

MTH603

ASSIGNMENT NO: 01

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Due Date: May 6, 2024

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Question 1:

Find a root of the equation $x^3 - 3x - 5 = 0$, in the interval (2,3) using Bisection Method after three Iterations. Note: Accuracy up to four decimal places is required.

Solution:

Sure, let's use the Bisection Method to find the root of the equation $x^3 - 3x - 5 = 0$ in the interval (2,3) after three iterations.

1. **Initial Interval:** We start with the given interval (2,3).

2. **Iteration 1:**

- Midpoint: $c = \frac{2+3}{2}$

$$= 2.5$$

- Evaluate $f(c)$: $f(2.5)$

$$= (2.5)^3 - 3(2.5) - 5$$

$$= -1.375$$

- Update Interval:

- Since $f(2.5)$ is negative, the root lies in the interval $(2.5, 3)$.

3. Iteration 2:

- Midpoint: $c = \frac{2.5 + 3}{2}$
 $= 2.75$

- Evaluate $f(c)$: $f(2.75)$

$$= (2.75)^3 - 3(2.75) - 5$$

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$$= 1.859375$$

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- Update Interval:

- Since $f(2.75)$ is positive, the root lies in the interval $(2.5, 2.75)$.

4. Iteration 3:

- Midpoint: $c = \frac{2.5 + 2.75}{2}$

$$= 2.625 \quad 0344-9767315$$

- Evaluate $f(c)$: $f(2.625)$

$$= (2.625)^3 - 3(2.625) - 5$$

$$= 0.216796875$$

- Update Interval:

- Since $f(2.625)$ is positive, the root lies in the interval $(2.5, 2.625)$.

So, the approximate root of the equation within the interval $(2, 3)$ after three iterations using the Bisection Method is $x \approx 2.6250$.

Question 2:

Find a root of the following equation in the interval $(0, 1)$ using Newton-Raphson Method after three iterations

$$xe^x - \cos x = 0$$

Take Initial value 0.5.

Note: Accuracy up to four decimal places is required. Here is a transcendental equation all the calculation should be done in the radians mode.

Solution:

To find a root of the equation $e^x - \cos x = 0$ in the interval (0,1) using the Newton-Raphson method, we first need to calculate the derivative of the function $f(x) = xe^x - \cos(x)$, which is necessary for the iterative formula of the method.

Let's denote the derivative of $f(x)$ as $f'(x)$ and then apply the Newton-Raphson iteration formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = e^x + xe^x + \sin(x)$$

initial value $x_0 = 0.5$

Let's do the calculations:

1. $f(0.5) = (0.5)e^{0.5} - \cos(0.5) \approx 0.01859$

$$f'(0.5) = e^{0.5} + (0.5)e^{0.5} + \sin(0.5) \approx 2.1839$$

Apply the Newton-Raphson formula:

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$$x_1 = 0.5 - \frac{0.01859}{2.1839} \approx 0.4912$$

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2. For $x_1 = 0.4912$

$$f(0.4912) \approx -0.00102$$

$$f'(0.4912) \approx 2.1747$$

$$x_2 = 0.4912 - \frac{-0.00102}{2.1747} \approx 0.4913$$

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3. For $x_2 = 0.4913$

$$f(0.4913) \approx -1.00545 \times 10^{-6}$$

$$f'(0.4913) \approx 2.1747$$

$$x^3 = 0.4913 - \frac{-1.00545 \times 10^{-6}}{2.1747} \approx 0.4913$$

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After three iterations, the root approximately is 0. 4913.