Significance Tests: Two Proportions - Key

1. Two types of medication for hives are being tested to determine if there is a difference in the percentage of adult patient reactions. Twenty out of a random sample of 200 adults given medication A still had hives 30 minutes after taking the medication. Twelve out of another random sample of 200 adults given medication B still had hives 30 minutes after taking the medication. Test at a 1% level of significance.

Let p_A = the proportion of adults given medication A and p_B = the proportion of adults given medication B that still have hives after 30 minutes.

H_0: $p_A - p_B = 0$

H_a: $p_A - p_B \neq 0$

Two-Sample z Test for Proportions

Two random samples of 200 adults.

10% Condition is met: $10n_A = 10(200) = 2000 < N$ and $10n_B = 10(200) = 2000 < N$

The large counts condition is met.

$$n_A\hat{p}_A = 200(.1) = 20 \geq 10 \text{ and } n_A(1 - \hat{p}_A) = 200(1 - .1) = 180 \geq 10$$
$$n_B\hat{p}_B = 200(.06) = 12 \geq 10 \text{ and } n_B(1 - \hat{p}_B) = 200(1 - .06) = 188 \geq 10$$

2-PropZTest

$z = 1.4744$ p-value = 0.1404

Our p-value of 0.1404 is greater than $\alpha = 0.01$, thus we fail to reject the null hypothesis in favor of the alternative hypothesis. There is insufficient evidence to suggest that the difference in p_A = the proportion of adults given medication A and p_B = the proportion of adults given medication B that still have hives after 30 minutes is not zero.

2. Are men more likely to be left handed than women? Previous research potentially suggests so. To investigate a possible relation between gender and handedness, a random sample of 320 adults was taken, with the following results:

	Men	**Women**
Sample Size	168	152
Number of Left-Handed	24	10

Run the appropriate test to determine if the proportion of men who are left-handed is greater than the proportion of women who are left-handed at the 5% significance level.

Let p_M = the proportion of men who are left-handed and p_W = the proportion of women who are left-handed

H_0: $p_M - p_W = 0$

H_a: $p_M - p_W > 0$

Two-Sample z Test for Proportions

Random samples of 168 men and 152 women.

10% Condition is met: $10n_M = 10(168) = 1680 < N_M$ and $10n_W = 10(152) = 1520 < N_W$

The large counts condition is met(-ish).

$n_M\hat{p}_M = 168(0.1429) = 24 \geq 10$ and $n_M(1 - \hat{p}_M) = 168(1 - 0.1429) = 144 \geq 10$

$n_W\hat{p}_W = 152(0.0592) = 10 \geq 10$ and $n_W(1 - \hat{p}_W) = 200(1 - 0.0592) = 143 \geq 10$

2-PropZTest

$z = 2.234$ p-value = 0.0127

Our p-value of 0.0127 is less than $\alpha = 0.05$, thus we reject the null hypothesis in favor of the alternative hypothesis. There is sufficient evidence to suggest that the difference in p_M = the proportion of men who are left-handed and p_W = the proportion of women who are left-handed is greater than zero.

3. A local school board member randomly sampled private and public high school teachers in his district to compare the proportions of highly qualified teachers in the faculty. Of the public school teachers sampled 15% were highly qualified teachers, and of the private school teachers sampled 17.5% were highly qualified teachers. The standard error of the sampling distribution for the difference in public and private school teachers was found to be 0.0453. Run the appropriate test at the 10% significance level. Assume the conditions for inference are met.

Let p_1 = the proportion of highly qualified public school teachers and p_2 = the proportion of highly qualified private school teachers

$H_0: p_1 - p_2 = 0$

$H_a: p_1 - p_2 \neq 0$

Two-Sample z Test for Proportions

$$z = \frac{\hat{p}_1 - \hat{p}_2}{SE_{\hat{p}_1 - \hat{p}_2}} = \frac{0.15 - 0.175}{0.0453} = -0.5519$$

p-value = 0.581

Our p-value of 0.581 is greater than α = 0.1, thus we fail to reject the null hypothesis in favor of the alternative hypothesis. There is insufficient evidence to suggest that the difference in p_1 = the proportion of highly qualified public school teachers and p_2 = the proportion of highly qualified private school teachers is not zero.

1. Manufacturers are required to test for Staphylococcus aureus (a potentially harmful bacteria) during cheese production. The Barnes Cheese Factory knows that the amount of Staphylococcus aureus in its aged cheddar cheese follows a Normal distribution with a mean of 8,000 FCU/g (colony-forming units per gram) and a standard deviation of 750 FCU/g.

   a. The FDA requires that Staphylococcus aureus levels remain below 10,000 FCU/g (colony-forming units per gram). What is the probability that a gram of aged cheddar from the Barnes Cheese Factory is over the FDA limit?

      normalcdf(10000, large number, mean = 8000, sd = 750) = 0.0038

   b. Assume the Barnes Cheese Factory produces 500 grams of aged cheddar each day. What is the expected number of grams that are over the FDA limit?

      $\mu = np = 500(0.0038) = 1.9 \; grams$

   c. Calculate the standard deviation of the number of grams that are over the FDA limit.

      $\sigma = \sqrt{np(1-p)} = \sqrt{500(0.0038)(1-0.0038)} = 1.3758$

   d. The Barnes Cheese Factory has a policy that requires the production line to shut down and be sanitized if more than 1% of the grams produced each day are over the FDA limit. Calculate the probability that the production line will shut down on a given day.

$P(x>5) = 1 - P(x \leq 5) = 1 - binomcdf(trials = 500, \; p = 0.0038, \; x = 5) = 1 - 0.987 = 0.013$