A IAL Pure Maths 4
Coordinate Geometry
QP

1.June 2024

4.

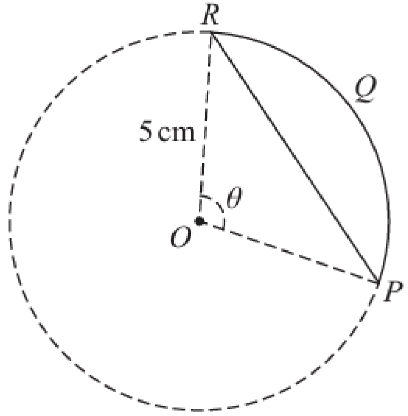

Figure 1

Figure 1 shows a sketch of a segment *PQRP* of a circle with centre *O* and radius 5 cm.

Given that

- angle *POR* is θ radians
- θ is increasing, from 0 to π, at a constant rate of 0.1 radians per second
- the area of the segment *PQRP* is A cm^2

(a) show that

$$\frac{\mathrm{d}A}{\mathrm{d}\theta} = K\left(1 - \cos\theta\right)$$

where K is a constant to be found.

(2)

(b) Find, in cm^2 s^{-1}, the rate of increase of the area of the segment when $\theta = \frac{\pi}{3}$

(4)

2.Oct 2024

4.

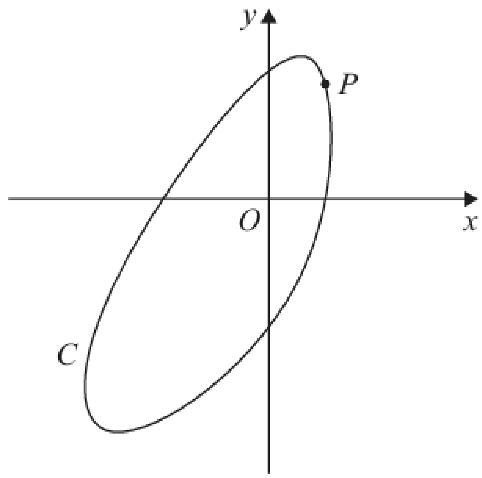

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$3x^2 + 2y^2 - 4xy + 8^x - 11 = 0$$

The point P has coordinates (1, 2).

(a) Verify that P lies on C.

(1)

(b) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The normal to C at P crosses the x-axis at a point Q.

(c) Find the x coordinate of Q, giving your answer in the form $a + b\ln 2$ where a and b are integers.

(3)

3.June 2023

2.

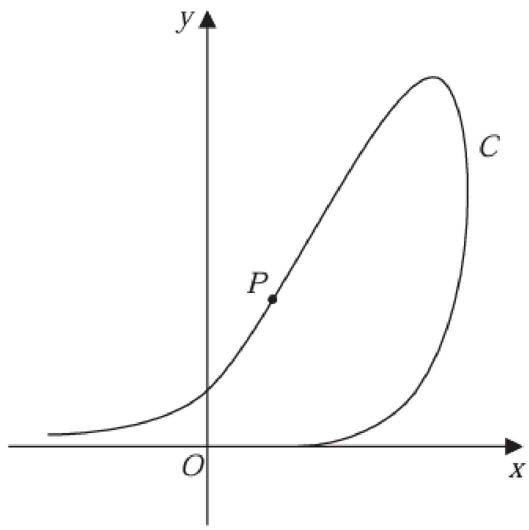

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$2^x - 4xy + y^2 = 13 \qquad y \geqslant 0$$

The point P lies on C and has x coordinate 2

(a) Find the y coordinate of P.

(2)

(b) Find $\dfrac{dy}{dx}$ in terms of x and y.

(5)

The tangent to C at P crosses the x-axis at the point Q.

(c) Find the x coordinate of Q, giving your answer in the form $\dfrac{a\ln 2 + b}{c\ln 2 + d}$ where a, b, c and d are integers to be found.

(3)

4.Jan 2022

1. The curve C has equation

$$xy^2 = x^2y + 6 \qquad x \neq 0 \quad y \neq 0$$

Find an equation for the tangent to C at the point $P(2, 3)$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

5.Oct 2022

11.

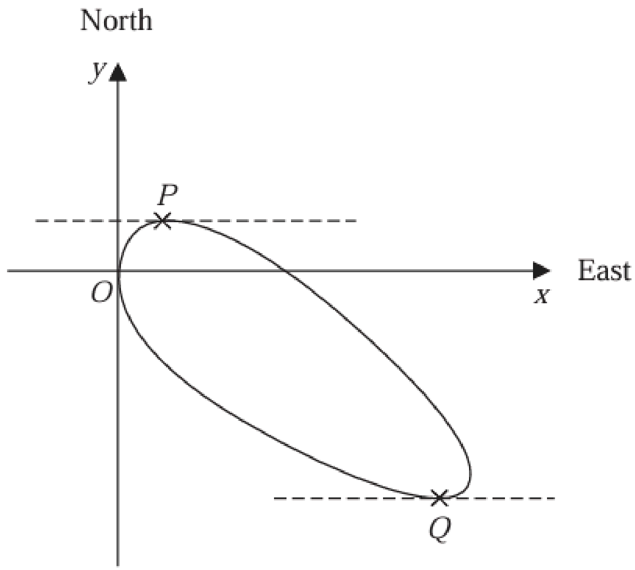

Figure 4

Figure 4 shows a sketch of the closed curve with equation

$$(x+y)^3 + 10y^2 = 108x$$

(a) Show that

$$\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2}$$

(5)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest north and furthest south of the origin O, as shown in Figure 4.

Using the result given in part (a),

(b) find how far the point Q is south of O. Give your answer to the nearest 100 m.

(4)

6.June 2021

6.

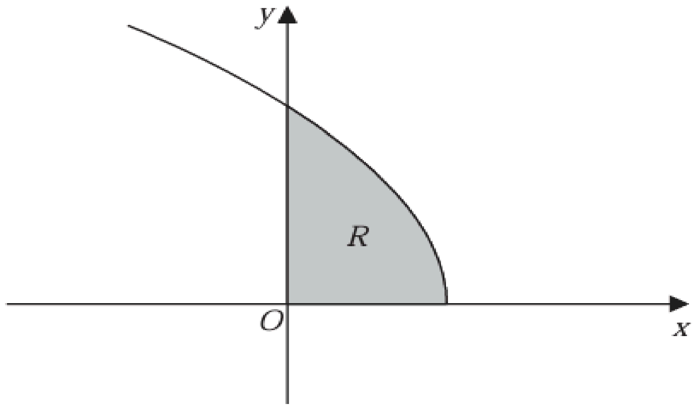

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = 4\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the y-axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{4}} 32\sin^2 t\cos t\,\mathrm{d}t$

(ii) Hence find, by algebraic integration, the exact value of the area of R.

(6)

(b) Show that all points on C satisfy $y = \sqrt{ax+b}$, where a and b are constants to be found.

(3)

The curve C has equation $y = \mathrm{f}(x)$ where f is the function

$$\mathrm{f}(x) = \sqrt{ax + b} \qquad -2 \leqslant x \leqslant 2$$

and a and b are the constants found in part (b).

(c) State the range of f.

(1)

7.Oct 2021

1. The curve C has equation

$$2x - 4y^2 + 3x^2 y = 4x^2 + 8$$

The point $P(3, 2)$ lies on C.

Find the equation of the normal to C at the point P, writing your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(7)

8.Oct 2021

3. $$g(x) = \frac{3x^3 + 8x^2 - 3x - 6}{x(x+3)} \equiv Ax + B + \frac{C}{x} + \frac{D}{x+3}$$

(a) Find the values of the constants A, B, C and D.

(5)

A curve has equation

$$y = g(x) \qquad x > 0$$

Using the answer to part (a),

(b) find $g'(x)$.

(2)

(c) Hence, explain why $g'(x) > 3$ for all values of x in the domain of g.

(1)

9.Oct 2021

5.

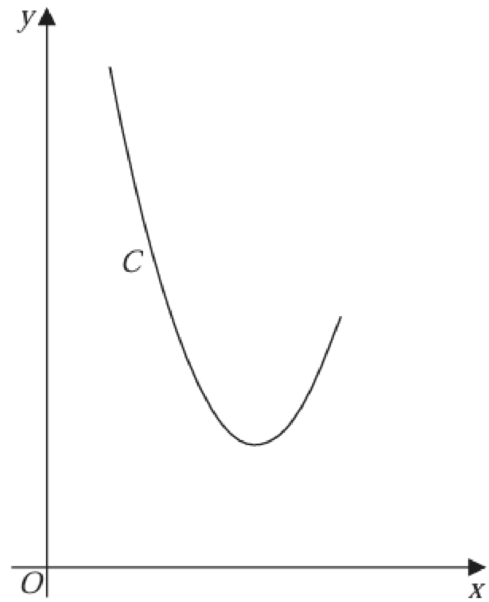

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 5 + 2\tan t \qquad y = 8\sec^2 t \qquad -\frac{\pi}{3} \leqslant t \leqslant \frac{\pi}{4}$$

(a) Use parametric differentiation to find the gradient of C at $x = 3$

(4)

The curve C has equation $y = \mathrm{f}(x)$, where f is a quadratic function.

(b) Find $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants to be found.

(3)

(c) Find the range of f.

(2)

10.Oct 2020

2. (a) Use the binomial expansion to expand

$$(4 - 5x)^{-\frac{1}{2}} \qquad |x| < \frac{4}{5}$$

in ascending powers of x, up to and including the term in x^2 giving each coefficient as a fully simplified fraction.

(4)

$$f(x) = \frac{2 + kx}{\sqrt{4 - 5x}} \qquad \text{where } k \text{ is a constant and } |x| < \frac{4}{5}$$

Given that the series expansion of $f(x)$, in ascending powers of x, is

$$1 + \frac{3}{10}x + mx^2 + \dots \qquad \text{where } m \text{ is a constant}$$

(b) find the value of k,

(2)

(c) find the value of m.

(2)