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B.Sc. (HONS.) Math (Semester – 3<sup>rd</sup>)

MATHEMATICAL STATISTICS

Subject Code: BMATS1302

Paper ID: [19131212]

Time: 03 Hours

Maximum Marks: 60

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of 2 marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A

(2 marks each)

Q1. Attempt the following:

- a) Write the classical definition of probability.
- b) Write the three axioms of probability.
- c) Four cards are drawn at random from a pack of 52 cards. Find the probability that they are a king, a queen, a jack, and an ace.
- d) Define random variable with an example.
- e) Write the properties of cumulative distribution function.
- f) A continuous random variable  $X$  has a pdf  $f(x) = kx(1 - x)$ ;  $0 \leq x \leq 1$ . Find the value of  $k$ .
- g) Write the mean and variance of negative binomial distribution.
- h) Let  $P(X = x) = e^{-2} \frac{2^x}{x!}$ ;  $x = 0, 1, 2, \dots$  Find  $E(X)$ .
- i) Define 'critical region'.
- j) Write the applications of Student's t-distribution.

Section – B

(5 marks each)

Q2. If  $A$  and  $B$  are any two events of sample space  $S$  and both are not disjoint. Show that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Q3. Define moment generating function and write its important properties.

Q4. Let  $X \sim \text{Bin}(n, p)$  and  $Y \sim \text{Bin}(m, p)$  and both are independent. Show that

$$X + Y \sim \text{Bin}(n + m, p).$$

Q5. If  $X$  and  $Y$  are independent Poisson variates such that  $P(X = 1) = P(X = 2)$  and

$$P(Y = 2) = P(Y = 3). \text{ Find the variance of } X - 2Y.$$

Q6. Explain the concepts of Type-I and Type-II errors in hypothesis testing.

Section – C

(10 marks each)

Q7. State and prove Baye's Theorem.

Q8. The joint pdf of random variables of  $X$  and  $Y$  is given by:

$$f(x, y) = \left\{ \begin{array}{l} \frac{(6-x-y)}{8}, \quad 0 \leq x \leq 2, \quad 2 \leq y \leq 4, \quad 0, \\ \text{elsewhere. } \end{array} \right\}$$

Find

- (a)  $P(X < 1, Y < 3)$
  - (b)  $P(X + Y < 3)$
  - (c)  $P(X < 1/Y < 3)$
  - (d)  $P(2X + Y > 3)$ .
- Q9. Define the sampling distribution of Chi-square and derive the expression for its moment generating function.