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B. Sc. (Hons.) Math (Semester – 5th)
DIFFERENTIAL GEOMETRY
Subject Code: BMATS1-522
Paper ID: 22131221

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Discuss the nature of geodesic on a plane.

b. Show that the involutes of a circular helix are plane curves.

c. Prove that the tangent plane to the surface $xyz = a^3$ and the co-ordinate planes bound a tetrahedron of constant volume.

d. Calculate the fundamental magnitudes for the right helicoid $x = u\cos v,\ y = u\sin v,$ $z = cv.$

e. Prove that straight lines on a surface are the only asymptotic lines which are geodesics.

f. Show that a necessary and sufficient condition that a curve be a straight line is that $\kappa = 0$ at all points.

g. Show that the principal normal to a curve is normal to the locus of centre of curvature at those points where the curvature is stationary.

h. Find the length of the curve given as the intersection of the surfaces $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,\ x = \cosh\left(\frac{z}{a}\right).$

i. Find the radii of curvature and torsion of the helix $x = a\cos u,\ y = a\sin u,\ \ z = au\tan\alpha.$

j. Consider the curve $u = u(t),\ v = v(t)$. Find the direction coefficients of the tangent to the curve.

Section – B **(5 marks each)**

Q2. Prove that the curves of the family $\frac{v^3}{u^2} = \text{constant},$ are geodesics on a surface with metric $v^2du^2 - 2uv\ dudv + 2u^2dv^2$; $(u > 0, v > 0)$.

Q3. Find the lines that have four point contact at (0, 0, 1) with the surface $x^4 + 3xyz + x^2 - y^2 - z^2 + 2yz - 3xy - 2y + 2z = 1$.

Q4. Discuss the geometrical interpretation of the second fundamental form.

Q5. If two curves have the same intrinsic equations then show that they are congruent.

Q6. Prove that for any curve $\left[t^{/}, t^{//}, t^{///}\right] = \left[r^{//}, r^{///}, r^{////}\right] = \kappa^3\left(\kappa\tau^{/} - \kappa^{/}\tau\right) = \kappa^5 \frac{d}{ds}\left(\frac{\tau}{\kappa}\right)$.

Section – C **(10 marks each)**

Q7. State and prove Gauss Bonnet theorem.

Q8. **(a)** Define the normal plane to a space curve at a point and find its equation. (7)

(b) Show that a real surface for which the equations $E/L = F/M = G/N$ hold is either plane or spherical. (3)

Q9. If C represents a space curve and C_1, the locus of the centre of osculating sphere, then prove that the tangent, principal normal and binormal to C_1 are parallel respectively to the binormal, principal normal and tangent to C at the corresponding points.