
$n = 1 - {}^{3}/_{8}$	$n = {}^{5}/_{8}$	${}^{5}/_{8} + {}^{3}/_{8}$
$1 - {}^{9}/_{10} = n$	$n = {}^{1}/_{10}$	${}^{1}/_{10} + {}^{9}/_{10}$
$n = 1 - {}^{5}/_{6}$	$n = {}^{1}/_{6}$	${}^{1}/_{6} + {}^{5}/_{6}$

Learning Targets

- I can make sense of problems and persevere in solving them.
- I can add and subtract two fractions.
- I can subtract a fraction from one whole.

Math Congress – Ed, Chip and Roy (35 min)

Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

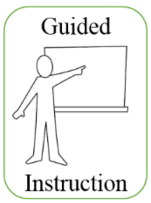
Example of Math Congress structure and sequence based on anticipated student strategies:

1. The number of eighths equivalent to one whole (all of the signs)
2. Strategies to create an equivalent fraction for Ed's signs (1/4 = 2/8)

3. Models to represent the situation
4. Strategies for adding and subtracting fractions

The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:

- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.



Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lesson 17 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Use visual models to add and subtract two fractions with the same units, including subtracting from one whole.

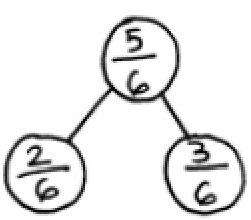
Application Problem (5 min)

Use a number bond to show the relationship between 2/6, 3/6, and 5/6. Then, use the fractions to write two addition and two subtraction sentences.

Note: This Application Problem reviews work from earlier grades using related facts. The number sentences could also be written with the sum or difference on the left. The process of creating number bonds to show the relationship between addition and subtraction helps to bridge to the beginning of today's lesson where students identify related fraction facts when 1 one is the whole.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Students working below grade level may benefit from drawing a tape diagram or another pictorial model of $\frac{5}{6}$, $\frac{2}{6}$, and $\frac{3}{6}$ in order to meaningfully derive two addition and two subtraction sentences from the number bond.

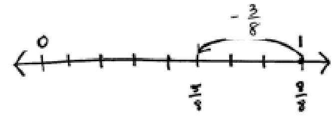


$\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

$\frac{5}{6} - \frac{3}{6} = \frac{2}{6}$

$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$



$\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$

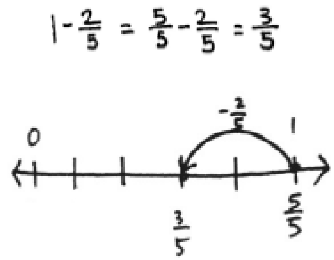
$1 - \frac{3}{8} = \frac{5}{8}$

Concept Development (20 min)

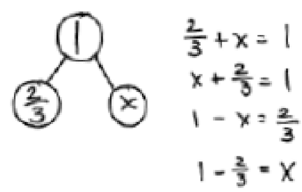
Materials: (S) Personal white board

Problem 1: Subtract a fraction from 1.

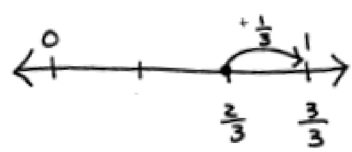
T: Let's find the value of 1 – 3/8. Are the units the same?
S: No. There are ones and eighths.
T: Rename 1 one as eighths.
S: 8 eighths.
T: 8 eighths minus 3 eighths is…?
S: 5 eighths.
T: Model the subtraction using a number line. To simplify our number lines, use hash marks to show the eighths. Label 0, 1, and the numbers used to solve.
T: Record your work from the number line as a number sentence.
S: (Write 8/8 – 3/8 = 5/8 or 1– 3/8 = 5/8.)
T: (Display 1– 2/5.) Discuss with your partner how to solve.
S: We have to make like units. 1 one is equal to 5 fifths. →5 fifths minus 2 fifths equals 3 fifths. →5/5 – 2/5 = 3/5. →1– 2/5 = 3/5.
T: Work with a partner to show 1– 2/5 is the same as 5/5 – 2/5 using a number line.



T: (Display 1– 2/3 = x.) Draw a number bond to show 2/3, x, and 1.
T: Write two subtraction and two addition sentences using 2/3, x, and 1.
S: $\frac{2}{3} + x = 1$. $x + \frac{2}{3} = 1$. $1 - \frac{2}{3} = x$. $1 - x = \frac{2}{3}$.



T: Draw a number line with endpoints 0 and 1. Partition and label thirds.
T: 2/3 + x = 1. Draw a point at 2/3. How many more thirds does it take to make 1?
S: 1 third.
T: We can think of subtraction as an unknown addend problem and count up.



Repeat with 1 – 7/12.

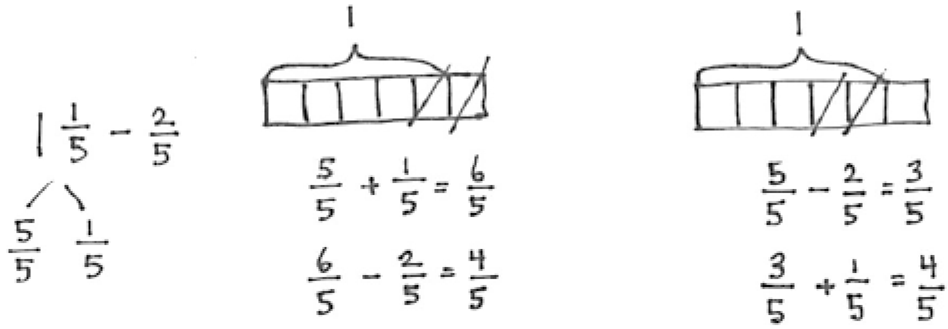
Problem 2: Subtract a fraction from a number between 1 and 2.

T: Let's solve 1 1/5 – 2/5. First, draw a number bond to decompose 1 1/5 into a whole number and fractional parts. Show the whole number as fifths.
S: (Show 1 1/5 decomposed as 5/5 and 1/5.)
T: I'm going to draw two tape diagrams to show the whole of 1 1/5 with 2/5 subtracted in different ways. (Draw two tape diagrams side by side. Cross off 2 fifths, as shown below, and write the related number sentences. See the illustration below.) Compare the methods with your partner.

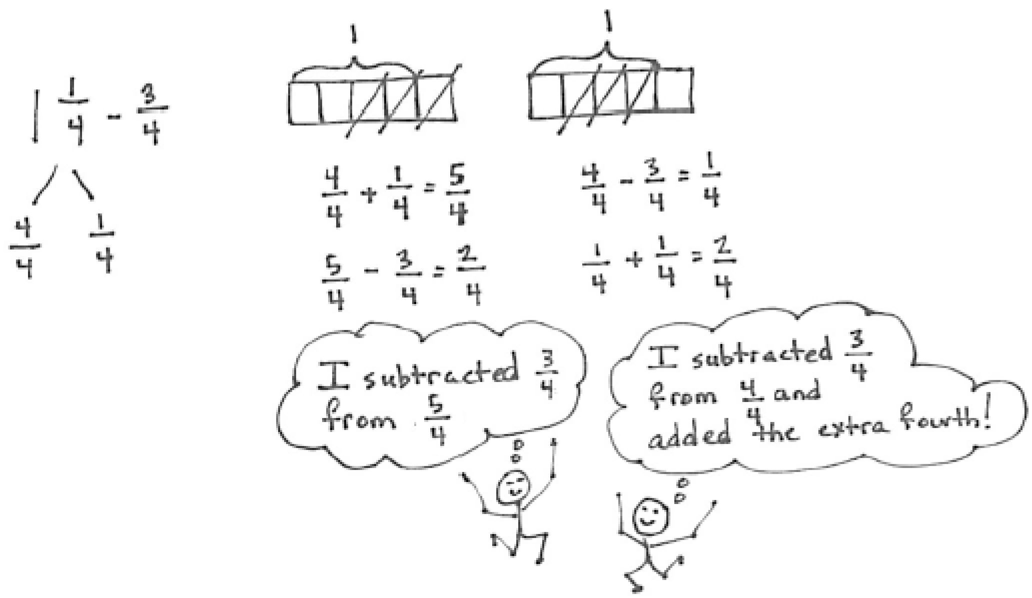
NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Student modeling of subtraction and addition on the number line may vary slightly depending on how students solve. For example, students working below grade level may model counting down with an arrow representing a series of hops. Encourage part–whole thinking and modeling by means of modeling with the number bond before the number line, if beneficial.

Unit 5 – Fractions



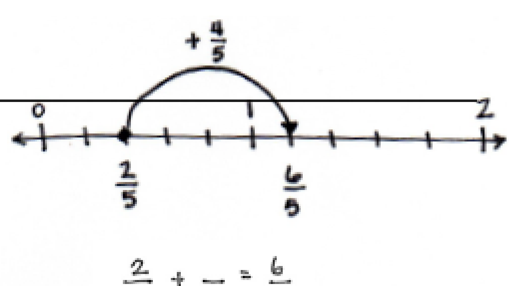
S: The solution on the left added 5 fifths and 1 fifth to get 6 fifths and then subtracted 2 fifths. →The second solution subtracted 2 fifths from 5 fifths and added that to 1 fifth. →That's how we learned how to subtract in Grades 1 and 2. When I subtract 8 from 13, I take it from the ten and add back 3.
T: Did both methods give the same answer?
S: Yes.
T: We can subtract from the total number of fifths, or we can subtract from 1, or take from 1, and add back the extra fifth.
T: Practice both methods using 1 1/4 – 3/4. Start by showing our number bond. Partner A, subtract from the total. Partner B, take from 1. Draw a tape diagram if it helps you.



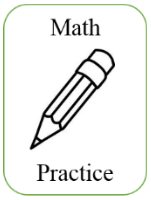
T: Try 1 3/8− 5/8, switching strategies with your partner.
S: (Solve.)
T: By the way, 13 – 8 can also be solved by thinking 8 + ____ = 13 and counting up. What number sentence shows counting up as a strategy for solving 1 1/5– 2/5? Talk to your partner.

S: $\frac{2}{5}$ + _____ = $\frac{6}{5}$. → $\frac{2}{5}$ + _____ = $1\frac{1}{5}$. → It's an unknown addend. → An unknown part.

T: Let's show it on the number line. (Draw the image to the right.)



T: I could also jump up to the whole number and add on.
T: The number line is a nice way to show counting up where the tape diagram was better for showing taking from the total and taking from 1. I chose to use the models that I thought would help you best understand. With your partner, take a moment to think about what subtracting from the total and subtracting from the whole number would look like on the number line.

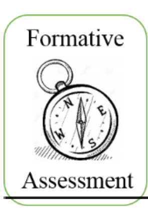


Problem Set (10 min)
Eureka Math – Lesson 17 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

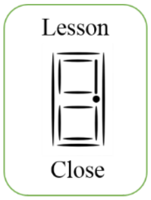
<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Use visual models to add and subtract two fractions with the same units, including subtracting from one whole.

You may choose to use any combination of the questions below to lead the discussion.
- For Problems 1(a) and (b), how did you determine the two addition and subtraction number sentences?
- Which strategy did you prefer for Problem 2(a–f)?
- What support does the number line offer you when solving problems such as these?
- Is the counting up strategy useful when solving subtraction problems? Explain.
- What extra step is there in solving when the fraction is written as a whole or mixed number instead of as a fraction?
- How is subtract from 1, or take from 1, similar to the take from 10 strategy?
- What role do fact families play in fractions? How are fraction fact families similar to whole number fact families?
- How did the Application Problem connect to today's lesson?



Formative Assessment (10 min)
- Eureka Math – Lesson 17 Exit Ticket (Module 5)



Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time they will continue exploring adding and subtracting fractions.

Homework

Eureka Math – Lesson 17 (Module 5)

Unit 5 – Fractions

DAY 21

Content Standard(s)

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Focus: Adding and Subtracting More than one Fraction (15 min)

	<u>Answer Key</u>	*Strategies that may be used*
$^{1}/_{5} + ^{1}/_{5} + ^{1}/_{5} = n$	$n = ^{3}/_{5}$	Helper Problem
$\frac{1}{4} + \frac{1}{4} + n = \frac{3}{4}$	$n = \frac{1}{4}$	$\frac{3}{4} - ^{2}/_{4}$
$n = ^{1}/_{3} + ^{1}/_{3} + ^{1}/_{3}$	$n = ^{3}/_{3}$	$^{2}/_{3} + ^{1}/_{3}$
$n = ^{7}/_{8} – ^{3}/_{8} – ^{1}/_{8}$	$n = ^{3}/_{8}$	$(^{7}/_{8} - ^{3}/_{8}) - ^{1}/_{8}$
$^{9}/_{10} – ^{3}/_{10} – ^{1}/_{10} = n$	$n = ^{5}/_{10}$ or $\frac{1}{2}$	$(^{9}/_{10} - ^{3}/_{10}) – ^{1}/_{10}$
$^{5}/_{6} = ^{1}/_{6} + n + ^{2}/_{6}$	$n = ^{6}/_{6}$	$^{5}/_{6} - ^{3}/_{6}$

Learning Targets

- I can make sense of problems and persevere in solving them.

Fraction Field Events (75 min)

(Adapted from Georgia Department of Education, https://www.georgiastandards.org/Georgia-Standards/Pages/Math-K-5.aspx)

The outcome of this task is to construct conceptual knowledge of adding and subtracting more than two fractions.

Carter Elementary School is having a field day. One of the events is the long jump. Participants in this event take a running start and then jump as far as they can into the pit which is ten feet long. The winner is determined by adding the distances jumped in three trials. The greatest total wins. Using the jump measuring below, determine the winner of this year's girls' and boys' long jump. Provide evidence to support your thinking.

1.

Name	1st Jump	2nd Jump	3rd Jump	Total Score
Kim	$7\ ^{3}/_{12}$ feet	$6\ ^{11}/_{12}$ feet	$6\ ^{5}/_{12}$ feet	
Amanda	$5\ ^{9}/_{12}$ feet	$6\ ^{1}/_{12}$ feet	$6\ ^{4}/_{12}$ feet	
Malaika	$7\ ^{9}/_{12}$ feet	$6\ ^{2}/_{12}$ feet	$7\ ^{11}/_{12}$ feet	
Mary	$8\ ^{1}/_{12}$ feet	$7\ ^{11}/_{12}$ feet	$8\ ^{3}/_{12}$ feet	
Freida	$7\ ^{10}/_{12}$ feet	$7\ ^{10}/_{12}$ feet	$8\ ^{2}/_{12}$ feet	

2. Who had the greatest total score for the girls' long jump?
3. Frieda wants to find how longer her second jump needed to be in order to win the event. In order to score greater than the winner, how far would Frieda need to jump? Explain your thinking using words, numbers, and pictures.
4.

Name	1st Jump	2nd Jump	3rd Jump	Total Score
Carlos	$8\ ^{1}/_{12}$ feet	$7\ ^{11}/_{12}$ feet	$8\ ^{6}/_{12}$ feet	
Emmett	$7\ ^{7}/_{12}$ feet	$6\ ^{10}/_{12}$ feet	$8\ ^{4}/_{12}$ feet	
Bob	$8\ ^{9}/_{12}$ feet	$9\ ^{2}/_{12}$ feet	$8\ ^{11}/_{12}$ feet	
Thomas	$6\ ^{7}/_{12}$ feet	$8\ ^{11}/_{12}$ feet	$8\ ^{3}/_{12}$ feet	
Gene	$7\ ^{10}/_{12}$ feet	$7\ ^{3}/_{12}$ feet	$8\ ^{5}/_{12}$ feet	

5. Who had the greatest total score for the boys' long jump?
6. Carlos wants to find how long his 2nd jump needed to be in order to win the event. In order to score greater than the winner, how far did Carlos need to jump? Explain your thinking using words, numbers and pictures.

<u>Answer Key</u>

1.

Name	1st Jump	2nd Jump	3rd Jump	Total Score
Kim	$7\ ^{3}/_{12}$ feet	$6\ ^{11}/_{12}$ feet	$6\ ^{5}/_{12}$ feet	$20\ ^{7}/_{12}$
Amanda	$5\ ^{9}/_{12}$ feet	$6\ ^{1}/_{12}$ feet	$6\ ^{4}/_{12}$ feet	$18\ ^{2}/_{12}$
Malaika	$7\ ^{9}/_{12}$ feet	$6\ ^{2}/_{12}$ feet	$7\ ^{11}/_{12}$ feet	$21\ ^{10}/_{12}$
Mary	$8\ ^{1}/_{12}$ feet	$7\ ^{11}/_{12}$ feet	$8\ ^{3}/_{12}$ feet	$24\ ^{3}/_{12}$
Freida	$7\ ^{10}/_{12}$ feet	$7\ ^{10}/_{12}$ feet	$8\ ^{2}/_{12}$ feet	$23\ ^{10}/_{12}$

2. Mary had the greatest total score for the girls' long jump.
3. Frieda needed a second jump greater than $8\ ^{3}/_{12}$ feet to win.
4.

Name	1st Jump	2nd Jump	3rd Jump	Total Score
Carlos	$8\ ^{1}/_{12}$ feet	$7\ ^{11}/_{12}$ feet	$8\ ^{6}/_{12}$ feet	$24\ ^{6}/_{12}$
Emmett	$7\ ^{7}/_{12}$ feet	$6\ ^{10}/_{12}$ feet	$8\ ^{4}/_{12}$ feet	$22\ ^{9}/_{12}$
Bob	$8\ ^{9}/_{12}$ feet	$9\ ^{2}/_{12}$ feet	$8\ ^{11}/_{12}$ feet	$26\ ^{10}/_{12}$
Thomas	$6\ ^{7}/_{12}$ feet	$8\ ^{11}/_{12}$ feet	$8\ ^{3}/_{12}$ feet	$23\ ^{9}/_{12}$
Gene	$7\ ^{10}/_{12}$ feet	$7\ ^{3}/_{12}$ feet	$8\ ^{5}/_{12}$ feet	$23\ ^{6}/_{12}$

5. Bob had the greatest total score for the boys' long jump.
6. Carlos needed a second jump greater than $10\ ^{3}/_{12}$ to win.

Supporting the Investigative Task

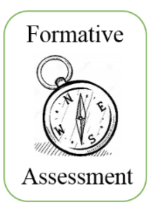
As students work, the teacher will confer with them making notes about generalizations students are making.

- Adding together whole number then fractional amounts
- Adding two fractions together first then adding the total to the third fraction
- Strategies for converting a fraction greater than one whole to a mixed number

Differentiation

- Support – If students are having difficulty, have them add together the whole number feet first then add the fractional amounts.
- Enrichment –

My jumping rope was cut in half
half was thrown away.
The other half was cut again
one third along the way.
The longer part (ten feet long)
is what I use to play.
How long was my jumping rope
when I began today?



Formative Assessment

- Check Me Cards (During Investigative Task)

  Check Me Cards are used by students to self-assess their work and obtain feedback during a task. Students place a clothespin next to the statement that best describes the status of their work. This allows the teacher to focus on the students in most need of help during the task. The teacher may also utilize the cards to provide feedback if a student(s) is not on task.

Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Fraction Field Events will be discussed in math congress, and they will continue adding and subtracting fractions.

DAY 22

Content Standard(s)

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Focus: Adding and Subtracting More than one Fraction (15 min)

	Answer Key	*Strategies that may be used*
$^{1}/_{6} + ^{1}/_{6} + ^{1}/_{6} = n$	$n = ^{3}/_{6}$ or ½	Helper Problem
$^{2}/_{12} + ^{3}/_{12} + n = ^{11}/_{12}$	$n = ^{6}/_{12}$ or ½	$^{11}/_{12} - ^{5}/_{12}$
$n = ^{3}/_{10} + ^{1}/_{10} + ^{4}/_{10}$	$n = ^{8}/_{10}$ or $^{4}/_{5}$	$(^{3}/_{10} + ^{1}/_{10}) + ^{4}/_{10}$
$n = ^{11}/_{12} - ^{4}/_{12} - ^{1}/_{12}$	$n = ^{3}/_{8}$	$(^{11}/_{12} - ^{4}/_{12}) - ^{1}/_{12}$
$^{5}/_{8} - ^{3}/_{8} - ^{2}/_{8} = n$	$n = ^{0}/_{8}$ or 0	$(^{5}/_{8} - ^{3}/_{8}) - ^{2}/_{8}$
$^{9}/_{12} = ^{1}/_{12} + n + ^{3}/_{12}$	$n = ^{5}/_{12}$	$^{9}/_{12} - ^{4}/_{12}$

Learning Targets

- I can make sense of problems and persevere in solving them.
- I can add and subtract more than two fractions.

Math Congress – Ed, Chip and Roy (35 min)

Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

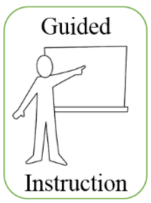
Example of Math Congress structure and sequence based on anticipated student strategies:

1. Strategies for adding and subtracting fractions
2. Adding together whole number then fractional amounts

3. Adding two fractions together first then adding the total to the third fraction
4. Strategies for converting a fraction greater than one whole to a mixed number

The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:

- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.



Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lesson 18 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Add and subtract more than two fractions.

<u>Concept Development (25 min)</u>
Materials: (S) Adding and subtracting fractions (Practice Sheet)

Note: Arrange students in groups of three to solve and critique each other’s work.
Exploration:

- Problems are sequenced from simple to complex and comprise addition and subtraction problems.
- All begin solving Problem A in the first rectangle.
- Students switch papers clockwise in their groups. Students analyze the solution in the first rectangle and critique it by discussing the solution with the writer. Then, students consider a different method to solve and record it in the second rectangle for Problem A.
- Students switch papers clockwise again for the second round of critiquing and third round of solving.
- Switching papers for the last time of the round, the original owner of the paper analyzes the three different methods used to solve the problem. A brief discussion may ensue as more than three methods could have been used within the group.
- The process continues as students solve Problems B through F.
- Some groups may not finish all problems during the time allotted, but the varied problems allow students to analyze and solve a wide variety of problems to prepare them for the Problem Set.
- Use the last five minutes of the Concept Development, prior to handing out the Problem Set, to review the many different solutions. The teacher may select one solution from three

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Exploration stations are sequenced from simple (Problem A) to complex (Problem F). To best guide student understanding, consider giving students working below grade level additional time to solve Problems A, B, and C, and then advance in order.

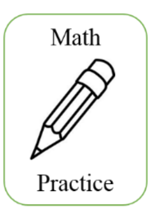
problems or three solutions from one problem to debrief. Identify common methods for solving addition and subtraction problems when there are more than two fractions.

Below are possible solutions for each problem. Students are encouraged to solve using computation through decomposition or other strategies.

Problem A:	$\frac{1}{8}+\frac{3}{8}+\frac{4}{8}$	
$\frac{1}{8}+\frac{3}{8}+\frac{4}{8}$ $\frac{4}{8}+\frac{4}{8}=\frac{8}{8}$	$\frac{1}{8}+\frac{3}{8}+\frac{4}{8}$ $\frac{1}{8}+\frac{7}{8}=\frac{8}{8}$	$\frac{1}{8}+\frac{3}{8}+\frac{4}{8}=\frac{8}{8}=1$
Problem B:	$\frac{1}{6}+\frac{4}{6}+\frac{2}{6}$	
$\frac{1}{6}+\frac{4}{6}+\frac{2}{6}$ $\frac{6}{6}+\frac{1}{6}=\frac{7}{6}$	$\frac{1}{6}+\frac{4}{6}+\frac{2}{6}$ $\frac{6}{6}=1$ $1+\frac{1}{6}=1\frac{1}{6}$	$\frac{1}{6}+\frac{2}{6}+\frac{4}{6}$ $\frac{3}{6}$ $\quad\frac{3}{6}$ $\frac{1}{6}$ $\frac{3}{6}+\frac{3}{6}+\frac{1}{6}=\frac{6}{6}+\frac{1}{6}=1\frac{1}{6}$
Problem C:	$\frac{11}{10}-\frac{4}{10}-\frac{1}{10}$	
$\frac{11}{10}-\frac{4}{10}-\frac{1}{10}$ $\frac{11}{10}-\frac{5}{10}=\frac{6}{10}$	$\frac{11}{10}-\frac{4}{10}=\frac{7}{10}$ $\frac{7}{10}-\frac{1}{10}=\frac{6}{10}$ $\frac{6}{10}=\frac{3}{5}$	$\frac{11}{10}-\frac{1}{10}=\frac{10}{10}$ $\frac{10}{10}-\frac{4}{10}=\frac{6}{10}$

Problem D:	$1-\frac{3}{12}-\frac{5}{12}$	
$1-\frac{3}{12}-\frac{5}{12}$ $\frac{12}{12}-\frac{8}{12}=\frac{4}{12}$	$\frac{12}{12}-\frac{3}{12}=\frac{9}{12}$ $\frac{9}{12}-\frac{5}{12}=\frac{4}{12}$	$\frac{3}{12}+\frac{5}{12}=\frac{8}{12}$ $\frac{8}{12}+\frac{4}{12}=\frac{12}{12}$ $\frac{4}{12}=\frac{1}{3}$

Problem E:	$\frac{5}{8}+\frac{4}{8}+\frac{1}{8}$	
$\frac{5}{8}+\frac{4}{8}+\frac{1}{8}=\frac{10}{8}$ $\frac{8}{8}$ $\frac{2}{8}$ $=1\frac{2}{8}$	$\frac{5}{8}+\frac{4}{8}+\frac{1}{8}$ $\frac{3}{8}$ $\frac{1}{8}$ $\frac{8}{8}+\frac{2}{8}=1+\frac{2}{8}=1\frac{2}{8}$	$\frac{1}{8}+\frac{5}{8}+\frac{4}{8}$ $\frac{2}{8}$ $\frac{2}{8}$ $\frac{8}{8}+\frac{2}{8}=1\frac{2}{8}=1\frac{1}{4}$
Problem F:	$1\frac{1}{5}-\frac{2}{5}-\frac{3}{5}$	
$1\frac{1}{5}-\frac{2}{5}-\frac{3}{5}$ $1-\frac{5}{5}=0$ $0+\frac{1}{5}=\frac{1}{5}$	$\frac{6}{5}-\frac{2}{5}=\frac{4}{5}$ $\frac{4}{5}-\frac{3}{5}=\frac{1}{5}$	$1\frac{1}{5}-\frac{2}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $=1-\frac{4}{5}$ $=\frac{1}{5}$



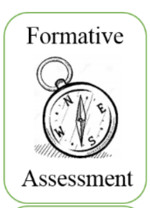
Problem Set (10 min)
Eureka Math – Lesson 18 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Add and subtract more than two fractions.

You may choose to use any combination of the questions below to lead the discussion.

- In Problem 1(h), the total is a mixed number. Was it necessary to change the mixed number to a fraction in this case? Explain.
- Discuss your solution strategy for Problem 1(i). Grouping fractions to make 1 is a strategy that can help in solving problems mentally. Solving for 2/12 + 10/12 and 5/12 + 7/12 can lead to the solution more rapidly.
- For Problem 2, did you agree with Monica or Stuart? Explain why you chose that strategy. Do you see a different method?
- Consider how you solved Problem 1(c) and the other solution for it in Problem 3. Would this solution be accurate?
  (Display $\frac{5+7+2}{7} = \frac{14}{7} = 2$.)
  Explain why this representation for addition of fractions is correct.
- Observe your solution to Problem 1(d). Is my solution correct? Why? Explain. Display:
  $\frac{7-3-1}{8} = \frac{3}{8}$.)
- Explain in words how we add or subtract more than two fractions with like units.
- When is it necessary to decompose the total in a subtraction problem into fractions? Give an example.

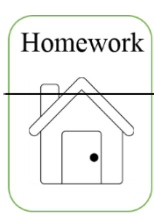


Formative Assessment (10 min)

- Eureka Math – Lesson 18 Exit Ticket (Module 5)

Closing the Lesson

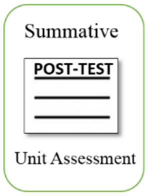
- Revisit the learning targets with students.
- Inform students that next time they will take an assessment to demonstrate what they have learned so far in this unit.



Homework

Eureka Math – Lesson 18 (Module 5)

DAY 23



Mid-Unit Assessment
Scantron = Test ID #PO473
The on-line summative unit assessment should be completed at the end of the unit. The objective is to evaluate student learning outcomes compared to standards and benchmarks. In addition, this data will be reviewed at an aggregate level to reflect on overall unit performance and instruction.

Student Materials
- Scantron Student ID Number
- Work paper
- Pencils
- Privacy folder for computer

Student Online Assessment Directions
1. Open Google Chrome web browser.
2. Go to www.achievementseries.com.
3. Select "Student Login".
4. Enter Site ID:
   Cahaba Elementary: 62-3426-3084
   Paine Elementary: 30-6732-4169
   Magnolia Elementary: 27-9923-2868
5. Enter Student ID.
6. Select the "green checkmark" to verify student name.
7. If message is displayed to allow pop-ups, select "OK".
8. Select "Start Test".
9. Enter Test ID.
10. Read the instructions: Put your name on your work paper. Number each problem, and show all work before entering an answer on the computer.
11. Select "Continue" to begin assessment.
12. After answering all assessment questions, review your responses and work paper.
13. Select "Turn in Test".

Unit 5 – Fractions

DAY 24

Content Standard(s)

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Focus: Decomposing Fractions, Adding and Subtracting Fractions (15 min)

	<u>Answer Key</u>	*Strategies that may be used*
$10 = n + {}^{5}/_{5}$	n = 9	Helper Problem
$10 - {}^{5}/_{5} = n$	n = 9	$9 + {}^{5}/_{5} = 10$
$10 - {}^{1}/_{5} = n$	n =	$9\ {}^{4}/_{5} + {}^{1}/_{5} = 10$
$10 - {}^{4}/_{5} = n$	n =	$9\ {}^{1}/_{5} + {}^{4}/_{5} = 10$
${}^{4}/_{5} + n = 10$	n =	$9\ {}^{1}/_{5} + {}^{4}/_{5} = 10$
${}^{1}/_{5} + n = 10$	n =	$9\ {}^{4}/_{5} + {}^{1}/_{5} = 10$

Learning Targets

- I can make sense of problems and persevere in solving them.

Going the Distance (60 min)

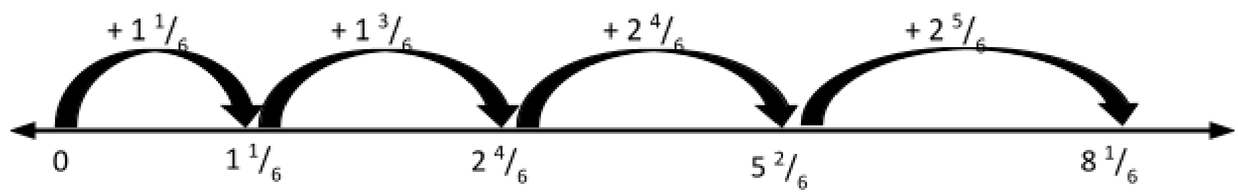
(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)
The outcome of this task is to construct conceptual knowledge of subtracting a fraction less than one whole from a whole number and adding a fraction less than one who to a whole number.

In order to train for the Girls on the Run 5K, the girls' running team at Lincoln Elementary School runs the following distances.

Week	Distance
Week 1	1 and 1/6 miles
Week 2	1 and 3/6 miles
Week 3	2 and 4/6 miles
Week 4	2 and 5/6 miles

Part 1
Draw a number line to show the distance that the girls ran each week.
Part 2
How far did the girls run in all? Write an equation that matches the story.
Part 3
The girls at Jefferson Elementary School ran ten miles total during the same time. How much father did they run than the girls at Lincoln Elementary School? Provide evidence to support your thinking.

Answer Key
Part 1:

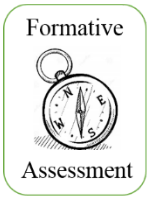


Part 2: The girls ran 8 and 1/6 miles. 1 1/6 + 1 3/6 + 2 4/6 + 2 5/6 = 8 1/6.
Part 3: The girls at Jefferson Elementary School ran 1 5/6 miles farther than the girls at Lincoln Elementary School.

Supporting the Investigative Task
As students work, the teacher will confer with them making notes about generalizations students are making.
- Use of a number line to add fractions
- Regrouping when fractional sums are greater than one whole
- Intentionally adding together the fractions that equal a whole first then adding the remaining fractions
- Strategies for subtracting fractions from a whole number

Differentiation
- Support – If students are having difficulty, have them begin by adding together the total miles for Week 1 and Week 4.
- Enrichment – Mr. Cal Q. Later has created a game called Jellybean Fractions to play with his students. Here's how it works. Each student takes a handful of jelly beans from the big gourmet jar on Mr. Later's desk (he encourages them to take their favorite number). Mr. Later asks the students to make groups of jellybeans representing half their total. If they can

make two equal groups without leftovers, they score a point. Mr. Later then asks students to rearrangement the jellybeans, this time representing one third of their total. If they can make three equal groups without leftovers, they score another point. Mr. Later continues with one fourth, one fifth, and so on. Find the best number to pick that is less than sixty.



Formative Assessment (15 min)
(Mathematics Formative Assessment by P. Keeley & C. Tobey)
This formative assessment may be administered during a gallery walk.

- Strategy Harvest

  "In this strategy, students complete a problem solving task and then circulate among their peers to find students who used a strategy different from theirs to solve the problem. Students record the other strategies and describe how the strategy is different from the one they used. During the process, students give feedback to each other on their strategy."

My Strategy	___________'s Strategy
___________'s Strategy	___________'s Strategy

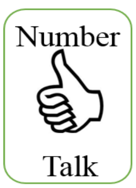
Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Going the Distance will be discussed in math congress, and they will continue adding and subtracting fractions.
-

Unit 5 – Fractions

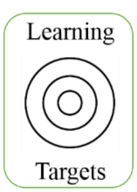
DAY 25

Content Standard(s)
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

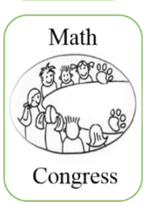


Focus: Decomposing Fractions, Adding and Subtracting Fractions (15 min)

	Answer Key	*Strategies that may be used*
3 – ¼ = n	n = 2 ¾	Helper Problem
2 ¾ + n = 3	n = ¼	2 ¾ + ¼ = 3
6 – ¼ = n	n = 5 ¾	5 ¾ + ¼ = 6
5 ¾ + n = 6	n = ¼	5 ¾ + ¼ = 6
6 – ¾ = n	n = 5 ¼	5 ¼ + ¾ = 6
¾ + n = 6	n = 5 ¼	5 ¼ + ¾ = 6



Learning Targets
- I can add fractions.
- I can subtract a fraction from a whole number.



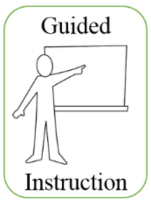
Math Congress – Going the Distance (40 min)
Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

Example of Math Congress structure and sequence based on anticipated student strategies:
1. Use of a number line to add fractions
2. Strategies for converting a fraction greater than one whole
3. Adding two fractions that equal a whole first then adding the remaining fractions

4. Strategies for subtracting fractions from a whole number

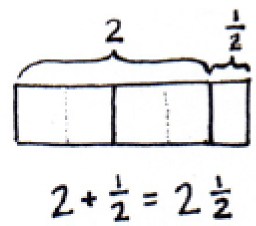
The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:

- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.



Guided Instruction/Mini-Lesson (20 min)
Eureka Math – Lesson 22 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Add a fraction less than 1 to, or subtract a fraction less than 1 from, a whole number using decomposition and visual models.

<u>Concept Development (20 min)</u>
Materials: (S) Personal white board



Problem 1: Add a fraction less than 1 to a whole number using a tape diagram.
T: Answer in mixed units: 2 meters + 5 centimeters is…?
S: 2 meters 5 centimeters.
T: 2 hours + 5 minutes is…?
S: 2 hours 5 minutes.
T: 2 ones + 5 eighths is…?
S: 2 ones and 5 eighths.
T: (Display 2 + 12.) Draw a tape diagram to show 2 ones. To know how large to draw 1/2, let's partition each whole number into 2 halves.
T: (Demonstrate partitioning the 2 ones with dotted lines.)
T: Partition the ones, and extend your model to add 1/2. Say a number sentence that adds the whole number to the fraction.
S: 2 + 1/2 = 2 1/2.
T: In this case, 2 ones plus 1 half gave us a sum that is a mixed number. We have seen mixed numbers often when working with measurement and place value, like when we added hundreds and tens, which are two different units.

Repeat the process with 3 + 2/3 = 3 2/3.

Problem 2: Subtract a fraction less than 1 from a whole number using a tape diagram.

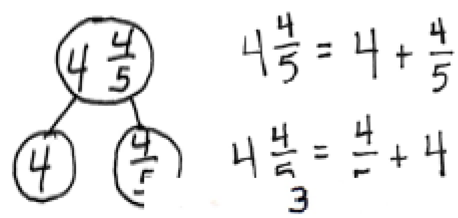
T: (Display 3 – 1/4.) Draw a tape diagram to represent 3, partitioned as 3 ones. Watch as I subtract 1/4. (Partition a one into 4 parts. Cross off 1/4. Trace along the tape diagram with a finger to count the remaining parts.)
T: What is remaining?
S: 2 and 3 fourths. →2 ones and 3 fourths.
T: Say the complete subtraction sentence.
S: 3 – 1/4 = 2 3/4.
T: Subtract 3 – 2/3. Draw a tape diagram with your partner. Discuss your drawing with your partner.
S: I drew a tape diagram 3 units long. I partitioned the last unit into thirds, and then I crossed off 2 thirds.
T: Say the entire number sentence.
S: 3 – 2/3 = 2 1/3.
T: Discuss what you see happening to the number of ones when you subtract the fraction.
S: It gets smaller. →There are fewer ones. If we started with 3, the answer was 2 and some parts. →Right, so if we had a big number such as 391 – 2/3, we know the whole number would be 1 less, 390, and some parts.
T: What relationship do you see between the fraction being subtracted and the fraction in the answer?
S: They are the same unit. →They are part of one of the whole numbers. →They add together to make 1. That's why the whole number is 1 less in the answer. →Right. In the last problem, we took away 2/3, and the fraction in the answer was 1/3. Those add to make 1.

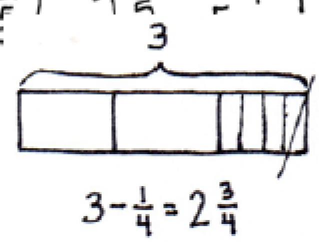
NOTES ON MULTIPLE MEANS OF REPRESENTATION:

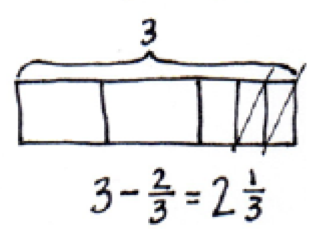
Clarify for English language learners multiple meanings for the term *whole*. *Whole* can mean the total or sum as modeled in a number bond. Use *whole number* when referring to a unit in the ones, tens, hundreds, etc.

Problem 3: Given three related numbers, form fact family facts.

T: Write 4, 4 4/5, and 4/5. These numbers are related. Draw a number bond to show the whole and the parts. Write two addition facts and two subtraction facts that use 4, 4 4/5, and 4/5. Make a choice as to whether to write your sums and differences to the right or to the left of the equal sign.







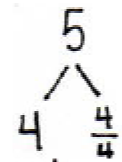
S: $4 + \frac{4}{5} = 4\frac{4}{5}$. → $\frac{4}{5} + 4 = 4\frac{4}{5}$. → $4\frac{4}{5} - \frac{4}{5} = 4$. → $4\frac{4}{5} - 4 = \frac{4}{5}$.

T: We can add and subtract ones and fractions just like we have always done. One number represents the whole, and the other two numbers represent the parts. For each of the following sets of related numbers, write two addition facts and two subtraction facts.

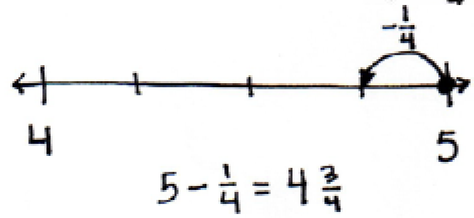
$\frac{3}{4}, 6\frac{3}{4}, 6$ $\quad$ $5, 4\frac{1}{3}, \frac{2}{3}$ $\quad$ $\frac{2}{5}, 4\frac{3}{5}, 5$

Problem 4: Subtract a fraction less than 1 from a whole number using decomposition.

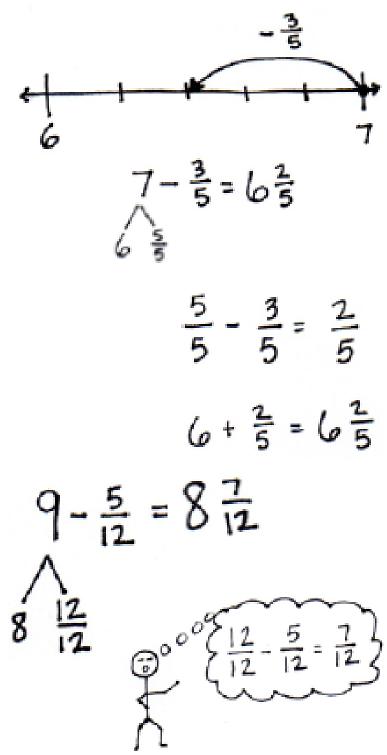
T: Write the expression 5 – 1/4. Discuss a strategy for solving this problem with your partner.
S: We can rename 1 one as 4 fourths, so we have 4 4/4 – 1/4. →We can make a mixed so the total is 4 and a fraction. →It's like unbundling a ten to subtract some ones.
T: Draw a number bond for 5 decomposed into two parts, 4 and 4 fourths or 4 and 1. (Allow students time to draw the bond.)



T: Construct a number line to represent 5 – 1/4 with 4 and 5 as endpoints. We are subtracting from 4/4, so our answer will be more than 4 and less than 5. Draw an arrow to represent 5 – 1/4. Write the number sentence under your number line.
S: (Write 5 – 1/4 = 4 3/4.)



T: Subtract 7 – 3/5. Solve with your partner, drawing a number bond and number line. (Allow students time to solve.)
T: Let's show your thinking using a number sentence. 7 decomposed is…?
S: 6 and 5/5.
T: (Record the bond under the number sentence.) How many ones remain?
S: 6.
T: (Record 6 in the number sentence.) 5/5 – 3/5 is…?
S: 2/5.
T: So, 2/5 remains. Add that to 6. The difference is…?
S: 6 2/5.
T: Subtract 9 – 5
12. Twelfths are a lot to partition on a number line. Solve this using just a number sentence and a number bond to decompose the total.
S: 9 – 5/12 = 8 7/12.



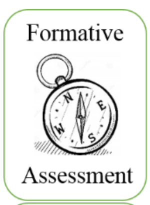
Problem Set (10 min)
Eureka Math – Lesson 22 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Add a fraction less than 1 to, or subtract a fraction less than 1 from, a whole number using decomposition and visual models.

You may choose to use any combination of the questions below to lead the discussion.

- Why is it necessary to decompose the total into ones and a fraction before subtracting? How does that relate to a subtraction problem such as 74 – 28?
- How did knowing how to subtract a fraction from 1 prepare you for this lesson?
- Describe how the whole number is decomposed to subtract a fraction. Use Problem 3(b) to discuss.
- How were number lines and number bonds helpful in representing how to find the difference?
- How did the Application Problem connect to today's lesson?



Formative Assessment (10 min)

- Eureka Math – Lesson 22 Exit Ticket (Module 5)

Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time they will extend their knowledge of adding and subtracting fractions by participating in an investigation called Pasta Party.

Homework
Eureka Math – Lesson 22 (Module 5)

Unit 5 – Fractions

DAY 26

Content Standard(s)

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Adding Fraction, Multiplying Fractions, Decomposition (15 min)

	Answer Key	*Strategies that may be used*
½ + ½ + ½ = n	n = $^{3}/_{2}$ or 1 ½	Helper Problem
3 x ½ = n	n = $^{3}/_{2}$ or 1 ½	½ + ½ + ½
$^{1}/_{3} + ^{1}/_{3} + ^{1}/_{3} + ^{1}/_{3}$ = n	n = $^{4}/_{3}$ or 1 $^{1}/_{3}$	Helper Problem
$^{1}/_{3}$ x 4 = n	n = $^{4}/_{3}$ or 1 $^{1}/_{3}$	$^{1}/_{3} + ^{1}/_{3} + ^{1}/_{3} + ^{1}/_{3}$
$^{1}/_{5} + ^{1}/_{5} + ^{1}/_{5} + ^{1}/_{5} + ^{1}/_{5}$ = n	n = $^{5}/_{5}$ or 1	Helper Problem
5 x $^{1}/_{5}$ = n	n = $^{5}/_{5}$ or 1	$^{1}/_{5} + ^{1}/_{5} + ^{1}/_{5} + ^{1}/_{5} + ^{1}/_{5}$

Learning Targets

- I can make sense of problems and persevere in solving them.

Pasta Party (60 min)
(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)
The outcome of this task is to extend previous understandings of multiplication to multiply a fraction by a whole number.

Part 1: Katie makes ¼ pound of pasta for each person at her dinner party. If seven people attend the party, how many pounds of pasta will be needed for her guests? Write an **addition** equation to show this situation. Show your answer with a number line or an area model. Use numbers or words to explain how your model shows addition.

Part 2: Write a multiplication equation to show this situation. Show your answer with a number line or an area model. Use numbers or words to explain how your model show multiplication.

Part 3: How are your addition and multiplication equations alike? Different? Would you use one over the other? Why or why not?

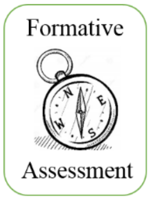
Answer Key
Part 1: If seven people attend the party 1 ¾ pounds of pasta will be needed for her guests.
Part 2: $\frac{1}{4} \times 7 = {}^{7}/_{4} = 1\,{}^{3}/_{4}$
Part 3: Answers may vary. Example: Both the addition and multiplication equations produce the same product/sum and represent the same situation. The equations are different because they use different operations.

Supporting the Investigative Task
As students work, the teacher will confer with them making notes about generalizations students are making.

- Use of number line or area model to represent addition and multiplication of fractions
- The denominator represents the size and/or the total number of pieces
- Models to represent the relationship between the addition and multiplication equations
- Use of addition and multiplication equation to represent the same situation

Differentiation

- Support – If students are having difficulty, have them solve the situation using ½ of a pound of pasta then relate their thinking to solve the situation using ¼ of a pound of pasta.
- Enrichment –The serving size is now ${}^{3}/_{8}$ of a pound of pasta. How many pound of pasta will be needed for her guests? Is this more or less than when the serving size was ¼ of a pound? How much more or less?



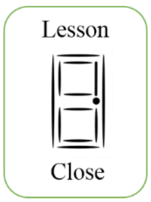
Formative Assessment (15 min)

(Mathematics Formative Assessment (Volume 2) by P. Keeley & C. Tobey)

- Flip the Question

  "Flip the Question is a questioning strategy that involves starting with questions of the form 'If I give you this information _____, you calculate this result ______.' and revising them into questions of the form 'If you have this result____, what information would you have started with?'"

Result	Information
If you have the result 5 ¼, what information would you have started with?	



Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Pasta Party will be discussed in math congress, and they will continue adding and multiplying fractions.

Unit 5 – Fractions

DAY 27

Content Standard(s)

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.

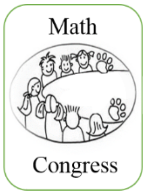
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Adding Fraction, Multiplying Fractions, Decomposition (15 min)

	Answer Key	*Strategies that may be used*
${}^{2}/_{3} + {}^{2}/_{3} = n$	$n = {}^{4}/_{3}$ or $1\,{}^{1}/_{3}$	Helper Problem
$2 \times {}^{2}/_{3} = n$	$n = {}^{4}/_{3}$ or $1\,{}^{1}/_{3}$	${}^{2}/_{3} + {}^{2}/_{3}$
${}^{2}/_{6} + {}^{2}/_{6} + {}^{2}/_{6} + {}^{2}/_{6} = n$	$n = {}^{8}/_{6},\ 1\,{}^{2}/_{6}$ or $1\,{}^{1}/_{3}$	Doubling/Halving
$4 \times {}^{2}/_{6} = n$	$n = {}^{8}/_{6},\ 1\,{}^{2}/_{6}$ or $1\,{}^{1}/_{3}$	${}^{2}/_{6} + {}^{2}/_{6} + {}^{2}/_{6} + {}^{2}/_{6}$
${}^{8}/_{12} + {}^{8}/_{12} + {}^{8}/_{12} = n$	$n = {}^{24}/_{12}$ or 2	Helper Problem
${}^{8}/_{12} \times 3 = n$	$n = {}^{24}/_{12}$ or 2	${}^{8}/_{12} + {}^{8}/_{12} + {}^{8}/_{12}$

Learning Targets

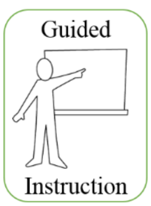
- I can add and multiply unit fractions.
- I can decompose fractions into whole numbers or mixed numbers.



Math Congress – Pasta Party (40 min)
Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

Example of Math Congress structure and sequence based on anticipated student strategies:
1. Use of number line or area model to represent addition and multiplication of fractions
2. The denominator represents the size and/or the total number of pieces
3. Models to represent the relationship between the addition and multiplication equations
4. Use of addition and multiplication equation to represent the same situation

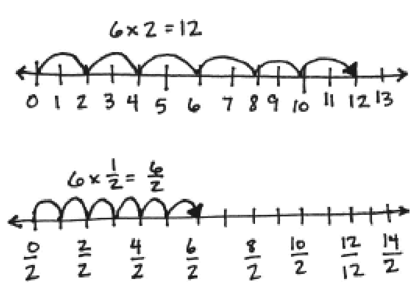
The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:
- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.



Guided Instruction/Mini-Lesson (20 min)
Eureka Math – Lesson 23 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Add and multiply unit fractions to build fractions greater than 1 using visual models.

<u>Concept Development (20 min)</u>
Materials: (S) Personal white board

Problem 1: Multiply a whole number times a unit fraction.
T: Write 6 × 2 as an addition sentence showing six groups of 2.
S: (Write 2 + 2 + 2 + 2 + 2 + 2 = 12.)
T: Draw a number line to show 6 twos.
S: (Draw a number line as shown to the right.)
T: Write 6 × 1/2 as an addition sentence showing six groups of 1/2.
S: (Write 1/2 + 1/2 + 1/2 + 1/2 + 1/2 + 1/2 = 6/2.)
T: Draw a number line to show 6 halves.
S: (Draw a number line as shown to the right.)
T: Work with your partner to draw parentheses, grouping halves to make ones.
S: We know 1/2 + 1/2 = 1, so maybe we can make three groups of that. →Yeah, let's draw parentheses around three separate groups of 2 halves.

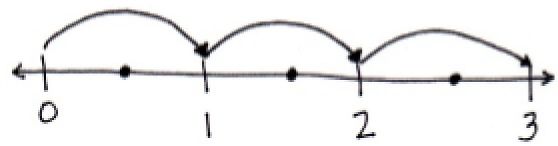


T: (Place parentheses.) 3 × 2
2 (point to the number sentence) is equal to…?
S: 3.
T: True or false? 6 × 1/2 = 3 × 2/2. Discuss with your partner.

$$6 \times \frac{1}{2} = \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{2} + \frac{1}{2}\right)$$

$$6 \times \frac{1}{2} = 3 \times \frac{2}{2}$$

Problem 2: Multiply a whole number times a unit fraction using the associative property.
T: Let's solve 6 × 1/2 using unit form. 6 × 1/2 is 6 halves.
T: (Display the number line as pictured.) Do you see three groups of 2 halves?
S: Yes.

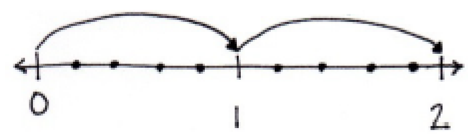


T: (Display: 6 halves = (3 × 2) halves = 3 × (2 halves) = 3 × $\left(\frac{2}{2}\right)$ = 3 × 1 = 3.)

T: Discuss this with your partner.

S: It tells us 6 halves equals 3 or 6 × $\frac{1}{2}$ = 3. → 3 × (2 halves) and 3 × $\left(\frac{2}{2}\right)$ shows the 3 ones really clearly. → 2 halves make 1, and 3 × 1 = 3.

T: But why did it start with (3 × 2) halves? Why not (2 × 3) halves? Or (1 × 6) halves?
S: Because we want to make ones. 2 halves make 1.
T: How many groups of 2 halves are in 6 halves?
S: 3.
T: So, 6 halves equals 3.
T: (Display 10 × 1/5.) Solve for 10 fifths using unit form.
S: We want to make groups of 5 fifths to make ones.
→10 fifths is the same as (2 × 5) fifths.
2 × (5 fifths) = 2 × $\left(\frac{5}{5}\right)$ = 2 × 1 = 2.



$$10 \times \frac{1}{5} = 2 \times \frac{5}{5} = 2$$

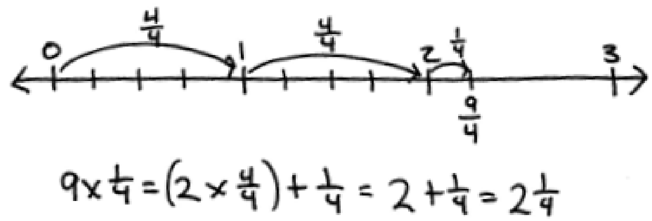
T: Support your answer with a number line.
S: I can make 10 slides of a fifth. → My arrows show 2 slides of 5/5. That is equal to 2. →10 × 1/5 = 2 × 5/5 = 2.

Repeat with 8 × 1/4.

Problem 3: Express the product of a whole number times a unit fraction as a mixed number.
T: (Display: 9 copies of 1/4.) 9 fourths. How many fourths make 1?
S: 4 fourths.
T: To make ones, how many groups of 4 fourths are in 9 fourths?
S: 2.
T: Two groups of 4 fourths makes 8 fourths. There is 1 fourth remaining.

Display: $9 \times \frac{1}{4} = \left(2 \times \frac{4}{4}\right) + \frac{1}{4} = 2 + \frac{1}{4} = 2\frac{1}{4}$.

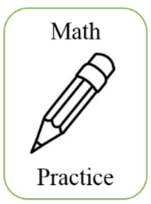
T: Draw a number line with endpoints 0 and
3. Label the ones, and partition fourths. With your partner, show (2 × 4/4) + 1/4. (Allow students time to draw two slides of 4/4 and then a slide of 1/4 as pictured to the right.)



T: With your partner, solve for 8 copies of 1 third.
S: There are two groups of 3 thirds in 8 thirds. That leaves 2 thirds remaining.

$\rightarrow 8 \times \frac{1}{3} = (2 \times \frac{3}{3}) + \frac{2}{3} = 2\frac{2}{3}$.

Repeat with $7 \times \frac{1}{2}$, $13 \times \frac{1}{5}$, and $17 \times \frac{1}{6}$.



Problem Set (10 min)
Eureka Math – Lesson 23 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

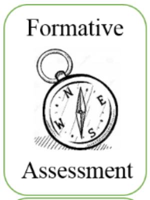
<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Add and multiply unit fractions to build fractions greater than 1 using visual models.

You may choose to use any combination of the questions below to lead the discussion.

- How is your work in Problem 1(a) related to your work in Problem 3(a)? How is adding like-unit fractions related to multiplying unit fractions? Is this true for Problems 1(b) and 3(b)?
- Using Problem 3(a), explain how 6 × 1/3 is the same as 2 × 3/3 .
- Explain why Problems 3(b) and 3(c) equal the same whole number. → Which is greater, 6 × 1/3 or 6 × 1/2?
- How are parentheses helpful as you solve Problem 2?
- Look at Problem 2 and Problem 3. Is there a way to tell when the product will be a whole number before multiplying? Explain your thinking.

- How did the Application Problem connect to today's lesson?



Formative Assessment (5 min)
- Eureka Math – Lesson 23 Exit Ticket (Module 5)

Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time they will extend their knowledge of comparing fractions by participating in an investigation called Fraction Fish.

Homework
Eureka Math – Lesson 23 (Module 5)

Unit 5 – Fractions

DAY 28

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Focus: Comparing Fractions (15 min)

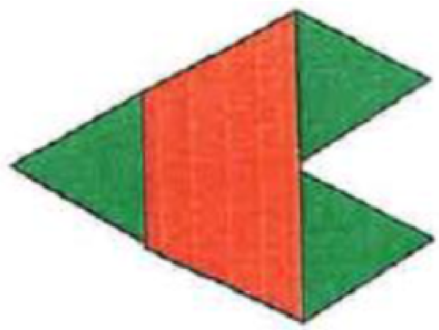
What fraction of a dollar would you rather have?	<u>Answer Key</u>	*Strategies that may be used*
½ or ¼	½	Halves are larger than fourths
$^{1}/_{5}$ or $^{3}/_{4}$	¾	$^{1}/_{5} < ½$ and $^{3}/_{4} > ½$
$^{4}/_{5}$ or ¾	$^{4}/_{5}$	$^{4}/_{5}$ is closer to one whole
$^{9}/_{10}$ or $^{4}/_{5}$	$^{9}/_{10}$	$^{9}/_{10}$ is closer to one whole
$^{3}/_{5}$ or $^{4}/_{10}$	$^{3}/_{5}$	$^{3}/_{5} > ½$ and $^{4}/_{10} < ½$
$^{3}/_{5}$ or $^{6}/_{10}$	$^{3}/_{5} = {}^{6}/_{10}$	Twice as many pieces that are half the size

Learning Targets

- I can make sense of problems and persevere in solving them.

Fraction Fish #1 - Comparisons (75 min)
(Ongoing Assessment Project (OGAP)/Delivered by AMSTI/UAB)
The outcome of this task is to construct conceptual knowledge of comparing fractions.

Ask students to build the fish below using pattern blocks.



Explain that this design equals one whole fish or 1 fish.

Have students respond to the following:
Part 1
List all of the fractional values for:
- Red trapezoid –
- Green triangle—
- Blue rhombus—
- Yellow Hexagon—

Part 2
Use the pattern blocks and the information about Fractions Fish #1 to solve these problems:
- Which is larger, 2/3 or 5/6?
- Which is larger, 1/6 or 1/2?
- Which is larger, 3/2 or 5/6?
- Which is larger, 13/6 or 5/2?
- Place the following fractions in order from smallest to largest: 1/3, 5/6, 2/3, 1/6, 1/2.
- Place the following improper fractions and mixed numbers in order from smallest to largest: 9/6, 1 1/6, 13/6. 2 1/3, 1 2/3, 9/3, 11/6, 5/6.

<u>Answer Key</u>
Part 1
List all the fractional values for:
- Red trapezoid = 1/2
- Green triangle = 1/6
- Blue rhombus = 1/3
- Yellow hexagon = 1

Part 2

Use the pattern blocks and the information about the values of each piece to solve these problems. Sketch the blocks used to explain why your answers make sense.

- 5/6 is larger
- 1/2 is larger
- 3/2 or 1 ½ is larger
- 5/2 or 2 ½ is larger
- 1/6, 1/3, ½, 2/3, 5/6
- 5/6, 1 $^{1}/_{6}$, 9/6, 1 $^{2}/_{3}$, 11/6, 13/6, 2 $^{1}/_{3}$, 9/3

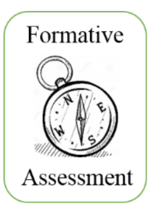
<u>Supporting the Investigative Task</u>

As students work, the teacher will confer with them making notes about generalizations students are making.

- Replacing blocks with common or like pieces (common denominator)
- Use of models to compare fractions
- Expressing the fraction that is greater than 1 whole fish.
- The simplest way to write the answer.

<u>Differentiation</u>

- Support – If students are struggling with comparing, have them build the two fractions then compare the models. If they are struggling with ordering, have them build all fractions then order them from smallest to largest.
- Enrichment – Create a larger fraction fish using no more than ten total pieces with at least one yellow hexagon, one red trapezoid, one blue rhombus, and one green triangle. List the fractional value for each piece. Write and solve at least five new comparison statements.



Formative Assessment

- <u>Check Me Cards (During Investigative Task)</u>

  Check Me Cards are used by students to self-assess their work and obtain feedback during a task. Students place a clothespin next to the statement that best describes the status of their work. This allows the teacher to focus on the students in most need of help during the task. The teacher may also utilize the cards to provide feedback if a student(s) is not on task.

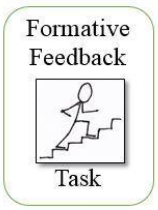
Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time their strategies for solving Fraction Fish #1 will be discussed in math congress, and they will continue comparing fractions greater than one whole.

Unit 5 – Fractions

DAY 29

Content Standard(s)
Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.



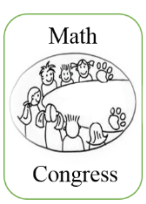
Formative Feedback Task (15 min)
This task should be used solely for the purpose of gathering individual student data, providing feedback, and ultimately advancing each student's learning. It should never be graded. This task should be completed with no assistance from the teacher or peers. The teacher will provide the student oral and written feedback through assessing and advancing questions. The ultimate goal of this feedback is to advance the individual learning of each student.

Focus: Adding and Subtracting Fractions
(Adapted from Howard County Schools, https://jsangiovanni.wikispaces.hcpss.org/)

Name ____ Date ________________ #________

1. George's mother gave him six glasses of ice tea each filled 4/5 of the way. George wants as many glasses as possible to be completely filled. He re-pours some iced tea to make as many full glasses as possible. How many glasses would George be able to completely fill? Use a model to represent the full glasses.

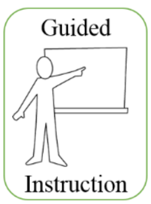
2. George is hosting a family dinner and has 10 people that are coming. How much more ice tea does he need to make to completely fill up each glass.



Math Congress – Fraction Fish #1 (40 min)
Pre-select strategies for discussion based on teacher notes during Investigative Task. Scaffold these ideas through the organization and structure of math congress.

Example of Math Congress structure and sequence based on anticipated student strategies:
1. Replacing blocks with common or like pieces (common denominator)
2. Use of models to compare fractions
3. Expressing the fraction that is greater than 1 whole fish.
4. The simplest way to write the answer.

The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:
- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.

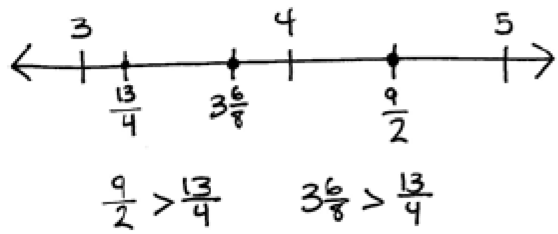


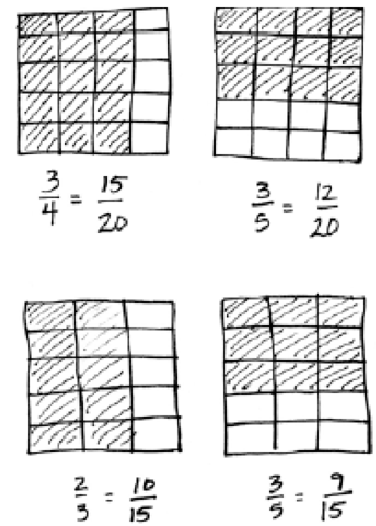
Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lesson 26 & 27 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Compare fractions greater than 1 by reasoning using benchmark fractions.

<u>Concept Development (20 min)</u>
Materials: (S) Personal white board

Problem 1: Compare mixed numbers and fractions on a number line using benchmark fractions.
T: Barbara needed 13/4 cups of flour, her friend Jeanette needed 9/2 cups, and her friend Robert needed 3 6/8 cups. Let's compare the amounts using a number line.
T: Draw a number line with the endpoints of 3 and 5. In the Application Problem, we found that 13/4 equals 3 1/4. Find 3 on the number line. Imagine the fourths. Mark 1/4 past 3. That shows where 3 + 1/4 is located. Label 13/4.
T: Plot 9/2 on the number line. Work with a partner. How many ones are in 9/2? How many remaining halves?
S: There are four groups of 2 halves in 9. →There are 4 ones and 1/2 more. →We can find 4 on the number line and then mark 1/2 past the 4.
T: Label 9/2. Is 9 halves greater than or less than 13 fourths?
S: Greater than, of course. There are 4 ones in 9/2. There are only 3 ones in 13/4.
T: Plot and label 3 6/8. Explain to a partner how this is done.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:
Some learners may benefit from scaffolded questioning to convert $\frac{9}{2}$ to a mixed number. Ask, "How many halves make 1?" Then, say, "Count by 2 halves. 2 halves, 4 halves, 6 halves, 8 halves. Stop. We only have $\frac{9}{2}$. Decompose $\frac{9}{2}$ using a bond with $\frac{8}{2}$ and the remaining fraction."

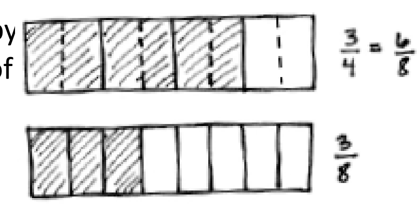




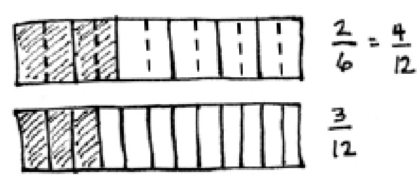
S: We can find the ones, 3, and then picture in our minds where 6/8 more would be. 6/8 is 2/8 greater than 1/2 since 4/8 = 1/2. →3 6/8 is between 3 1/2 and 4. →6/8 = 3/4. →3 6/8= 3 3/4.
T: Compare 3 6/8 and 13/4.
S: 3 6/8 is greater than 3 1/2. 13/4 is less than 3 1/2. →3 6/8 is greater than 13/4.

Repeat with 58/8, 7 5/8, and 30/4.

Problem 2: Model, using a tape diagram, the comparison of two mixed numbers having related denominators.
T: (Display 3 3/8 and 3 3/4.) Look at the mixed numbers from the Application Problem. You compared fractions by thinking about the size of units. Can you remember another way to compare fractions?
S: We can use common denominators.
T: Yes! Four is a factor of 8. We can convert fourths to eighths by make eighths. Draw a tape diagram to model the comparison of



S: (Draw as shown to the right.) $\frac{3}{4}=\frac{6}{8}$. $\frac{6}{8}>\frac{3}{8}$. So, $3\frac{3}{8}<3\frac{3}{4}$.
T: With your partner, draw a tape diagram to compare 2 2/6 and 2 3/12.
S: 2/6 = 4/12. So, 2 2/6 > 2 3/12.

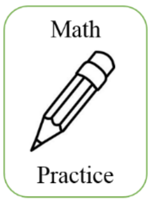


Repeat, using a number line to make like units, to compare 4 1/3 and 4 2/9 and then 5 1/4 and 5 3/8.

Problem 2: Compare two fractions with unrelated denominators.
T: Discuss a strategy to use to compare 4 3/4 and 23/5 .
S: We need to convert 23/5 to a mixed number. 23/5 = 4 3/5 . Both mixed numbers have the same number of ones, so I can use the denominators to compare because the numerators are the same. Fourths are bigger than fifths, so 3 fourths would be greater than 3 fifths. 4 3/4 > 23/5.
T: Yes. That is the same strategy we used in the Application Problem. This time, use the area model to show 3/4 is greater than 3/5. Draw two same-sized rectangles representing 1 one.

Partition one area into fourths using vertical lines. Partition one area into fifths using horizontal lines. Make like denominators.
S: I'll draw fifths horizontally on the fourths, and fourths vertically on the fifths. We made twentieths!
T: Compare the twentieths to prove 4 3/4 > 4 3/5.
S: $\frac{3}{4}=\frac{15}{20}$ and $\frac{3}{5}=\frac{12}{20}$. $\frac{15}{20}>\frac{12}{20}$. So, $4\frac{3}{4}>4\frac{3}{5}$.

Repeat, using the area model to make like units, to compare 2 2/3 and 2 3/5.



Problem Set (10 min)
Eureka Math – Lesson 27 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Compare fractions greater than 1 by creating common numerators or denominators.

You may choose to use any combination of the questions below to lead the discussion.
- How did the tape diagram help to solve Problem 1 (a) and (b)? Why is it important to make sure the whole for both tape diagrams is the same size?
- Who converted to a mixed number or a fraction greater than 1 before finding like units for Problem 3(c)? Was it easier to compare mixed numbers or fractions greater than 1 for this particular problem? (Note: Finding mixed numbers first, one could use a benchmark fraction of 1 half to compare 6/10 to 2/5 without needing to find like units.) Is it more efficient to compare fractions greater than 1 or mixed numbers?
- In Problem 3(e), the added complexity was that the denominators were not related, as in the previous. What strategy did you use to solve? Did you solve by finding like numerators or by drawing an area model to find like denominators?
- Were there any problems in Problem 3 that you could compare without renaming or without drawing a model? How were you able to mentally compare them?
- How did having to compare a mixed number to a fraction add to the complexity of Problem 2(a)?

- How did the Application Problem connect to today's lesson?

Formative Assessment (10 min)

- Eureka Math – Lesson 26 & 27 Exit Ticket (Module 5)

Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time they will extend their knowledge of line plots by participating in an investigation called Kitten Weights.

Homework

Eureka Math – Lesson 26 & 27 (Module 5)

Unit 5 – Fractions

DAY 30

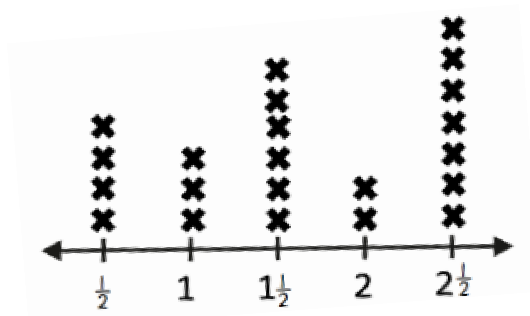
Content Standard(s)

Represent and interpret data.

4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.

Focus: Line Plots, Adding Fractions, Subtracting Fractions (15 min)

Stickers Length (in inches)



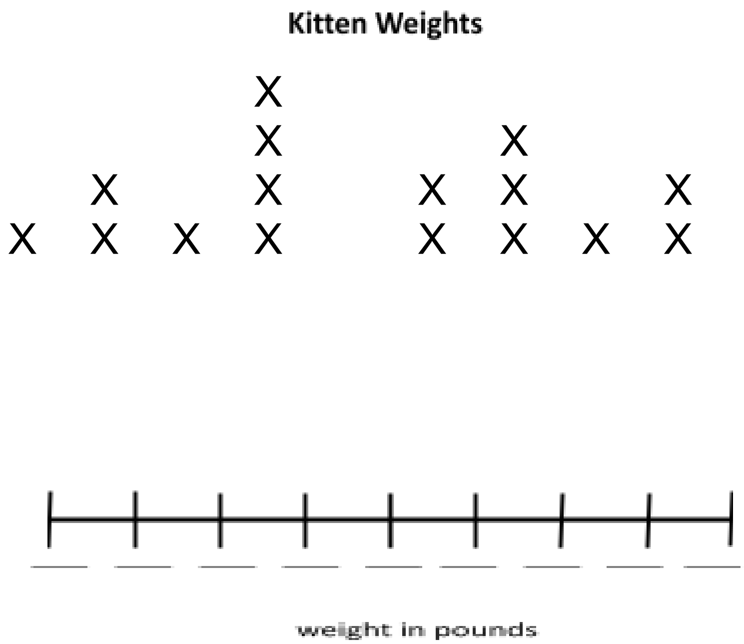
Questions	Answer Key
Emily took the stickers she has the most of and combined their measurement. What is the combined measurement?	17 ½ inches
How much longer are the combined measurement of the longest stickers than the combined measurement of the shortest stickers?	15 ½ inches
What is the combined length of the longest and shortest stickers?	19 ½ inches

Learning Targets

- I can create a line plot to display a data set of measurements in fractions of a unit.
- I can add and subtract fractions presented in line plots.

Kitten Weights (35 min)
(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)
The outcome of this task is to extend knowledge of creating line plots to display a data set of measurements in fractions of a unit.

Jasmine volunteers at an animal shelter. One of her jobs is weighing the kittens and keeping track of their weights. When Jasmine visited the shelter today, she weighed the kittens and made the line plot below to show the weights of the sixteen kittens.



Part 1
When Jasmine got home, she realized she forgot to write the numbers on the scale below the line plot. She remembered, though, that the smallest kitten weighed three pounds and that four of the kittens weighed 3 ¾ pounds. Fill in the blanks to complete the scale of the line plot. Explain how you figured out what numbers to place on the scale of the line plot.

Part 2
Two of the kittens that Jasmine weighed, Butterscotch and Cocoa, are brothers, but they don't weigh the same amount. Together, the two kittens weigh 9 pounds. What could the kittens' weights be?

Part 3
Milkshake and Oreo are the heaviest kittens that Jasmine weighed. Last week, Milkshake weighed 1 ¼ pounds less than he did today. What was Milkshake's weight last week?

<u>Answer Key</u>
Part 1: The scale would include the numbers 3, 3 ¼, 3 ½, 3 ¾, 4, 4 ¼, 4 ½, 4 ¾, and 5 from left to right.

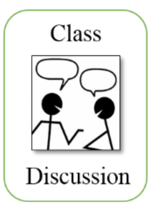
Part 2: The kittens could weigh 4 ¼ pounds and 4 ¾ pounds.
Part 3: Last week, Milkshake weighed 3 ¾ pounds.

<u>Supporting the Investigative Task</u>
As students work, the teacher will confer with them making notes about generalizations students are making.

- The denominator represents the size and/or the total number of pieces/sections of the line plot
- Decomposing a whole into fractional parts
- Strategies for subtracting fractions

<u>Differentiation</u>

- Support – If students are having difficulty, have them construct a number line that begins with three pounds and is divided into the same number of sections as the line plot. Next, have students label 3 ¾ on the number line and connect this model to the line plot.
- Enrichment – The shelter ordered a new scale to weigh all of the kittens at once. Jasmine noticed the scale has a weight limit of eighty-pounds. Will all of the kittens fit on the scale at one time without going over the weight limit? How close are the kittens' weight to the weight limit?

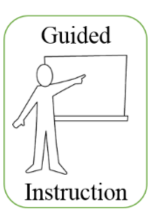


Class Discussion – Kitten Weights (20 min)
Guide students in a reflection of the lesson and discussion of discoveries, ideas, strategies, and connections during the investigative task.

Students will respond to the same questions included in the investigative task. Ask students to provide evidence of their responses.

The following instructional practices should be utilized to reinforce big ideas and strategies communicated by students:

- Have students use hand signals to represent agreement/disagreement.
- Ask students to turn and talk to cement understanding.
- Ask students to restate an idea communicated by another student. This strategy is helpful when students need to hear an idea again or in a different way to gain understanding.



Guided Instruction/Mini-Lesson (20 min)
Eureka Math – Lesson 28 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Solve word problems with line plots.

Concept Development (25 min)
Materials: (S) Personal white board, Problem Set

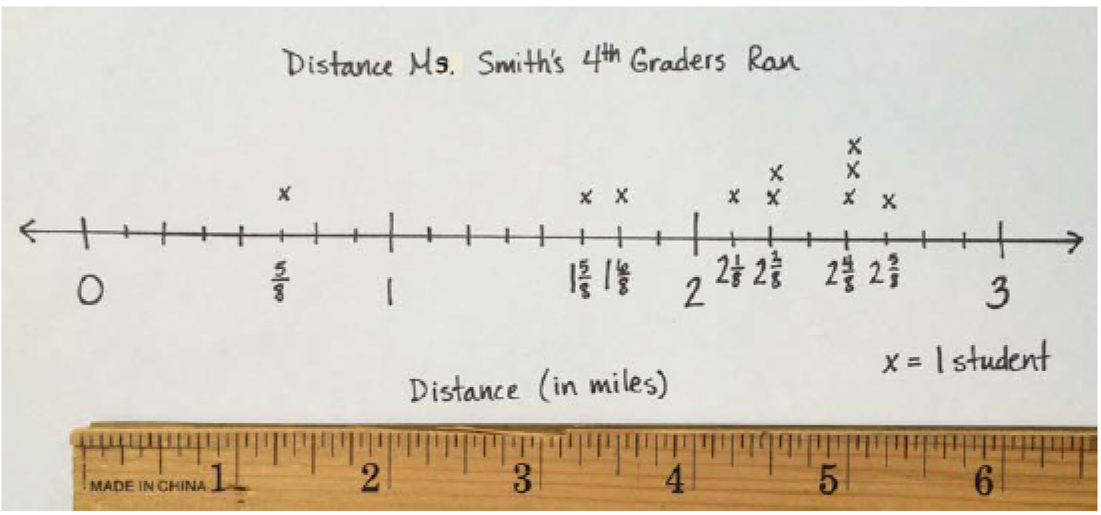
Note: Today's Problem Set is used throughout the Concept Development. The teacher guides the construction and interpretation of a line plot. As students complete each problem, the teacher might debrief with students about their solutions. Students have had prior exposure to creating and interpreting line plots in Grades 2 and 3.

Problem 1

Student	Distance (in miles)
Joe	$2\frac{1}{2}$
Arianna	$1\frac{3}{4}$
Bobbi	$2\frac{1}{8}$
Morgan	$1\frac{5}{8}$
Jack	$2\frac{5}{8}$
Saisha	$2\frac{1}{4}$
Tyler	$2\frac{2}{4}$
Jenny	$\frac{5}{8}$
Anson	$2\frac{2}{8}$
Chandra	$2\frac{4}{8}$

Display the table from the Problem Set.
T: This table shows the distance that Ms. Smith's fourth graders were able to run before stopping for a rest. Tell your partner what you notice about the data.
S: It has the names of students and the distances they ran as a mixed number. →Some of the fractions have different denominators.
→I can see fractions that are equivalent. →The distance is measured in miles.
T: Let's create a **line plot** to show the information. Discuss with your partner what you might remember about line plots from Grade 3: How does a line plot represent data?
S: It's like a number line. →We don't put points on the line, but we make marks above the line. →Yeah. The X's go above the line because sometimes there are a lot of X's at one number.
→It's like a bar graph because the tallest column shows the most.
T: Discuss with your partner what the endpoints will be for the number line.
S: The largest fraction is 2 5/8, and the smallest is 5/8, so we could use 0 and 3.
T: To create a number line using a ruler, we need to decide what measurement on the ruler we can use to mark off the distances students ran. What is the smallest unit of measurement in the chart?
S: 1 eighth mile.
T: Let's see. If I mark off eighth miles from 0 to 3 using an eighth of an inch on a ruler, the increments are very small! Discuss with your partner another length unit that we could use to mark the eighth miles.
S: Let's use inches. Those are nice and big! →There are 24 eighths between 0 and 3. Our paper isn't 24 inches wide. →What if we double the eighth inch to fourth inch marks?

T: Draw a line, and make hash marks at every 1/4 inch to represent each eighth mile. Then, label the whole numbers. (Allow students time to work.)
T: Below the line, write "Distance (in miles)" to tell what unit our line plot shows. (Allow students time to work.)
T: The line plot needs a title to tell what it shows. Tell your neighbor a title for this line plot and record it above the line, leaving some space for the data.
S: We can title it "Distance Ms. Smith's Fourth Graders Ran". (Record the title.)
T: Data on a line plot is marked with an X. We need to tell what each X will represent by providing a key. Below the line plot, record "X = 1 student". (Allow students time to work.)
T: Now, mark each student's distance using an X above a point on the number line that shows the distance they ran in miles. Label that point on the number line with the unit eighths. Tell your partner what you notice.
S: One student ran almost 3 miles! →Some students ran the same distance. →Some distances were measured using different fractional units. I converted fourths and halves to eighths. →Most students ran between 2 and 3 miles.



Problem 2

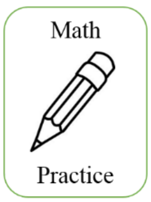
Circulate as students work. When the class is ready, stop students and debrief Problem 2. If preferred, ask questions such as the following:
T: For Problems 2(a) and 2(b), did you refer to the table or the line plot?
T: For Problem 2(b), make a comparison statement for the distance Jack ran compared to Jenny.
T: What strategy did you use for Problem 2(c)? Did you count on the number line or use renaming a fraction to solve?

T: What previous knowledge about subtracting fractions or subtracting mixed units helped you to solve Problem 2(d)?
T: The line plot works just like a number line. I can tell that Arianna ran farther than Morgan. For Problem 2(e), how can you confirm that?
T: For Problem 2(g), comparing eighths and tenths requires a large denominator, like fortieths or eightieths. Using what you know about equivalent fractions to eighths, how could renaming Ms. Smith's distance to fourths make the comparison to Mr. Reynolds's distance simpler?

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Scaffold the word problems on the Problem Set for students working below grade level with questioning. For example, for Problem 2(d) ask, "What was the longest distance run? The shortest? What is the difference, in miles, between the longest and shortest distance run?"

Additionally, students may benefit from organizing data in a table before solving, for example, Problem 2(b).



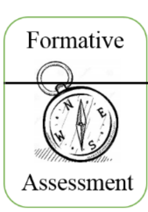
Problem Set (10 min)
Eureka Math – Lesson 28 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Solve word problems with line plots.

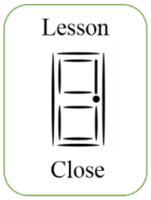
You may choose to use any combination of the questions below to lead the discussion.

- For Problem 2(g), which strategy did you use to compare the two distances? Would you be able to determine the correct answer if you answered Problem 2(f) incorrectly? Why or why not?
- Let's share some of the questions that you wrote for Problem 3. Were there similarities in the questions that you and your partner wrote? Were there differences? Explain.
- How is a line plot useful in showing data? By simply looking at the line plot, what can you tell about the distances that students ran?
- What might be some reasons to use a **line plot** to display data rather than using a chart or table?



Formative Assessment (10 min)

- Eureka Math – Lesson 28 Exit Ticket (Module 5)



Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time they will continue their work on fractions by participating in a math workshop (where they will rotate to five different centers).

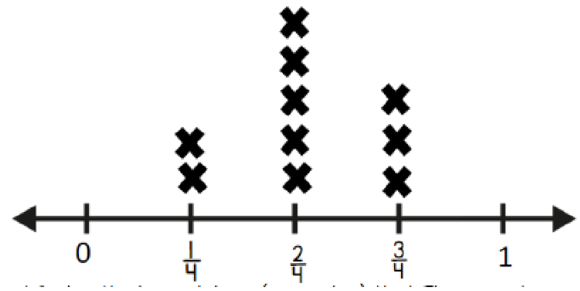
DAY 31

Content Standard(s)

Solve problems involving measurement and conversion of measurements from a
Extend understanding of fraction equivalence and ordering.
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
Represent and interpret data.
4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Focus: Line Plots, Adding Fractions, Subtracting Fractions (15 min)

Homework Time (in hours)



Elena kept record of the amount of time it took her to complete her homework for the last 10 days. The line plot below shows the data.

Questions	Answer Key
What is the longest time (in minutes) that Elena spent on homework?	45 minutes
What is the shortest time (in minutes) that Elena spent on homework?	15 minutes
How much time (in hours and minutes) did Elena spend on homework in the last 10 days?	5 hours 15 minutes

Learning Targets

- I can add fractions.
- I can compare two or more fractions.
- I can convert fractions greater than one whole to mixed numbers.
- I can create a line plot to display a data set of measurements in fractions of a unit.
- I can add and subtract fractions presented in line plots.

Math Workshop – Centers (75 min)
Students will rotate through five different centers and complete designated tasks. Students will work independently, even though they will rotate as a group. Each rotation will last approximately 10-15 minutes. If a student finishes before the rotation time ends, he will complete any teacher specified unfinished problem set practice. The teacher will conduct an invitational group as one of the centers. This instruction should be differentiated based on the group needs. The teacher may choose to use debrief questions from Eureka lessons already completed and/or tasks related to learning targets. Differentiated instruction may include, but not be limited to debrief questions, tasks, problem set review, homework review, and cumulative review.

Center 1: Sprint (Adding Fractions, Decomposing Fractions)
(Eureka Math, Lesson 22, Module 5)

A STORY OF UNITS — Lesson 22 Sprint 4•5

A

Add Fractions

Number Correct: ______

1.	$1 + 1 =$		23.	$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} =$	$1\frac{}{5}$
2.	$\frac{1}{5} + \frac{1}{5} =$		24.	$3 + 3 + 3 =$	
3.	$2 + 1 =$		25.	$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} =$	eighths
4.	$\frac{2}{5} + \frac{1}{5} =$		26.	$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} =$	$1\frac{}{8}$
5.	$2 + 2 =$		27.	$\frac{5}{8} + \frac{5}{8} + \frac{5}{8} =$	$1\frac{}{8}$
6.	$\frac{2}{5} + \frac{2}{5} =$		28.	$1 + 1 + 1 =$	
7.	$3 + 2 =$		29.	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$	halves
8.	$\frac{3}{5} + \frac{2}{5} =$	fifths	30.	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$	$1\frac{}{2}$
9.	$\frac{5}{5} =$		31.	$4 + 4 + 4 =$	
10.	$\frac{3}{5} + \frac{2}{5} =$		32.	$\frac{4}{10} + \frac{4}{10} + \frac{4}{10} =$	tenths
11.	$3 + 2 =$		33.	$\frac{4}{10} + \frac{4}{10} + \frac{4}{10} =$	$1\frac{}{10}$
12.	$\frac{3}{8} + \frac{2}{8} =$		34.	$\frac{6}{10} + \frac{6}{10} + \frac{6}{10} =$	$1\frac{}{10}$
13.	$3 + 2 + 2 =$		35.	$2 + 2 + 2 =$	
14.	$\frac{3}{8} + \frac{2}{8} + \frac{2}{8} =$		36.	$\frac{2}{6} + \frac{2}{6} + \frac{2}{6} =$	sixths
15.	$\frac{3}{8} + \frac{3}{8} + \frac{2}{8} =$	eighths	37.	$\frac{2}{6} + \frac{2}{6} + \frac{2}{6} =$	
16.	$\frac{8}{8} =$		38.	$\frac{3}{6} + \frac{3}{6} + \frac{3}{6} =$	$1\frac{}{6}$
17.	$\frac{3}{8} + \frac{3}{8} + \frac{2}{8} =$		39.	$\frac{5}{12} + \frac{2}{12} + \frac{4}{12} =$	
18.	$2 + 1 + 1 =$		40.	$\frac{4}{12} + \frac{4}{12} + \frac{4}{12} =$	
19.	$\frac{2}{3} + \frac{1}{3} + \frac{1}{3} =$	thirds	41.	$\frac{5}{12} + \frac{5}{12} + \frac{7}{12} =$	$1\frac{}{12}$
20.	$\frac{2}{3} + \frac{1}{3} + \frac{1}{3} =$	$1\frac{}{3}$	42.	$\frac{7}{12} + \frac{9}{12} + \frac{7}{12} =$	$1\frac{}{12}$
21.	$2 + 2 + 2 =$		43.	$\frac{7}{15} + \frac{8}{15} + \frac{7}{15} =$	$1\frac{}{15}$
22.	$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} =$	fifths	44.	$\frac{12}{15} + \frac{8}{15} + \frac{9}{15} =$	$1\frac{}{15}$

A STORY OF UNITS — Lesson 22 Sprint 4•5

B

Add Fractions

Number Correct: ______

Improvement: ______

1.	$1 + 1 =$		23.	$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} =$	$1\frac{}{8}$
2.	$\frac{1}{6} + \frac{1}{6} =$		24.	$1 + 1 + 1 =$	
3.	$3 + 1 =$		25.	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$	halves
4.	$\frac{3}{6} + \frac{1}{6} =$		26.	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$	$1\frac{}{2}$
5.	$3 + 2 =$		27.	$2 + 2 + 2 =$	
6.	$\frac{3}{6} + \frac{2}{6} =$		28.	$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} =$	fifths
7.	$4 + 2 =$		29.	$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} =$	$1\frac{}{5}$
8.	$\frac{4}{6} + \frac{2}{6} =$	sixths	30.	$\frac{3}{5} + \frac{3}{5} + \frac{3}{5} =$	$1\frac{}{5}$
9.	$\frac{6}{6} =$		31.	$6 + 6 + 6 =$	
10.	$\frac{4}{6} + \frac{2}{6} =$		32.	$\frac{6}{10} + \frac{6}{10} + \frac{6}{10} =$	tenths
11.	$5 + 2 =$		33.	$\frac{6}{10} + \frac{6}{10} + \frac{6}{10} =$	$1\frac{}{10}$
12.	$\frac{5}{8} + \frac{2}{8} =$		34.	$\frac{5}{10} + \frac{5}{10} + \frac{5}{10} =$	$1\frac{}{10}$
13.	$5 + 1 + 1 =$		35.	$2 + 2 + 2 =$	
14.	$\frac{5}{8} + \frac{1}{8} + \frac{1}{8} =$		36.	$\frac{2}{6} + \frac{2}{6} + \frac{2}{6} =$	sixths
15.	$\frac{5}{8} + \frac{2}{8} + \frac{1}{8} =$	eighths	37.	$\frac{2}{6} + \frac{2}{6} + \frac{2}{6} =$	
16.	$\frac{8}{8} =$		38.	$\frac{3}{6} + \frac{3}{6} + \frac{3}{6} =$	$1\frac{}{6}$
17.	$\frac{3}{8} + \frac{3}{8} + \frac{2}{8} =$		39.	$\frac{5}{12} + \frac{3}{12} + \frac{3}{12} =$	
18.	$1 + 1 + 2 =$		40.	$\frac{5}{12} + \frac{5}{12} + \frac{2}{12} =$	
19.	$\frac{1}{3} + \frac{1}{3} + \frac{2}{3} =$	thirds	41.	$\frac{6}{12} + \frac{5}{12} + \frac{6}{12} =$	$1\frac{}{12}$
20.	$\frac{1}{3} + \frac{1}{3} + \frac{2}{3} =$	$1\frac{}{3}$	42.	$\frac{8}{12} + \frac{10}{12} + \frac{5}{12} =$	$1\frac{}{12}$
21.	$3 + 3 + 3 =$		43.	$\frac{7}{15} + \frac{7}{15} + \frac{8}{15} =$	$1\frac{}{15}$
22.	$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} =$	eighths	44.	$\frac{13}{15} + \frac{9}{15} + \frac{7}{15} =$	$1\frac{}{15}$

Center 2: 4.NF.2 Task(s)
(Howard County Schools, https://jsangiovanni.wikispaces.hcpss.org/)

Name: ______________________________ Date: ___________ #:_____

Center 2

1. The chart shows the amount of sugar used to make one quart of different flavored ice cream.

   Which list shows the amount of sugar in each ice cream in order from least to greatest?

   A Chocolate, Vanilla, Cookie Dough, Mint

   B Chocolate, Cookie Dough, Mint, Vanilla

   C Mint, Vanilla, Chocolate, Cookie Dough

   D Mint, Vanilla, Cookie Dough, Chocolate

Flavor	Amount of Sugar in Cups
Chocolate	$3\frac{1}{2}$ cups
Vanilla	$\frac{4}{5}$ cups
Cookie Dough	$2\frac{1}{4}$ cups
Mint	$\frac{1}{8}$ cups

2. Strawberry ice cream has more sugar than the other ice cream flavors. How much sugar could be in strawberry ice cream?

   Strawberry ice cream could have __________ cups of sugar

3. Rocky Road ice cream has less sugar than cookie dough ice cream but more sugar than vanilla ice cream. How much sugar could be in Rocky Road ice cream?

   Rocky Road ice cream could have __________ cups of sugar.

Center 3: Lesson 24 & 25 Problem Set (4.NF.3)

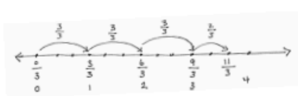
(Eureka Math, Lesson 24 & 25, Module 5)

A STORY OF UNITS — Lesson 24 Problem Set 4•5

2. Convert each fraction to a mixed number. Show your work as in the example. Model with a number line.

   a. $\frac{11}{3}$

   $\frac{11}{3} = \frac{3 \times 3}{3} + \frac{2}{3} = 3 + \frac{2}{3} = 3\frac{2}{3}$



   b. $\frac{9}{2}$

   c. $\frac{17}{4}$

3. Convert each fraction to a mixed number.

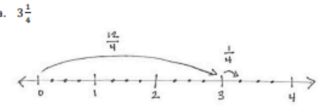
a. $\frac{9}{4}$ =	b. $\frac{17}{5}$ =	c. $\frac{25}{6}$ =
d. $\frac{30}{7}$ =	e. $\frac{38}{8}$ =	f. $\frac{48}{9}$ =
g. $\frac{63}{10}$ =	h. $\frac{84}{10}$ =	i. $\frac{37}{12}$ =

A STORY OF UNITS — Lesson 25 Problem Set 4•5

Name ______________________________ Date ______________

1. Convert each mixed number to a fraction greater than 1. Draw a number line to model your work.

   a. $3\frac{1}{4}$



   $3\frac{1}{4} = 3 + \frac{1}{4} = \frac{12}{4} + \frac{1}{4} = \frac{13}{4}$

   b. $2\frac{4}{5}$

   c. $3\frac{5}{8}$

   d. $4\frac{4}{10}$

   e. $4\frac{7}{9}$

Center 4: 4.MD.4 Task(s)

(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)

Name: ______________________________ Date: ____________ #: ____

How High Did it Bounce?

A class measures how high a bouncy ball will bounce compared to the height of the wall. Based on the data, make a line plot to display the data.

3/8	5/8	7/8	6/8	5/8	6/8
6/8	5/8	4/8	4/8	2/8	6/8
5/8	7/8	5/8	6/8	4/8	5/8

0 1/8 2/8 3/8 4/8 5/8 6/8 7/8 1

Part 1: How many bouncy balls went halfway up the wall or higher?

Part 2: How may bouncy balls went $^3/_4$ of the wall or higher?

Part 3: If the wall is 8 feet high, what is the combined height of all of the heights of the bouncy balls?

Center 5: Invitational Group
The teacher will meet with each group during their Center 5 Rotation, where differentiated instruction should occur (providing each group with an experience based on need). The teacher may choose to reteach a concept from a formative feedback task or formative assessment, expand on a guided instruction/mini-lesson previously taught, expand on a problem set using debrief questions, review homework, reteach a concept, conduct a cumulative review, or provide other tasks specific to student needs.

Lesson Close

Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time they will continue investigating fractions and begin working on a math menu.

Unit 5 – Fractions

DAY 32

Content Standard(s)
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

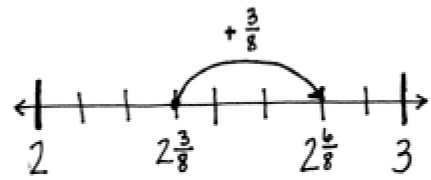
Focus: Adding Mixed Numbers (15 min)

	Answer Key	*Strategies that may be used*
$^{3}/_{5} + ^{4}/_{5} = n$	$n = ^{7}/_{5}$ or $1\ ^{2}/_{5}$	Helper Problem
$1\ ^{3}/_{5} + ^{4}/_{5} = n$	$n = 1\ ^{7}/_{5}$ or $2\ ^{2}/_{5}$	$(^{3}/_{5} + ^{4}/_{5}) + 1$
$2\ ^{4}/_{5} + 1\ ^{3}/_{5} = n$	$n = 3\ ^{7}/_{5}$ or $4\ ^{2}/_{5}$	$(1\ ^{3}/_{5} + ^{4}/_{5}) + 2$
$^{3}/_{4} + n = 1\ ^{3}/_{4}$	n = 1	Helper Problem
$^{3}/_{4} + n = 2$	n = 1 ¼ or $^{5}/_{4}$	1 + ¼
$n + ^{3}/_{4} = 2$ ½	n = 1 ¾	1 ¼ + $^{2}/_{4}$

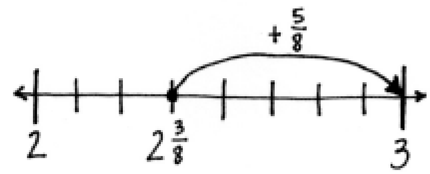
Learning Targets
- I can make sense of problems and persevere in solving them.
- I can add a fraction and a mixed number.
- I can add mixed numbers.
- I can decompose fractions greater than one whole into whole numbers or mixed numbers.

Guided Instruction/Mini-Lesson (20 min)
Eureka Math – Lesson 30 & 31 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Add a mixed number and a fraction.

<u>Concept Development (20 min)</u>
Materials: (S) Personal white board



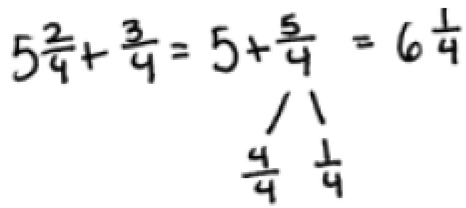
Problem 1: Use unit form and the number line to add a mixed number and a fraction having sums of fractional units less than or equal to 1.
T: Write 2 3/8 + 3/8.
T: Say the expression using unit form.
S: 2 ones 3 eighths + 3 eighths.
T: What are the units involved in this problem?
S: Ones and eighths.
T: When we add numbers, we add like units. (Point to the mixed numbers and demonstrate.) How many ones are there in all?



2 ones 3 eighths + 3 eighths = 2 ones 6 eighths

S: 2 ones.
T: How many eighths are there in all?
S: 6 eighths.
T: 2 ones + 6 eighths is…?
S: 2 6/8.
T: Show the addition using a number line. Start at 2 3/8, and then add 3/8 more. Notice how the ones stay the same and the fractional units are simply added together since their sum is less than 1.
T: Write 2 3/8 + 5/8. Add like units. How many ones? How many eighths?
S: 2 ones and 8 eighths.
T: Show the addition using a number line. Start at 2 3/8. Add 5/8 more.
S: Hey! When I add 5/8 more, it equals 3.
T: The fractional units have a sum of 1. 3/8 + 5/8 = 8/8 = 1.

Problem 2: Decompose a sum of a mixed number and a fraction with sums of fractional units greater than 1.
T: (Write 5 2/4 + 3/4.) Right away, we see that the sum of the fourths is greater than 1.
T: The sum of the ones is…?
S: 5.
T: The sum of the fourths is…?
S: 5 fourths.
T: Decompose 5 fourths to make one. Use a number bond.
S: 5/4 = 4/4 + 1/4.
T: (Write the following.)



$5\frac{2}{4}+\frac{3}{4}=5+\frac{5}{4}=5+\frac{4}{4}+\frac{1}{4}=6\frac{1}{4}.$

T: Explain to your partner how we got a sum of 6 1/4.

S: We added like units. We added ones to ones and fourths to fourths. We changed 5 fourths to make 1 and 1 fourth and added 5 + 1 1/4. The sum is 6 1/4.

Let students practice adding like units to find the sum using the following:

$7\frac{2}{5}+\frac{4}{5}$ and $3\frac{5}{12}+1\frac{11}{12}$.

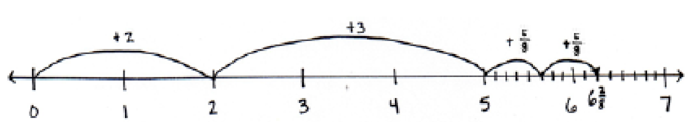
Problem 3: Add mixed numbers when the sum of the fractional units is greater than 1 by combining like units.

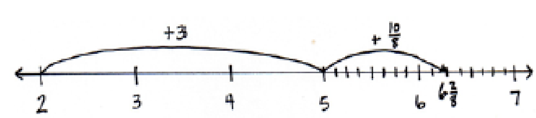
T: (Write 2 5/8 + 3 5/8.) Right away, we see that the sum of the eighths is greater than 1.
T: The sum of the ones is…?
S: 5.
T: The sum of the eighths is…?
S: 10 eighths.
T: Take out 8 eighths to make one.
S: 1 2/8. →8/8 and 2/8. (Record with a number bond.)
T: (Write the following.)

$$\begin{aligned} 2\frac{5}{8}+3\frac{5}{8} &= 5+\frac{10}{8} \\ &= 5+\frac{8}{8}+\frac{2}{8} \\ &= 6\frac{2}{8} \end{aligned}$$

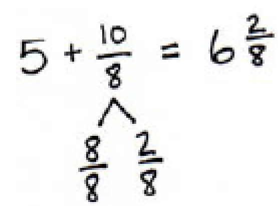
T: Explain to your partner how we got a sum of 6 2/8.
S: We added like units. We added ones to ones and eighths to eighths. Then, we changed 10 eighths to make 1 and 2 eighths and added 5 + 1 2/8 = 6 2/8.
T: Use a number line to model the addition of like units.
Students may show slides on the number line in different ways depending on their fluency with the addition of like units. Accept representations that are logical and follow the path of the number sentence.

Two samples are shown.



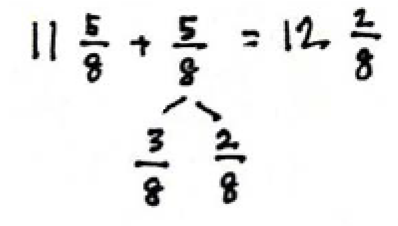


Let students practice with the following: 2 2/5 + 2 4/5 and 3 5/12 + 1 11/12 . Allow students to work mentally to solve, if they can, without recording the breakdown of steps.

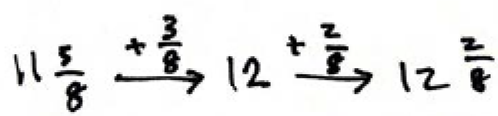


Problem 4: Add mixed numbers when the sum of the fractional units is greater than 1 by making one.

T: (Write 5 5/8 + 6 5/8.) We can also add the ones first and decompose to make one in the same way we learned to make ten in the first and second grades.
T: 5 and 6 is…?
S: 11.
T: (Write 11 5/8 + 5/8.) How much does 5 eighths need to make one?
S: 3 eighths. (Decompose 5/8 as 3/8 and 2/8 as shown to the right.)



T: We can use the arrow way to show this clearly. Instead of drawing a number line, we can draw arrows to show the sum. 11 5/8 + 3/8 is…? (Model the arrow way while speaking.)
S: 12. (Record 12, and draw the next arrow.)
T: 12 + 2/8 is…? (Record as modeled to the right.)
S: 12 2/8.



T: $5\frac{5}{8} + 6\frac{5}{8} = 11\frac{5}{8} + \frac{5}{8}$
$= 11\frac{5}{8} + \frac{3}{8} + \frac{2}{8}$
$= 12\frac{2}{8}$

Let students practice with 3 7/8 + 4 3/8 and 9 11/12 + 10 5/12 . Again, students may want to add more steps in the recording, e.g., 5 5/8 + 6 5/8 = 11 5/8 + 5/8 = 11 8/8 + 2/8 = 12 2/8. Gently encourage them to stop recording the steps they are able to easily complete mentally.
T: (Write 4 2/3 + 3 1/3 + 5 2/3.) The sum of the ones is…?
S: 12.
T: The sum of the thirds is…?
S: 5 thirds.
T: Record your work.
S: $4\frac{2}{3} + 3\frac{1}{3} + 5\frac{2}{3} = 12 + \frac{5}{3}$
$= 13\frac{2}{3}.$

Please note that this is not the only way to record this sum. Students might break the problem down into more or fewer steps, use a number bond, or do mental math.

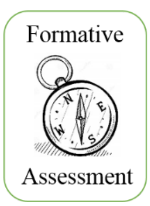
Math Practice

Problem Set (10 min)
Eureka Math – Lesson 31 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

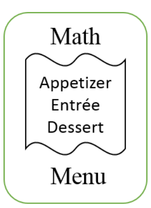
<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Add a mixed number and a fraction.

You may choose to use any combination of the questions below to lead the discussion.
- Explain how decomposing mixed numbers helps you find their sum.
- Explain how you solved Problem 1(c).
- Explain the methods you chose for solving Problems 4(a), 4(b), and 4(c). Did you use the same methods as your partner?
- How is adding 4 tens 7 ones and 6 tens 9 ones like adding 4 ones 7 twelfths and 6 ones 9 twelfths? How is it different?
- If you were unsure of any answer on this Problem Set, what could you do to see if your answer is reasonable? Would drawing a picture or estimating the sum or difference be helpful?
- How did the Application Problem connect to today's lesson?

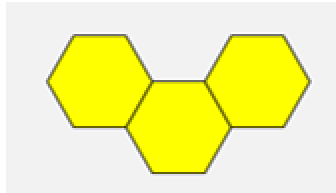


Formative Assessment (10 min)
- Eureka Math – Lessons 30 &31 Exit Ticket (Module 5)



Math Menu (40 min)
A math menu of differentiated tasks provides students with opportunities to apply concepts learned in a variety of contexts. New tasks will be introduced each day. Students are expected to complete a minimum of four total tasks upon conclusion of math menu, one being a required task for all students.

Task 1: Dividing Up the Land (Required)
(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)
There is a plot of land shaped like the figure below. Each hexagon has a value of 1 whole unit. The plot of land, therefore, has a value of 3 whole units.



Determine how to use pattern blocks to divide the shape up into the following ways. For each way, make a picture and write an equation.

Part 1
The land owner will only sell the land in sections that are one-third of a unit. The following people buy land:

Taylor: 2 sections
Bill: 1 section
Nick: 4 sections

Use your pattern blocks to make a picture of how the land was divided up. Is there any land left? If so, how much? Write an equation to show how the land was split up by the land owner. Include any unsold land.

Part 2
The land owner will only sell the land in sections that are one-sixth of a unit. The following people buy land:

Tony: 3 sections
Susan: 2 sections
Bob: 4 sections
Mallory: 1 section
Wes: 6 sections

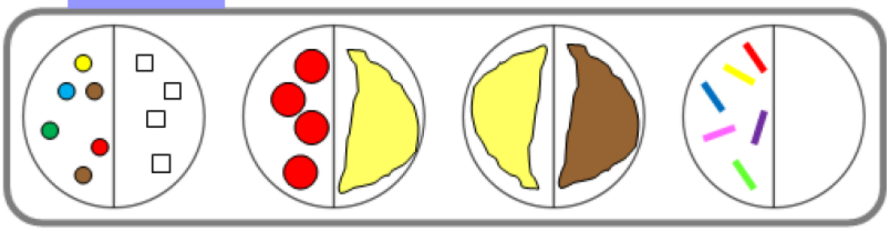
Use your pattern blocks to make a picture of how the land was divided up. Is there any land left? If so, how much? Write an equation to show how the land was split up by the land owner. Include any unsold land.

<u>Answer Key</u>
Part 1: There are 2 sections left or 2/3 + 1/3 + 4/3 + 2/3 = 3.
Part 2: There are 2 sections left or 2/6 of a unit. 3/6 + 2/6 + 4/6 + 1/6 + 6/6 + 2/6

Task 2: Fraction Cookies Bakery
(Adapted from Georgia Department of Education, https://www.georgiastandards.org/Georgia-Standards/Pages/Math-K-5.aspx)
You own a bakery that specializes in fraction cookies. Customers place orders from all over the country for your unique cookies. You recently received the orders shown below. Before making the cookies to fill the order, you need to confirm each order by sending a confirmation notice to each customer. Use the circle template to show how you would create each cookie order with the correct fractional amount of cookies. Customers expect you to use the fewest number of

cookies possible to complete each order. No part of a cookie should be without a topping except for one. You may split a topping between two cookies as shown below (the vanilla icing was shared between two cookies rather than covering both halves of one cookie with vanilla icing.

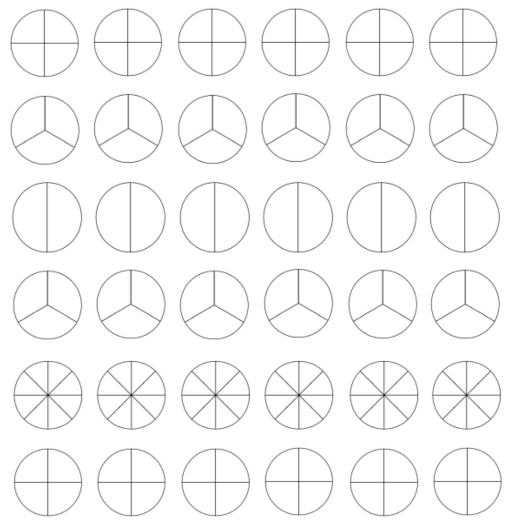
Example:



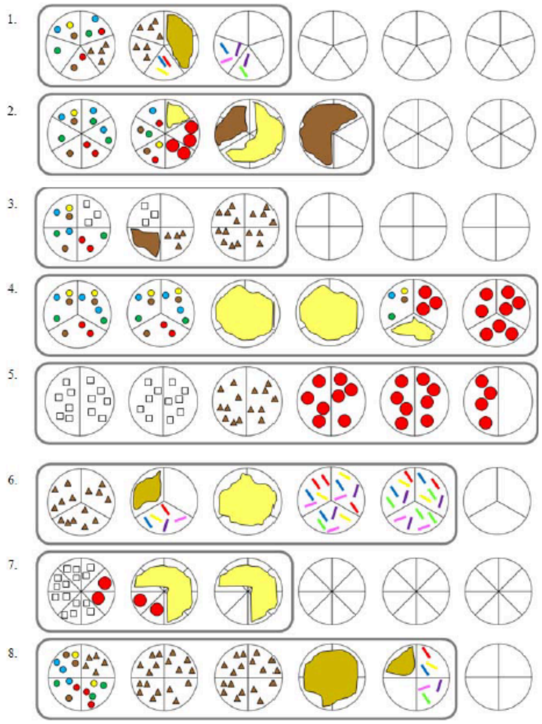
Order Form

Order Number	Toppings								Order Totals		
	M & M's	Walnuts	Chocolate Chips	Raspberries	Peanut Butter	Vanilla Icing	Chocolate Icing	Sprinkles	Improper fraction	Mixed Number	Number of Whole Cookies
Example	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$		$\frac{2}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$3\frac{1}{2}$	4
#1	$\frac{4}{5}$		$\frac{3}{5}$		$\frac{2}{5}$			$\frac{3}{5}$			
#2	$\frac{9}{6}$			$\frac{2}{6}$		$\frac{5}{6}$	$\frac{6}{6}$				
#3	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{4}{4}$				$\frac{1}{4}$				
#4	$\frac{7}{3}$			$\frac{4}{3}$		$\frac{7}{3}$					
#5		$\frac{4}{2}$	$\frac{2}{2}$	$\frac{5}{2}$							
#6			$\frac{4}{8}$		$\frac{1}{8}$	$\frac{3}{8}$		$\frac{7}{8}$			
#7		$\frac{6}{8}$		$\frac{4}{8}$		$\frac{12}{8}$					
#8	$\frac{8}{4}$		$\frac{9}{4}$		$\frac{5}{4}$			$\frac{2}{4}$			

Circle Templates



Answer Key



Order #	Improper Fraction	Mixed Number	# of Whole Cookies
1	$^{12}/_{5}$	$2\ ^{2}/_{5}$	3
2	$^{22}/_{6}$	$3\ ^{4}/_{6}$	4
3	$^{10}/_{4}$	$2\ ^{2}/_{4}$	3
4	$^{18}/_{3}$	6	6
5	$^{11}/_{2}$	5 ½	6
6	$^{15}/_{3}$	5	5
7	$^{22}/_{8}$	$2\ ^{6}/_{8}$	3
8	$^{19}/_{4}$	4 ¾	5

Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time they will continue investigating fractions and be introduced to two additional math menu choices.

Homework

Eureka Math – Lesson 30 & 31 (Module 5)

Unit 5 – Fractions

DAY 33

Content Standard(s)

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Focus: Subtracting Mixed Numbers (15 min)

	Answer Key	*Strategies that may be used*
$2\,{}^{3}/_{5} - {}^{1}/_{5} = n$	$n = 2\,{}^{2}/_{5}$	Helper Problem
$2\,{}^{3}/_{5} - {}^{3}/_{5} = n$	$n = 2$	Helper Problem
$2\,{}^{3}/_{5} - {}^{4}/_{5} = n$	$n = 1\,{}^{4}/_{5}$	$(2\,{}^{3}/_{5} - {}^{3}/_{5}) - {}^{1}/_{5}$
$2\,{}^{3}/_{5} - 1\,{}^{4}/_{5} = n$	$n = {}^{4}/_{5}$	$(2\,{}^{3}/_{5} - 1\,{}^{3}/_{5}) - {}^{1}/_{5}$
$3\,{}^{1}/_{3} - 1\,{}^{2}/_{3}$	$n = 1\,{}^{2}/_{3}$	$(3\,{}^{1}/_{3} - 1\,{}^{1}/_{3}) - {}^{1}/_{3}$

Learning Targets

- I can make sense of problems and persevere in solving them.
- I can subtract mixed numbers.
- I can recognize and generate equivalent fractions.

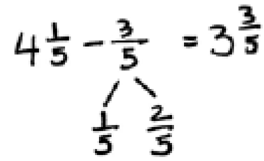
Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lesson 32 & 33 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Subtract a fraction from a mixed number.

<u>Concept Development (20 min)</u>
Materials: (S) Personal white board

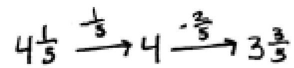
Unit 5 – Fractions

Problem 1: Subtract a fraction less than 1 from a whole number by decomposing the subtrahend.

T: (Write 4 1/5 — 3/5.) Do we have enough fifths to subtract 3 fifths?
S: No!
T: (Show 3/5 decomposed as 1/5 and 2/5 as pictured to the right.)



T: Does 1/5 + 2/5 have the same value as 3/5? (Point to the parts of the bond.)
S: Yes!
T: Now do we have enough fifths?
S: No. It's still 3/5. We can't take that from 1/5.
T: Look at the parts. Let's take away one part at a time. Draw a number line to model the subtraction.
T: Solve 4 1/5 — 1/5. Count back 1 fifth on the number line.



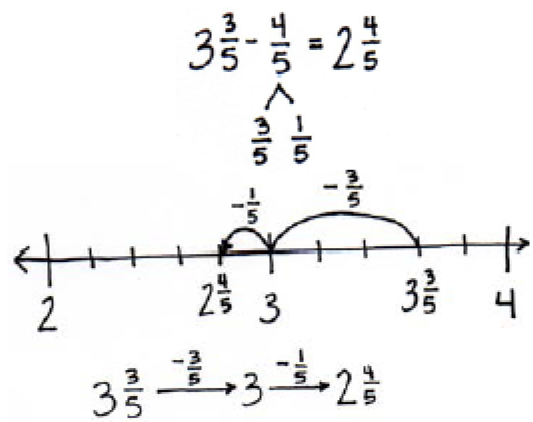
S: That's 4.
T: Now, subtract 2/5 from 4. Talk to your partner.
S: We already know how to do that,

$3\frac{5}{5} - \frac{2}{5} = 3\frac{3}{5}$. → $1 - \frac{2}{5}$ is $\frac{3}{5}$, so $4 - \frac{2}{5}$ is $3\frac{3}{5}$.

T: We can also use the arrow way. Start with 4 1/5, count back 1/5 to get to 4, and then count back 2/5 more to get 3 3/5. (Shown above to the right.)
T: Write 3 3/5 — 4/5. First, decompose 4/5 into two parts, count back to 3, and then subtract the other part.
S: I see. We take away one part of 4/5 at a time. 4/5 = 3/5 + 1/5.

$3\frac{3}{5} - \frac{3}{5} = 3$. $3 - \frac{1}{5} = 2\frac{4}{5}$.

T: Model on a number line, and then model using arrows.

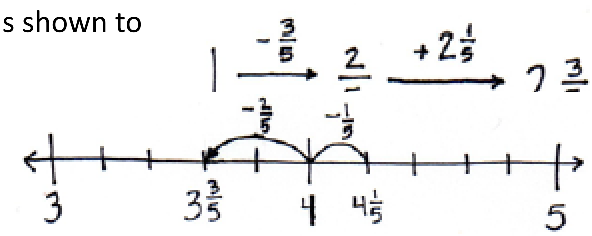


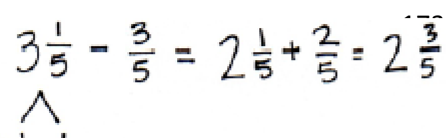
Let students practice with the following:

$4\frac{5}{10} - \frac{7}{10}$, $2\frac{2}{12} - \frac{7}{12}$, $3\frac{7}{10} - \frac{9}{10}$, $2\frac{1}{4} - \frac{3}{4}$.

Problem 2: Decompose the total to take out 1 when subtracting a fraction from a mixed number when there are not enough fractional units.

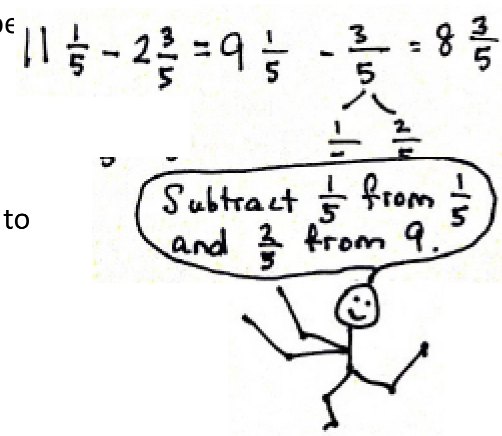
T: (Write 3 1/5 — 3/5, including the number bond as shown to the right.)
T: Do you have enough fifths to subtract 3/5?
S: No. →This is the same problem as before.
T: Let's try a different strategy to solve. Talk to your partner. Where can we get more fifths?



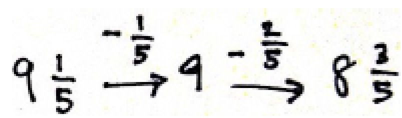


S: From 3 1/5.
T: Decompose 3 1/5 by taking out one. We have 2 1/5 and 1. (Record using a number bond.)
T: Take 3/5 from 1. How many are left?
S: 2/5.
T: We have 2/5 left plus 2 1/5, which is equal to 2 3/5. Let's show this using the arrow way. Let students practice with the following: 12 1/4 ─ 3/4 and 7 3/10 ─ 9/10.

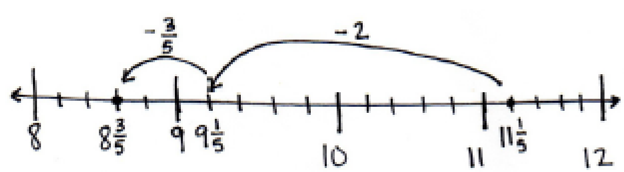
Problem 3: Subtract a mixed number from a mixed number when there are not enough fractional units by first subtracting the whole numbers and then decomposing the subtrahend.
T: (Write 11 1/5 + 2 3/5.) When we add mixed numbers, we add the like units. We could add the ones first and then the fifths.
T: (Write 11 1/5 – 2 3/5.) When we subtract mixed numbe
What subtraction expression remains?



S: 9 1/5 – 3/5.
T: Just like yesterday, decompose 3 fifths as 1/5 and 2/5 (as pictured to the right).
T: 9 1/5 – 1/5 is…? (Record using the arrow way, as seen to the right.)
S: 9.
T: Count back 2/5 from 9. 9 – 2/5 is…? (Record with the second arrow.)
S: 8 3/5.
T: (Write $9\frac{1}{5} - \frac{3}{5} = 9 - \frac{2}{5} = 8\frac{3}{5}$.)

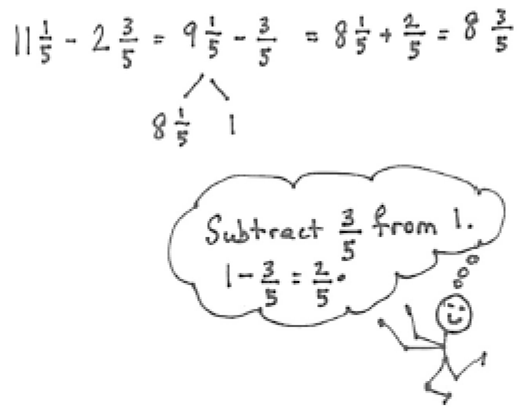


T: Explain to your partner why this is true.
S: It's like counting back! →We subtract a fifth from 9 1/5, and then we subtract 2/5 from 9. →First, we renamed 3/5 as 1/5 and 2/5. Then, we subtracted in two steps. → It looks like we subtracted 1/5 from both numbers and got 9 – 2/5, which is just easier.
T: Use a number line to model the steps of counting backward from 11 1/5 to subtract 2 3/5
S: (Draw as shown to the right, or draw to match the arrow way recording.) Let students practice with the following: 4 1/8 – 1 7/8 and 7 5/12 – 3 9/12. Those who struggle with subtracting from a whole number with automaticity can break apart the whole number using Lesson 32's strategy until gaining mastery, e.g., 4 1/8 – 2 7/8 = 2 1/8 – 7/8 = 1 9/8 – 7/8 = 1 2/8. Have them share their work with a partner, explaining their solution.

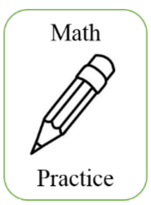


Problem 4: Subtract a mixed number from a mixed number when there are not enough fractional units by decomposing a whole number into fractional parts.

T: (Write 11 1/5 – 2 3/5.) Let's solve using a different strategy.
T: Subtract the whole numbers.
S: 11 1/5 – 2 3/5 = 9 1/5 – 3/5.
T: Decompose 9 1/5 by taking out one.
S: (Draw a number bond to show 8 1/5 and 1.)
T: 1 – 3/5 is…?
S: 2/5.
T: 8 1/5 + 2/5 is…?
S: 8 1/5 + 2/5 = 8 3/5. That's the same answer as before. We just found it in a different way.



Let students practice with the following: 4 1/8 – 1 7/8 and 7 5/12 – 3 9/12. Encourage students to practice this strategy of subtracting from 1, but don't belabor its use with students. Allow them to use any strategy that makes sense to them and enables them to correctly solve the problem, explaining the steps to their partner. Ask those who finish early to solve using an alternative strategy to strengthen their number sense.



Problem Set (10 min)
Eureka Math – Lesson 33 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

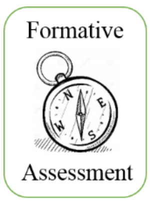
<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Subtract a mixed number from a mixed number

You may choose to use any combination of the questions below to lead the discussion.

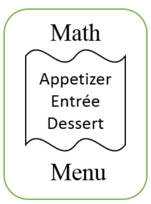
- Can you accurately subtract mixed numbers by subtracting the fraction first, or must you always subtract the whole numbers first? Give an example to explain.
- When subtracting mixed numbers, what is the advantage of subtracting the whole numbers first?

- Which strategy do you prefer to use, decomposing the number we are subtracting as we did in Problem 2 of the Concept Development or taking from 1, as we did in Problem 3? Discuss the advantages of the strategy as you explain your preference.
- Which strategies did you choose to solve Problem 4(a–d) of the Problem Set? Explain how you decided which strategy to use.
- What learning from Lesson 32 was used in this lesson? How can subtracting a mixed number from a mixed number be similar to subtracting a fraction from a mixed number?
- How did our Application Problem relate to today's lesson?



` **Formative Assessment (10 min)**

- Eureka Math – Lessons 32 & 33 Exit Ticket (Module 5)



Math Menu (40 min)
A math menu of differentiated tasks provides students with opportunities to apply concepts learned in a variety of contexts. New tasks will be introduced each day. Students are expected to complete a minimum of four total tasks upon conclusion of math menu, one being a required task for all students.

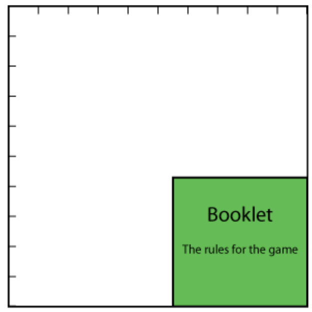
Task 3: Jumping Rope
(Math Forum – NCTM, Problem #2041)

My jumping rope was cut in half
half was thrown away.
The other half was cut again
one third along the way.
The longer part (ten feet long)
is what I use to play.
How long was my jumping rope
when I began today?

<u>Answer Key</u>
The rope was 30 feet when she began.

Task 4: Fractions in a Box

(University of Cambridge, http://nrich.maths.org)
We have a game which has a number of discs in seven different colors. They are kept in a flat square box with a square hole for each disc. There are 10 holes in each row and 10 in each column. So, there would be 100 discs altogether, except that there is a square booklet which is kept in a corner of the box in place of some of the holes.
We haven't drawn a grid to show all the holes because that would give the answer away!



There is a different number of discs of each of the seven colors.

- Half of the discs are red.
- One-fourth are black
- One-twelfth are blue.
- One complete row of ten holes of the box is filled with all the blue and green discs.
- One of the shortened rows (that is where the booklet is) is exactly filled with all the orange discs.
- Two of the shortened row are filled with some ofr the red discs and the rest of the red discs exactly fill a number of complete rows of 10 in the box.
- There is just one white disc and all the rest are yellow.

1. How many discs are there altogether?
2. What fraction of them are orange?
3. What fraction are green? Yellow? White?

<u>Answer Key</u>
1. There are eighty-four discs because the booklet takes up sixteen of the one hundred squares – forty-two discs are red, twenty-one are black, seven are blue, three are green, four are yellow, six are orange, and one is white.
2. One fourteenth of the discs are orange.
3. One twenty-eighth of the discs are green, one twenty-first are yellow, and one eighty-fourth are white.

Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time they will continue investigating fractions and be introduced to two additional math menu choices.

Homework
Eureka Math – Lessons 32 & 33 (Module 5)

Content Standard(s)
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Multiplying a Fraction by a Whole Number

	Answer Key	*Strategies that may be used*
½ x 2 = n	n = 1	½ + ½
2 x ½ = n	n = 1	½ + ½
¼ x 4 = n	n = 1	¼ + ¼ + ¼ + ¼
4 x ¼ = n	n = 1	¼ + ¼ + ¼ + ¼
¾ x 4 = n	n = 3	¾ + ¾ +¾ +¾
4 x ¾ = n	n = 3	¾ + ¾ +¾ +¾
$^2/_3$ x 6 = n	n = 4	$^2/_3 + ^2/_3 + ^2/_3 + ^2/_3 + ^2/_3 + ^2/_3$
6 x $^2/_3$ = n	n = 4	$^2/_3 + ^2/_3 + ^2/_3 + ^2/_3 + ^2/_3 + ^2/_3$

Learning Targets
- I can make sense of problems and persevere in solving them.
- I can multiply fractions by a whole number.
- I can recognize and generate equivalent fractions.
- I can decompose fractions into unit fractions.

Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lessons 35 & 36 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Represent the multiplication of n times a/b as (n × a)/b using the associative property and visual models.

<u>Concept Development (20 min)</u>
Materials: (S) Personal white board

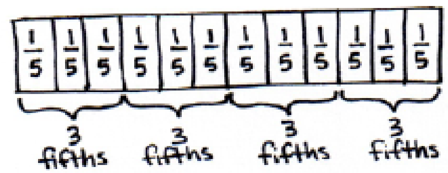
NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Adjust the Application Problem to challenge students working above grade level. For example, ask, "How many total hours and minutes did

Unit 5 – Fractions

Problem 1: Problem 1: Use the associative property to solve n × a/b in unit form.

T: Write a multiplication number sentence to show four copies of 3 centimeters.
S: (Write 4 × 3 centimeters = 12 centimeters.)
T: (Write 4 × (3 centimeters).) I put parentheses around 3 centimeters to show that 3 is telling the number of centimeters in one group, but to solve, we moved the parentheses. Show me where you moved them to.
S: (Write (4 × 3) centimeters = 12 centimeters.)
T: Yes, you used the associative property by associating the 3 with the number of groups rather than the unit of centimeters.
T: Write a multiplication number sentence to show four copies of 3 fifths in unit form.
S: (Write 4 × 3 fifths = 12 fifths.)
T: (Write 4 × (3 fifths) = (4 × 3) fifths.) Is this true?
S: Yes, that's the associative property.
T: Draw a tape diagram to show four copies of 3 fifths.
S: (Draw a tape diagram.)



4 × (3 fifths) = (4 × 3) fifths
= 12 fifths

Repeat with three copies of 5 sixths and four copies of 3 eighths, associating the factors and drawing a matching tape diagram.

Problem 2: Use the associative property to solve n × a/b numerically.

T: (Display 4 × 3/5.) Say this expression.
S: Four times 3 fifths.
T: Write it in unit form.
S: (Write 4 × 3 fifths.) We just did this problem!
T: (Write 4 × 3 fifths = 12 fifths and 4 × 3/5 = 12/5, as shown to the right.) Compare these number sentences. Are these true? Discuss with your partner.
S: Yes, the top was solved in unit form, and the bottom used numbers.

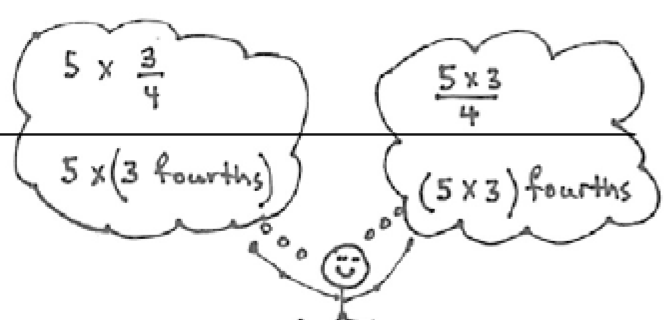
4 × 3 fifths = 12 fifths

$4 \times \frac{3}{5} = \frac{12}{5}$

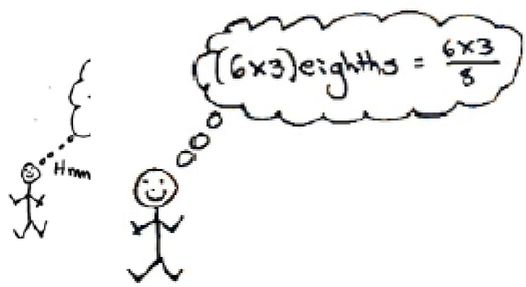
T: (Write $4 \times (3 \times \frac{1}{5}) = 4 \times 3$ fifths.)
We can say
4 × (3 × 1/5) = 4 × 3 fifths. On your personal board, move the parentheses to associate the factors of 4 and 3.
S: (Write (4 × 3) × 1/5.)
T: And the value is…?
S: 12/5.
T: (Write $4 \times (3 \times \frac{1}{5}) = (4 \times 3) \times \frac{1}{5} = \frac{12}{5}$.)
Is 4 groups of 3 fifths the same as 12 fifths?
S: Yes.
T: (Display 5 × 3/4.) Say this expression.
S: Five times 3 fourths.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

When using the associative property to solve $4 \times \frac{3}{5}$, some students may proficiently solve mentally, while others may need visual support to solve, including step-by-step guidance. For example, before asking for the value of $(4 \times 3) \times \frac{1}{5}$, it might be helpful to ask, "What is $12 \times \frac{1}{5}$?"



T: Keep the unit form in mind as you solve numerically. Record only as much as you need.
S: 5 × 3/4 = 15/4.
T: Yes, and as I thought of this as 5 times 3 fourths, I wrote down



$5 \times \frac{3}{4} = \frac{5 \times 3}{4} = \frac{15}{4}$.

Why is my number sentence true?
S: When you associated the factors, fourths became the unit, and we write the unit fourths as the denominator.
T: Yes. I think of 5 × (3 fourths) as 5 × 3/4 and (5 × 3) fourths as 5 × 3/4. Both have the same value—12 fourths.
Repeat with 8 × 2/3 and 12 × 3/10.

Problem 3: Rewrite a repeated addition problem as n times a/b.
T: Look back to the tape diagram we drew for the Application Problem. Say an addition sentence that represents this model.

S: $\frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6} = \frac{25}{6}$.

T: Write it as a multiplication sentence.
S: 5 × 5/6 = 25/6.

T: Which is more efficient? $\frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6}$ or $5 \times \frac{5}{6}$? Discuss with your partner.

S: 5 × 5/6. It doesn't take as long to write. →Multiplication is usually more efficient because making groups is easier than counting by fives, especially if there are a lot of copies.
T: How do we solve 5 × 5/6?
S: We know 5 × 5/6 can be solved like this:

$\frac{5 \times 5}{6} = \frac{25}{6}$.

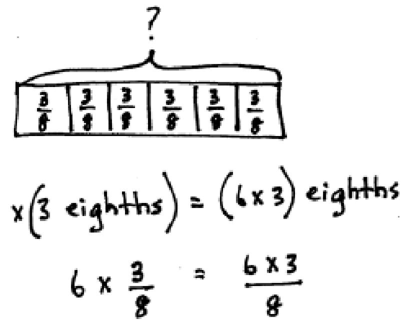
→It's 5 × 5 sixths, so that is 25 sixths.

Repeat with 3/5 + 3/5 + 3/5 + 3/5, drawing a tape diagram and solving using multiplication.

Problem 4: Solve n times a/b as (n × a)/b.
T: (Project 6 × 3/8.) Say this expression in unit form.
S: 6 × 3 eighths.

T: (6 × 3) eighths = $\frac{6 \times 3}{8}$, yes?

S: Yes!
T: Use this way of recording this time.



S: $6 \times \frac{3}{8} = \frac{6 \times 3}{8} = \frac{18}{8}$.

T: Rename as a mixed number.
S: 18/8 = 16/8 + 2/8 = 2 2/8.

Repeat with 3/8 × 5 and 9 × 4/5.

Problem Set (10 min)
Eureka Math – Lesson 36 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

Student Debrief
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Represent the multiplication of n times a/b as (n × a)/b using the associative property and visual models.

You may choose to use any combination of the questions below to lead the discussion.

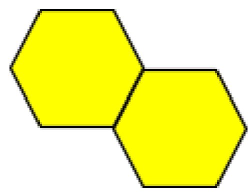
- Problem 4(d) is a good example of how multiplication is more efficient than repeated. Explain.
- Explain to your partner the method that you used to solve Problem 4(a–d).
- What was challenging about Problem 4(d)?
- Problem 4(b) results in a fraction greater than 1 with a large numerator. Watch as the fraction is renamed before multiplying. Discuss what you see with your partner. How does this method simplify the work done after the product is found?
- Try solving Problem 4(c) using a method similar to the one used above. (Note: Simplification is not a requirement in Grade 4 standards.)

Formative Assessment (10 min)

- Eureka Math – Lessons 35 & 36 Exit Ticket (Module 5)

Math Menu (40 min)
A math menu of differentiated tasks provides students with opportunities to apply concepts learned in a variety of contexts. New tasks will be introduced each day. Students are expected to complete a minimum of four total tasks upon conclusion of math menu, one being a required task for all students.

Task 5: Trading Blocks
(Public Schools of North Carolina, http://3-5cctask.ncdpi.wikispaces.net/)
The 2 pattern blocks below have a value of 1 whole.



Part 1:
- If the 2 pattern blocks above have the value of 1 whole, then what is the fractional value of 3 trapezoids? What is the fractional value of 1 trapezoid?
- If you have 3 trapezoids, how many green triangles would it take to cover the same area? If the 2 hexagons have a value of 1 whole, what is the fractional value of all the green triangles? What is the fractional value of one triangle?
- Since the number of trapezoids and the number of green triangles cover the dame space, they are equal. Write an equivalent fraction expressing the number of trapezoids to the number of green triangles.

Part 2:
- If 2 hexagons have a value of 1 whole, what is the value of 4 blue rhombuses? What is the value of 1 blue rhombus?
- If you have 4 rhombuses how many green triangles would it take to cover the same area? If the 2 hexagons have a value of 1 whole, what is the fractional value of all the green triangles? What is the value of 1 green triangle?
- Write an equivalent fraction expressing the number of rhombuses and the number of green triangles?
- Write a sentence explaining how you found equivalent fractions.

<u>Answer Key</u>
Part 1:
- The 3 trapezoids make up ¾ of the figure. One trapezoid is equal to ¼ of the figure.
- It would take 9 triangles to equal three trapezoids. $^{9}/_{12}$ would represent the fraction that is equal to the ¾ trapezoids.
- ¾ = $^{9}/_{12}$

Part 2
- The 4 rhombuses would equal $^{4}/_{6}$ or $^{2}/_{3}$ of the figure. One rhombus is equal to $^{1}/_{6}$.
- It would take 8 triangles to equal 4 rhombuses. $^{8}/_{12}$ would represent the fraction that is equal to the $^{4}/_{6}$ rhombuses.

- $^{4}/_{6} = ^{8}/_{12}$

Task 6: Jellybean Fractions
(Math Forum – NCTM, Problem #733)
Mr. Cal Q. Later has created a game called Jellybean Fractions to play with his students. Here's how it works:

- Each student takes a handful of jellybeans from the big gourmet jar on Mr. Later's desk. (He encourages them to take their favorite number.)
- Mr. Later asks the students to make groups of jellybeans representing ½ of their totals. If they can make two "nice" groups (i.e., there are no jellybeans that need to be cut apart), they score a point.
- Mr. Later then asks the students to rearrange the jellybeans, this time representing 1/3 of their totals. If they can make three "nice" groups, they score another point.
- Mr. Later continues ¼, 1/5, and so on.
- After continuing for a bit longer, Mr. Later asks the students to develop a strategy for scoring well in this game (while munching a few jellybeans for inspiration.)

This is your assignment as well, with a few more specific questions:
1. Briefly state one number of jellybeans that would be a poor choice and one that would be a good choice. Be sure to specify which is which.
2. Explain why these choices are poor and good, respectively.
Extension: Find the best number to pick that is less than sixty (this would be at least two handfuls, don't you think?)

<u>Answer Key</u>
1. Seven would be a poor choice and twenty-four would be a good choice.
2. Seven would be a poor choice because it is a prime number, which means its only factors are one and itself. Twenty-four is a good choice because it has several factors, which means it can be divided into several different sized groups.
Extension: The best number to pick that is less than sixty is forty-eight.

Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time they will continue investigating fractions and be introduced to two additional math menu choices.

Homework
Eureka Math – Lesson 35 & 36 (Module 5)

Unit 5 – Fractions

DAY 35

Content Standard(s)
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Focus: Multiplying a Whole Number by a Fraction

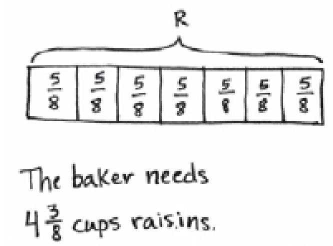
	Answer Key	*Strategies that may be used*
4 x ½ = n	n = 2	½ + ½ + ½ + ½
8 x ½ = n	n = 4	(4 x ½) x 2
16 x ½ = n	n = 8	(8 x ½) x 2
32 x ½ = n	n = 16	(16 x ½) x 2
4 x ¼ = n	n = 1	¼ + ¼ + ¼ + ¼
8 x ¼ = n	n = 2	(4 x ¼) x 2
16 x ¼ = n	n = 4	(8 x ¼) x 2
32 x ¼ = n	n = 8	(16 x ¼) x 2

Learning Targets
- I can multiply a mixed number by a whole number.
- I can create a line plot to display a data set of measurements in fractions of a unit.
- I can add and subtract fractions presented in line plots.

Guided Instruction/Mini-Lesson (25 min)
Eureka Math – Lesson 37 & 38 (Module 5)
(Script should serve as a guide only; modify as needed based on student needs and time allotment.)
Objective: Find the product of a whole number and a mixed number using the distributive property.

Application Problem (5 min)

The baker needs 5/8 cup of raisins to make 1 batch of cookies. How many cups of raisins does he need to make 7 batches of cookies?



Solution 1

$R = 7 \times \frac{5}{8} = 7 \times 5 \times \frac{1}{8}$

$= 35 \times \frac{1}{8}$

$= \frac{35}{8}$

$= 4\frac{3}{8}$

Solution 2

$R = 7 \times \frac{5}{8} = \frac{7 \times 5}{8}$

$= \frac{35}{8}$

$= 4\frac{3}{8}$

Solution 3

R = 7 × 5 eighths = 35 eighths

$= \frac{35}{8}$

$= 4\frac{3}{8}$

Note: This Application Problem reviews Lessons 35 and 36 of Topic G, where students learned to represent the product of a whole number and a fraction using the associative property. Notice that, although they can be used, parentheses are not modeled in the solutions. Students have already established that parentheses indicate the changed associations. Since the process has been established, parentheses are not necessary and can make notation cumbersome.

Concept Development (20 min)

Materials: (S) Personal white board

> **NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:**
>
> A gentle reminder and grid paper may help learners draw appropriately proportioned, though not meticulously precise, tape diagrams. Generally, the bar for 3 should be longer than the bar for $\frac{1}{5}$.

Problem 1: Draw a tape diagram to show the product of a whole number and a mixed number.

T: With me, draw a tape diagram showing 3 1/5 in two parts, the ones and the fractional part.

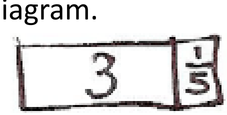
S: (Draw.)

T: Point to and say the two parts of your tape diagram.

S: (Point as saying each value.) 3. 1/5.

T: Draw one more copy of 3 1/5 as two parts on the same tape diagram.

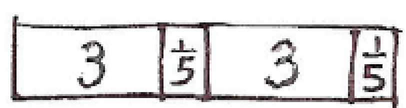
S: (Draw.)



T: There are two copies of 3 1/5. We can record this as 2 × 3 1/5. (Write 2 × 3 1/5 on the board.)

T: What are the 4 parts of your tape diagram?

S: 3, 1/5, 3, and 1/5. →2 threes and 2 fifths.



T: Make a new tape diagram of two groups of 3 1/5 the same length as your other tape diagram. This time, draw the threes on the left and the fifths on the right.

T: How many threes do we have?

S: 2 threes.

T: How many fifths do we have?

S: 2 fifths.

T: 2 × 3 1/5 is equal to 2 threes and 2 fifths. (Write 2 × 3 1/5 = (2 × 3) + (2 × 1/5).)

T: 2 times 3 is…? (Point to the expression.)

S: 6. (Write their response as shown to the right.)

$$2 \times 3\frac{1}{5} = (2 \times 3) + (2 \times \frac{1}{5})$$
$$= 6 + \frac{2}{5} = 6\frac{2}{5}$$

T: 2 times 1/5 is…? (Point to the expression.)
S: 2/5. (Write their response as shown to the right.)
T: The parts are 6 and 2/5. What is the total?
S: 6 + 2/5 = 6 2/5.
T: Let's try another one. Make a tape diagram to show four units of 5 2/10. Make another tape diagram to show how the whole numbers and fractional parts can be redistributed. Write a multiplication expression to represent your groups of 5 2/10 using the format we used to do two groups of 3 and a fifth.
S: (Write 4 × 5 2/10.)

Problem 2: Identify the distributive property to multiply a whole number and a mixed number.

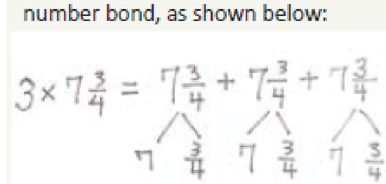
T: Express 5 2/10 as an addition expression. (Note that this is a c
S: 5 + 2/10.
T: (Write 4 × 5 2/10 = 4 × (5 + 2/10).) How many groups of 5 did you draw?
S: Four.
T: How many groups of 2 tenths?
S: Four.
T: There are four groups of 5 and four groups of 2/10. (Write (4 × 5) + (4 × 2/10).) We distribute our multiplication to both parts of our mixed number.
T: 4 × 5 is…?
S: 20.
T: 4 × 2/10 is…?
S: 8/10.
T: (Write = 20 + 8/10.) Our total product is…?
S: 20 8/10.
T: (Write 3 × 7 3/4.) With your partner, write a number sentence to multiply the whole number by each part.
(Pause.) What number sentence did you write?
S: 3 × 7 3/4 = (3 × 7) + (3 × 3/4).
T: Show the products for each part. What are the two products?
S: 21 and 9/4.
T: Rename 9/4 as a mixed number. 9/4 is…?
S: 2 1/4.
T: What is the product of 3 × 7 3/4?
S: 23 1/4.
T: You used the distributive property when you broke apart 7 3/4 and multiplied each part by 3.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

If students are reversing numerators and denominators, try using a color to distinguish them. For example, write the numerator in red. Have students consistently whisper-read fractions as they solve. Continue to use models for meaning-making. Frequently check for understanding, and guide students to offer personalized solutions.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

An additional step to solving $3 \times 7\frac{3}{4}$ that may scaffold understanding for students working below grade level is to model the decomposition of $7\frac{3}{4}$ as a number bond, as shown below:

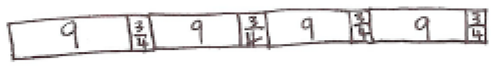


T: Try another. Solve 5 × 3 2/3. This time, imagine the distributive property in your head. Think out loud if you need to as you solve. Write only as much as you need to.
S: 5 × 3 2/3 = 15 + 10/3 = 18 1/3.

Problem 3: Use and share strategies for using the distributive pro whole number and a mixed number.
T: (Write 4 × 9 3/ 4 = ___.) Solve the problem on your personal white boards.

> **NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**
> Give everyone a fair chance to share their work and solutions by providing appropriate scaffolds. Demonstrating students may use translators, interpreters, or sentence frames to present. If the pace of the lesson is a consideration, prepare presenters beforehand.

Allow students about one to two minutes to solve.
T: What is 4 × 9 3/4?
S: 39.
T: Share your work with your partner.
S: I made a tape diagram showing four units of 9 3/4.
→I used the distributive property by writing four groups of 9 and four groups of 3/4. Then, I added those products and got 39. →I took a shortcut and wrote 36 + 12/4.



$$4 \times 9\tfrac{3}{4} = 36 + \tfrac{12}{4}$$
$$= 36 + 3$$
$$= 39$$

Have students work with a partner to solve the following problems:
$5\frac{6}{8} \times 4$, $12\frac{2}{6} \times 3$, and $9 \times 7\frac{5}{7}$.

Math Practice

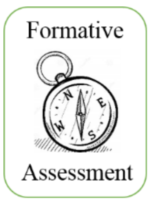
Problem Set (10 min)
Eureka Math – Lesson 38 (Module 5)
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving.

<u>Student Debrief</u>
During the problem set, the teacher will rotate around the room and assess student understanding. Look for misconceptions or misunderstandings that should be addressed. Students can be invited to review their solutions and the totality of the lesson experience. They should check work by comparing answers with a partner. If time permits, answers may be reviewed as a class. The debrief questions may also be used to guide instruction during math workshop, invitational groups and/or intervention.

Lesson Objective: Find the product of a whole number and a mixed number using the distributive property.

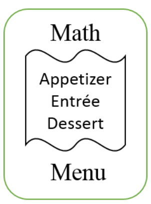
You may choose to use any combination of the questions below to lead the discussion.

- Explain how you knew what number was unknown from Problem 1.
- What method for solving did you use in Problem 2? Use a specific example from your Problem Set to explain.
- What did you do to solve the problems when the first factor was a mixed number?
- How did you solve Problem 2(e)? Turn and share with your partner.
- Why is it sometimes useful to see both a tape diagram and the numbers?
- How might you improve your work from today's Application Problem?



Formative Assessment (10 min)

- Eureka Math – Lessons 37 & 38 Exit Ticket (Module 5)



Math Menu (40 min)
A math menu of differentiated tasks provides students with opportunities to apply concepts learned in a variety of contexts. New tasks will be introduced each day. Students are expected to complete a minimum of four total tasks upon conclusion of math menu, one being a required task for all students.

Task 7: Pizza Dough
(Math Forum – NCTM, Problem #4003)
Mario is having 8 friends come to his house for homemade pizza. His favorite dough recipe serves 3 people. Here are the ingredients:

1 ¾ cups flour
¾ cup warm water
2 tsp yeast
1 tsp salt
1 tsp sugar

How much water should he use to make enough dough for him and his guests? Explain your thinking.

Extension: Mario has a 2-pound bag of special pizza flour. According the label on the bag, "1 pound of flour = 3 ½ cups." Does Mario have enough to make the pizzas? Explain how you know.

<u>Answer Key</u>
Mario needs 2 ¼ cups of warm water to make enough dough.
Extension: Yes, he does have enough to make the pizzas.

Task 8: Insect Collection

(MCS Fourth Grade,
http://mcsfourthgrade.wikispaces.com/MD-Represent+%26+Interpret+Data)
Kwon and Miguel have an insect collection. They have measured the lengths of all their insects. Their data shows that 4 insects are 1/8 inch long, 6 are 1/4 inch long, 8 are 1/2 inch long, 2 are 1/6 inch long, 1 is 1/12 inch long, and 5 are 1/3 inch long. Create a line plot that shows the data. How much longer is the longest insect from the shortest insect?

Answer Key
The longest insect is 5/12 inch longer than the shortest insect.

Closing the Lesson

- Revisit the learning targets with students.
- Inform students that next time they will participate in a class discussion on the required menu problem, complete an independent performance task, and wrap up menu.

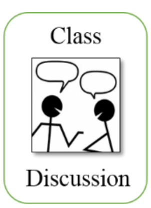
Homework
Eureka Math – Lesson 37 & 38 (Module 5)

Unit 5 – Fractions

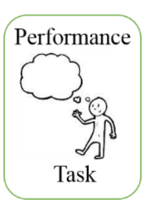
DAY 36

Content Standard(s)
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b.
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Learning Targets
- I can make sense of problems and persevere in solving them.



Class Discussion of Required Menu Problem:
Dividing up the Land (30 min)
Guide students in discussion of varied problem solving strategies used for the required menu problem.



Performance Task – Chocolate Bars (30 min)
This task will serve as an independent summative assessment of student learning. It should be completed with no assistance.

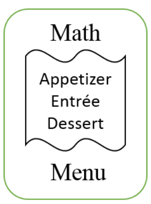
<u>Chocolate Bars</u>

Amy, Beth, Katie, Gretchen, and Deb love chocolate. One afternoon, they each had a 2 large chocolate bars. Each chocolate bar was the same size.
Here is what each girl ate:

- Amy: $^{8}/_{6}$ of her chocolate bars
- Beth: 1 $^{1}/_{6}$ of her chocolate bar
- Katie: $^{9}/_{6}$ of her chocolate bars
- Gretchen: 1 $^{5}/_{6}$ of her chocolate bars
- Deb: 1 $^{3}/_{6}$ of her chocolate bars

1. How could you show how many chocolate bars were eaten in total?
2. How much chocolate remains?

<u>Answer Key</u>
1. Seven and two-sixths chocolate bars were eaten.
2. There are 1 $^{4}/_{6}$ chocolate bars remaining.



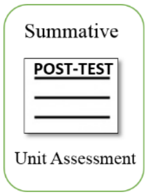
Math Menu (30 min)
A math menu of differentiated tasks provide students with opportunities to apply concepts learned in a variety of contexts. New tasks will be introduced each day. Students are expected to complete a minimum of four total tasks upon completion of math menu, one being a required task for all students.

Students will complete their selected math menu problems.

Closing the Lesson
- Revisit the learning targets with students.
- Inform students that next time they will take an assessment to demonstrate what they have learned in this unit.

Unit 5 – Fractions

DAY 37



Summative Unit Assessment
Scantron = Test ID #OE065
The on-line summative unit assessment should be completed at the end of the unit. The objective is to evaluate student learning outcomes compared to standards and benchmarks. In addition, this data will be reviewed at an aggregate level to reflect on overall unit performance and instruction.

<u>Student Materials</u>

- Scantron Student ID Number
- Work paper
- Pencils
- Privacy folder for computer

<u>Student Online Assessment Directions</u>

1. Open Google Chrome web browser.
2. Go to www.achievementseries.com.
3. Select “Student Login”.
4. Enter Site ID: 30-6732-4169.
5. Enter Student ID.
6. Select the “green checkmark” to verify student name.
7. If message is displayed to allow pop-ups, select “OK”.
8. Select “Start Test”.
9. Enter Test ID.
10. Read the instructions: Put your name on your work paper. Number each problem, and show all work before entering an answer on the computer.
11. Select “Continue” to begin assessment.
12. After answering all assessment questions, review your responses and work paper.
13. Select “Turn in Test”.

Unit 5 – Fractions
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