











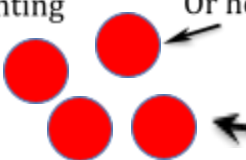
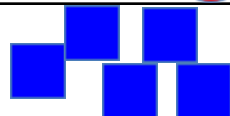
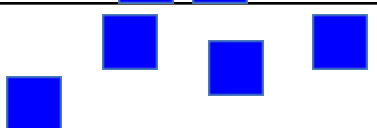








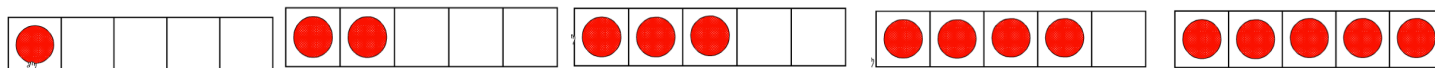
## Counting Principles and Subitizing

	Explanation	Example
<b>One-to-one correspondence</b>	Understanding that each object to be counted should be tagged with one and only one unique numeric tag	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">1  </div> <div style="text-align: center;">2  </div> <div style="text-align: center;">3  </div> </div>
<b>Stable-order principle</b>	Knowing the number-name list (i.e., one, two, three... ) must be used in a fixed order every time a group of objects is counted across all trials	Stable order: 1, 2, 3, 4, 5, 6 Not 1, 2, 3, 6, 5, 4
<b>Cardinal principle (cardinality)</b>	The number tag used for the last object in a count symbolizes the total number of objects in a set	While pointing at each object, count: <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">             "one"   </div> <div style="text-align: center;">             "two"   </div> <div style="text-align: center;">             (with emphasis)              "three"   </div> </div> "There are <i>three</i> (squares) here."
<b>Abstraction principle</b>	Any types of objects can be counted together in a set	<div style="display: flex; flex-direction: column; align-items: center;"> <div style="margin-bottom: 10px;">             3   </div> <div style="margin-bottom: 10px;"> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;">3</div>  </div> <div> <div style="display: flex; align-items: center;">  <div style="border: 1px solid black; padding: 10px; margin-left: 10px; font-size: 2em;">3</div> </div> </div> </div>
<b>Order irrelevance principle</b>	➔	Start counting here  Or here Or here <div style="border: 1px solid black; padding: 5px; float: right; margin-top: -40px;">4</div>
<b>Conservation Principle</b>	Understanding that the count for a set group of objects stays the same whether they are spread out or close together.	<div style="display: flex; align-items: center; margin-bottom: 10px;">  <div style="border: 1px solid black; padding: 5px; margin-left: 10px;">5</div> </div> <div style="display: flex; align-items: center;">  <div style="border: 1px solid black; padding: 5px; margin-left: 10px;">5</div> </div>
<b>Subitizing</b>	"Instantly seeing how many." Identifying the number of objects in a group without having to count them.	<div style="display: grid; grid-template-columns: repeat(3, 1fr); gap: 10px;"> <div></div> <div></div> <div></div> <div></div> <div></div> <div></div> </div>

**Table 1: The stages of cardinality development**

Stages of cardinality development	Stage description for answering <i>how many</i> ? (Adapted from Bermejo, 1996; and Clements & Sarama, 2009)
<b>Stage I: Pre-counters</b>	<ul style="list-style-type: none"> <li>Children do not understand the question <i>how many</i>, and so they provide random answers. These children are typically 1-2 years of age.</li> </ul>
<b>Stage II: Reciters</b>	<ul style="list-style-type: none"> <li>Children respond with a number-word sequence, but without tagging each item.</li> </ul>
<b>Stage III: Corresponders</b>	<ul style="list-style-type: none"> <li>Children respond to the question by completely recounting the set usually demonstrating one to one correspondence. Children are typically at this stage around 3 years of age.</li> </ul>
<b>Stage IV: Immature counters</b>	<ul style="list-style-type: none"> <li>Children answer with the last number-tag used even if inaccurate. These children are not mature enough yet to monitor their counting to ensure its accuracy.</li> </ul>
<b>Stage V: Rigid rule followers</b>	<ul style="list-style-type: none"> <li>Children answer with the largest number-tag included in the count but it may not have been the last tag used. These children are beginning to sleuth out the rules and patterns of how counting works but continue to make errors.</li> </ul>
<b>Stage VI: Counters</b>	<ul style="list-style-type: none"> <li>Children are able to monitor their own or someone else's counting for accuracy and provide the correct response to the <i>how many</i> question. Children reach this stage for the smaller quantities (1-5) around the age of four and for the larger quantities (6-10) around the age of five.</li> </ul>

## Five Frames













Five frames are a very useful tool for helping students learn to count and encourage development of the counting principles. Use manipulatives like counters, bears, square tiles, to reinforce the counting principle you are working on. These frames can also encourage students to develop the ability to subitize.

<https://www.k-5mathteachingresources.com/counting-centers.html>

<http://www.k-5mathteachingresources.com/kindergarten-math-activities.html>

<http://illuminations.nctm.org/Activity.aspx?id=3564>

**Table 4. Common counting errors**

Type of Counting Error	Example	Remedy
<b>SEQUENCE ERROR</b>		
Saying the number sequence out of order, skipping numbers, or using the same number more than once.	<div> <div>"1      2      3      6      10"</div> <div>      </div> <div>      </div> </div>	Practice reciting (or singing) the single-digit sequence, first focusing on one to ten, then later moving on to numbers greater than ten.
Struggling with the count sequence past twelve.	<p>Skips 15: "1...13, 14, 16, 17, 18."</p> <p>Uses incorrect words: "1...13, 14, fiveteen." "1...18, 19, 10-teen" or "1...29, 20-ten, 20-eleven."</p> <p>Stops at a certain number: "1...20" (stops) "1...20" (starts from 1 again)</p>	<p>Highlight and practice exceptions, such as <i>fif+teen</i>. Fifteen and thirteen are commonly skipped because they are irregular.</p> <p>Recognize that a nine signals the end of a series and that a new one needs to begin (e.g., nineteen marks the end of the teens).</p> <p>Recognize that each new series (decade) involves combining a decade and the single-digit sequence, such as twenty, twenty plus one, twenty plus two, etc.</p> <p>Recognize the decade term that begins each new series (e.g., twenty follows nineteen, thirty follows twenty-nine, and so forth). This involves both memorizing terms such as ten, twenty, and thirty by rote and recognizing a pattern: "add -ty to the single-digit sequence" (e.g., <i>six + ty</i>, <i>seven + ty</i>, <i>eight + ty</i>, <i>nine + ty</i>).</p>

## COORDINATION ERROR

Labeling an object with more than one number word.



Encourage the child to slow down and count carefully. Underscore that each item needs to be tagged only once with each number word.

Pointing to an object but not counting it.



Same as above.

## KEEPING TRACK ERROR

Recounting an item counted earlier.



Help the child devise strategies for sorting counted items from uncounted items. For movable objects, for instance, have the child place counted items aside in a pile clearly separated from uncounted items. For pictured objects, have him or her cross off items as counted.

## SKIM

No effort at one-to-one counting or keeping track.

Waves finger over the collection like a wand (or jabs randomly at the collection) while citing the counting sequence (e.g., "1, 2, 3...9, 10").

Underscore that each item needs to be tagged with one and only one number word and help the child to learn processes for keeping track. Model the counting.

## NO CARDINALITY RULE

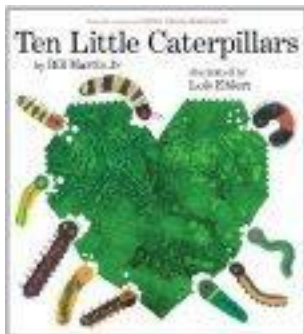
Not recognizing that the last number word used in the counting process indicates the total.

Asked how many, the child tries to recount the collection or simply guesses.

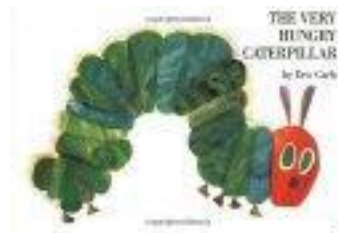
Play *Hidden Stars* with small collections of one to three items first and then somewhat larger collections of items.

## Counting Read Alouds

<http://astore.amazon.com/k5matteares-20?node=11&page=2>



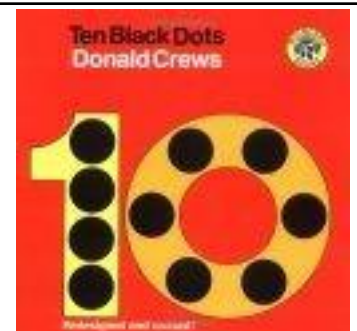
[Ten Little Caterpillars](#)  
by Bill Martin Jr.  
**\$13.53**



[The Very Hungry Caterpillar](#)  
by Eric Carle  
**\$6.07**

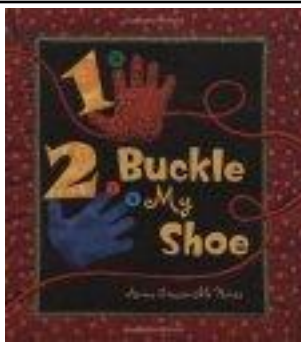


[Fish Eyes: A Book You Can Count On](#)  
by Lois Ehlert  
**\$6.30**

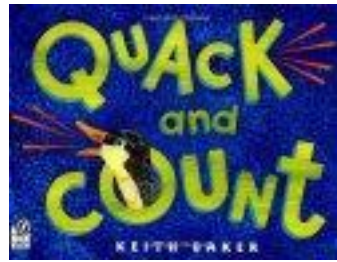


[Ten Black Dots](#)  
by Donald Crews  
**\$6.29**

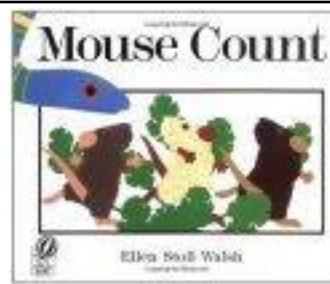




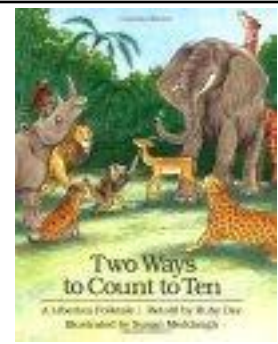
[1, 2, Buckle My Shoe](#)  
by Anna Grossnickle  
Hines  
\$12.97



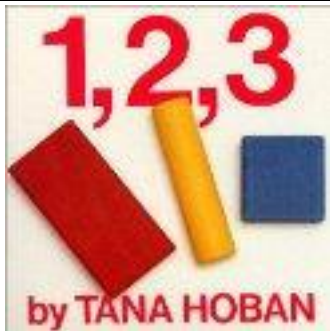
[Quack and Count](#)  
by Keith Baker  
**\$6.30**



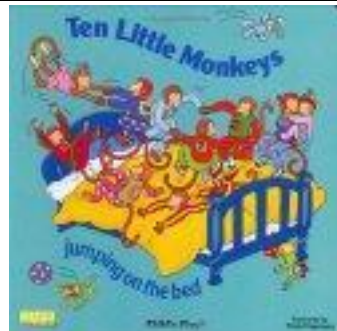
[Mouse Count](#)  
by Ellen Stoll Walsh  
**\$5.40**



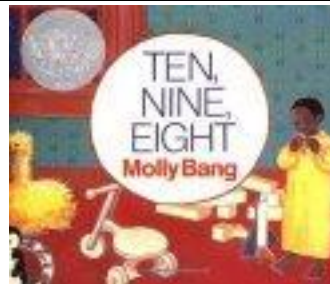
[Two Ways to Count to Ten: A Liberian Folktale](#)  
by Ruby Dee  
**\$7.19**



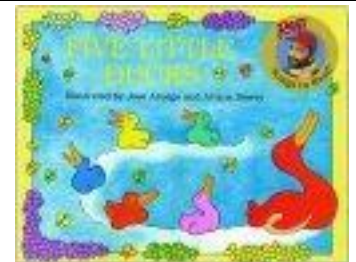
[1, 2, 3 Board Book](#)  
by Tana Hoban



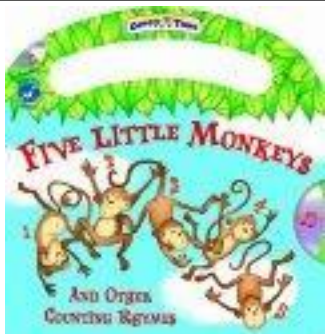
[The Right Number of Elephants](#)  
by Jeff Sheppard  
**\$6.29**



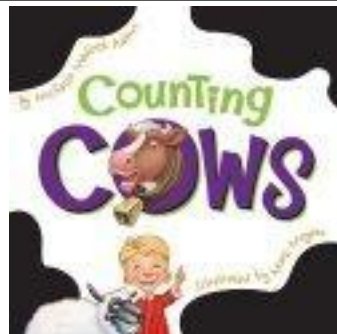
[Ten, Nine, Eight Board Book \(Caldecott Coll...\)](#)  
by Molly Bang  
\$6.29



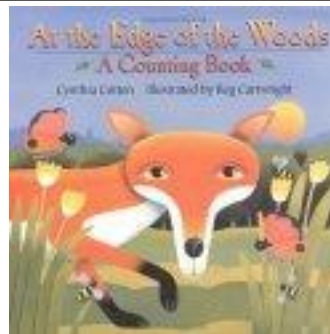
[Five Little Ducks \(Raffi Songs to Read\)](#)  
by Raffi  
\$6.29



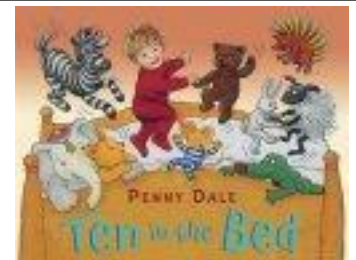
[Five Little Monkeys And Other Counting Rhym...](#)  
by Rebecca Elliott



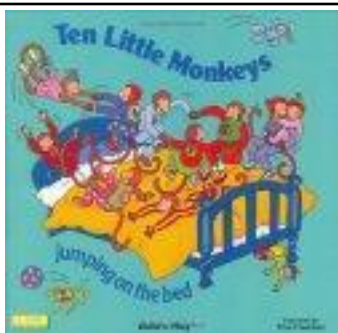
[Counting Cows](#)  
by Michelle Medlock  
Adams  
\$7.19



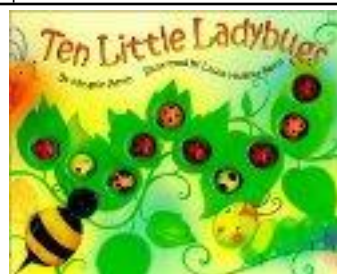
[At the Edge of the Woods: A Counting Book](#)  
by Cynthia Cotten  
\$16.08



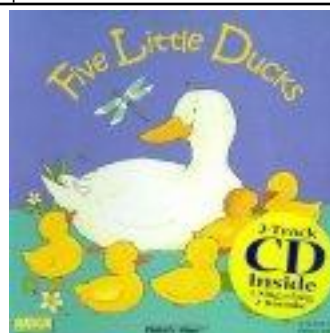
[Ten in the Bed](#)  
by Penny Dale  
\$5.39



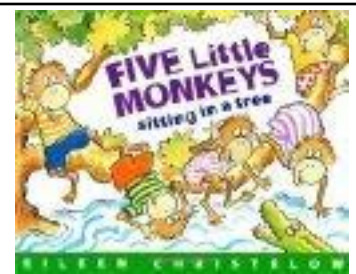
[Ten Little Monkeys: Jumping on the Bed \(Cla...](#)



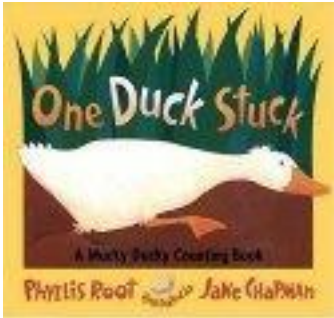
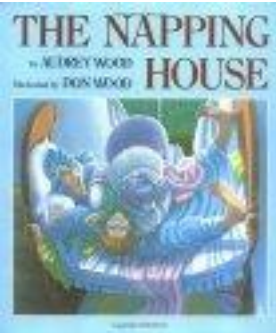
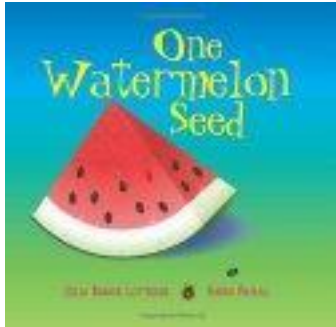
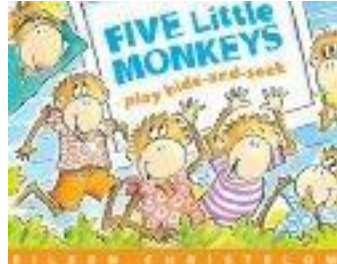
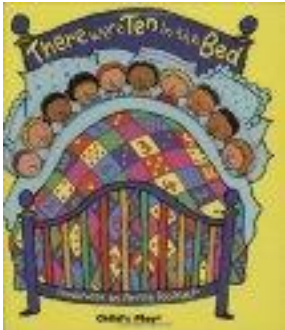
[Ten Little Ladybugs](#)  
by Melanie Gerth



[Five Little Ducks](#)  
\$5.58



[Five Little Monkeys Sitting in a Tree \(A Fi...](#)  
by Eileen Christelow  
\$6.29

<p>\$5.39</p>  <p><u>One Duck Stuck: A Mucky Ducky Counting Book</u> by Phyllis Root</p>	 <p><u>The Napping House</u> by Audrey Wood <b>\$12.40</b></p>	 <p><u>One Watermelon Seed</u> by Celia Barker Lottridge</p>	 <p><u>Five Little Monkeys Play Hide and Seek (A F...</u> by Eileen Christelow \$6.29</p>
 <p><u>There Were Ten in the Bed (Dial Books (Chil...</u> by Annie Kubler \$8.99</p>			

## Works Cited

Clements, Douglas H. "Subitizing: What Is It? Why Teach It?" Teaching Children Mathematics 5.7 (1999): 400. <http://gse.buffalo.edu/fas/clements/files/Subitizing.pdf>

Frye, D., Baroody, A. J., Burchinal, M., Carver, S. M., Jordan, N. C., & McDowell, J. (2013). *Teaching math to young children: A practice guide* (NCEE 2014-4005). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education. Retrieved from the NCEE website: <http://whatworks.ed.gov>

Sadler, Faith H. (2009). Help! They Still Don't Understand Counting. *TEACHING Exceptional Children Plus*, 6(1) Article 3. Retrieved 3/8/14 from <http://escholarship.bc.edu.education/tecplus/vol6/iss1/art3>