Welcome to the Consortium Math Diagnostic

Note to Teachers:

We are sharing with you a set of 11 possible questions or problems that you might use in this diagnostic. They represent different ideas and procedures within the world of mathematics. Each problem offers us an opportunity to learn about how our students think, strategize, reason and communicate their mathematical ideas. We recommend you use 4-5 questions that you believe will tell you the most important information about your students' thinking. You might use a week in the early school year where you use the first 10-15 minutes each day asking your students to work on one of these problems. We will have workshops in which we will learn together how to analyze the information we gather about our students. Please create a folder for each student so we can gather their work over the year to learn about their growth as mathematical thinkers. We also recommend you use YOUR STORY as the initial piece so your students can see that we are interested in getting to know them in every possible way.

Here are some questions we would like you to think about as you decide which problems you want to use:

- What would you like to learn about your students as mathematical thinkers?
- What would you like to learn about your students' understanding of mathematics?
- What would you like to learn about your students as conceptual thinkers?
- What would you like to learn about your students' understanding and ability to work with mathematical procedures?
- What would you like to learn about your students as mathematical problem solvers?

First Name: _____ Last Name:_____

Welcome to the Consortium Math Diagnostic

Note to Students:

This is not a test. The math teachers at your school want to learn as much as they can about you and all the students in your school so that your learning experiences in math class will be interesting and meaningful. You are going to be given a set of questions. You are going to work on them as best you can. Please don't worry if you are not sure about what to do. Share with us any ideas you have towards answering the question. Don't worry about being right or wrong. You will see that the teachers at your school are most interested in helping you learn how to think mathematically.

YOUR STORY

We want to know what you think about math. Here are two sets of questions.
Share with us as much information as you can so we can begin to get to know you.

a) Do you like mathematics? Why

b) What did you think of the math course you had last year? Was it easy or difficult? What did you like? Dislike? Are there any experiences that you had that helped you to understand some new idea in mathematics?

WHAT IS THE SOLUTION?

1. I am thinking of two numbers.

Here is some information about the numbers. **Can you figure out what the two numbers are?**

- Five times the smaller number minus two gives us the bigger number.
- Two times the smaller number plus sixteen gives us the bigger number.

Be sure to show all of your work. (*Graph paper is available at the back of this package if you would like to use it.)*

WHO IS CORRECT?

2. Taylor and Drake solved the following equation $4 - 2(x - 6) = 12 - 4x$

The presentation below shows their work :

Taylor's Solution	Drake's Solution
$4 - 2(x - 6) = 12 - 4x$ $2(x - 6) = 12 - 4x$ $2x - 12 = 12 - 4x$ $-4x \quad\quad -4x$ $-6x - 12 = 12$ $+12 \quad +12$ $-6x = 24$ $\frac{-6}{} \quad \frac{-6}{}$ $x = -4$	$4 - 2(x - 6) = 12 - 4x$ $4 - 2(x - 6) = 8x$ $4 - 2x + 12 = 8x$ $2x + 16 = 8x$ $-2x \quad\quad -2x$ $16 = 6x$ $\frac{6}{} \quad \frac{6}{}$ $2.67 = x$

What do you think about their work? Provide your observations. Do you notice any error in the student's thinking? If you disagree with both solutions, provide a written justification and solve it in the space provided below.

Your written observations:

YOUR solution if you disagree with Taylor and Drake's solution

$$4 - 2(x - 6) = 12 - 4x$$

FROM LEAST TO GREATEST

3. Compare these five (5) expressions and put them in order from *least to greatest*. ***Be sure to show all of your work.*** (*Graph paper is available at the back of this package.)*

$(\frac{1}{2})^3$	$(-1)^{25}$	-5^2	$4^2 \times 4^0 \times 4^{-4}$	$\frac{8^{16}}{8^{17}}$

THE PIGGY BANK

4. Abby and Bob both save their money in piggy banks at home. Abby and Bob's savings over time can each be described by a linear function. Abby's function has a larger ***y intercept*** than Bob's. Bob's function has a larger ***slope***. Describe and compare their savings over time. **What can you say *for sure*?** You can use graphs, equations, tables and text to support your reasoning. (*Graph paper is available at the back of this package.)*

THE BIG PARTY

5. You've rented Madison Square Garden to make a huge party for your friends and neighbors. The security guard lets your friends enter the Garden in groups every five minutes.

The first group to enter includes only one person.
The second group includes 3 people.
The third group includes 5 people.
The fourth group includes 7 people.
This pattern continues…

a. If 10 groups are admitted by the security guard, how many people would wind up at your party?

b. If 30 groups are admitted, how many people would wind up at your party?

c. If ***n*** number of groups are admitted, how many people would be at your party?

d. If 1,764 people are at your party, how many groups did the security guard allow to enter?

<u>Show all your work.</u> Explain your reasoning.

TABLE HOPPING

6. The school is having a big event in the gym and wants to use their rectangular tables. 6 people can sit together at 1 rectangular table. If 2 tables are placed together, 10 people can sit together. 14 people can sit if three tables are placed together. If the pattern continues….

a) How many people will could sit at four tables?
Draw a picture to show why your answer is correct.

b) How many people could sit if 30 tables were put together? Explain your thinking.

c) Would your pattern make it possible to seat 150 people without any empty seats? Justify your answer mathematically.

NUMBERS BETWEEN NUMBERS

7. How many numbers are there between ½ and ¼? Justify your answer as best you can.

How many numbers are there between any two numbers? Prove your answer is always true.

WHAT IS THE 147TH LETTER IN THIS PATTERN?

8. Look at the pattern below:

a,b,c,d,a,b,c,d,a,b,c,d,a,b,c,d,a,b,c,d,…

What would be the 147th letter in this pattern?

Show all your work and explain as best you can your thinking about this problem.

MR. O's ICE CREAM PARLOR

9. Mr. O's Ice Cream Parlor has just opened for business and offers 25 different flavors of ice cream. All the flavors started with an equal amount of ice cream. After three weeks Mr. O wanted to find out which of the following four flavors is most popular and which is least popular. He needs your help to figure this out. Here is the information you will need to help him:

- **3/5 of the chocolate ice cream remains in the container.**
- **Mr. O sold 1/4 of the vanilla ice cream.**
- **65% of the Cookies and Cream is still available.**
- **The strawberry ice cream still contains 0.80 of the original amount**

a) Which flavor is most popular? Justify your answer mathematically

b) Suppose after a busy week the following happened:

- **½ of the remaining chocolate ice cream was sold.**
- **25% of the vanilla ice cream was still available.**
- **Mr. O sold an additional 0.50 of the container of Cookies and Cream**
- **25% of the strawberry ice cream that remained was sold.**

Does your answer to, which flavor is the most popular change? Justify your answer mathematically.

THE STUDENT RACE

Here is some information about a race. Please read it carefully and determine the results of the race. There are six students in a bicycle race. Each student wore a different color. There were no ties.

1. Ashley beat the riders in orange and white.
2. The rider in green beat Dez, who beat the rider in silver and Ashley
3. The rider in yellow beat Sheena and Ashley.
4. The rider in blue beat Jessica, who beat the rider in silver.
5. The rider in yellow beat the rider in blue who beat the rider in white.
6. Maxine lost to Dez, but beat the rider in orange.
7. Hava didn't wear yellow, and Sheena didn't wear orange.
8. The rider in orange beat the rider in silver but lost to the rider in white.
9. Sheena lost to Jessica.

Arrange the names in the order of finish. Name which color each person wore

NAME. COLOR

1. _____ _____

2. _____ _____

3. _____ _____

4. _____ _____

5. _____ _____

6. _____ _____

Show your work and thinking process. Prove that your solutions are correct.

Whose Dog Is That?

Anaya, Bertha, Candy, Dodi, and Eddie showed up with their puppies at the Little Red School House for Dogs. During the morning break, they found that each of their pets had the same name as the roommate of one of the other owners.

In particular they noticed that

- Anaya's dog is named Georgie.
- Candy's dog is named Jamal.
- Eddie's dog is named Isabel.
- Dodi's dog is named Frank.
- Anaya's roommate is also named Frank.
- Bertha's roommate has the same name as Georgie's dog.
- Hani and his roommate, Candy, have the best-behaved dog.

What are the names of each owner's roommate and dog?

If you think there is more than one solution state your solutions how you found them. If you think there is only one solution, state your solution and explain why there are no others.

Garden Design: The Argument

Jennifer and Raul are each planning to plant a garden, but they are worried about animals getting in, so they want to fence the garden. The fencing they plan to use is available only in 5-foot sections, which cannot be broken, bent or cut.

Jennifer has 120 feet of fencing in 5-foot sections. Raul has 130 feet of fencing in 5-foot sections. They each want to use all of their 5-foot sections and need their gardens to be shaped as rectangles

.

Jennifer says that because Raul has more fencing than she does, he will be guaranteed to enclose more garden area than she. Raul says that who encloses more area depends on the dimensions of their designs.

PART I – WHO IS RIGHT?

A) Draw possibilities for Jennifer's design. Please be sure to include the measurements of each side and the area of each rectangle.

B) Draw possibilities for Raul's design. Please be sure to include the measurements of each side and the area of each rectangle.

C) Who is right? Support your reasoning using pictures, words and/or numbers.

PART II – NEW DILEMMAS

A. D'Andre is planning to buy the same fencing material in 5-foot sections for his rectangular garden. Describe the strategy he should use to design a garden with the greatest possible area.

B. Holly does not care if the fencing is rectangular or not. For her garden, she just wants to enclose the greatest possible area. If she has 150 feet of fencing in 5-foot sections, create a design that maximizes the enclosed garden area. Explain your thinking.

Angle Measurements of Polygons

1) Three students were asked to figure out the sum of the angle measurements of an octagon. Here is some of the discussion they had.

Maya: I know that the sum of the angles in a triangle is 180 degrees.

Jose: I think the sum of the angles in a 4 sided polygon (quadrilateral) is 360 degrees.

Kasim: Ms. Torres (our math teacher) told me that a pentagon (a 5 sided polygon)
Has five angles whose measurements add up to 540 degrees.

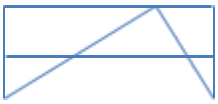
They started playing with this information to try to figure out the sum of the angle measurements of an octagon. Try to help them figure this out. Show all of your ideas.

2) Now try to come up with a strategy of figuring out the sum of the angle measurements of a polygon with ***any*** amount of sides?

Triangle Inside a Rectangular Box

A famous mathematician, Paul Lockhart posed the following problem. Think about it and share all your ideas.

"If I'm in the mood to think about shapes – and I often am – I might imagine a triangle inside a box.



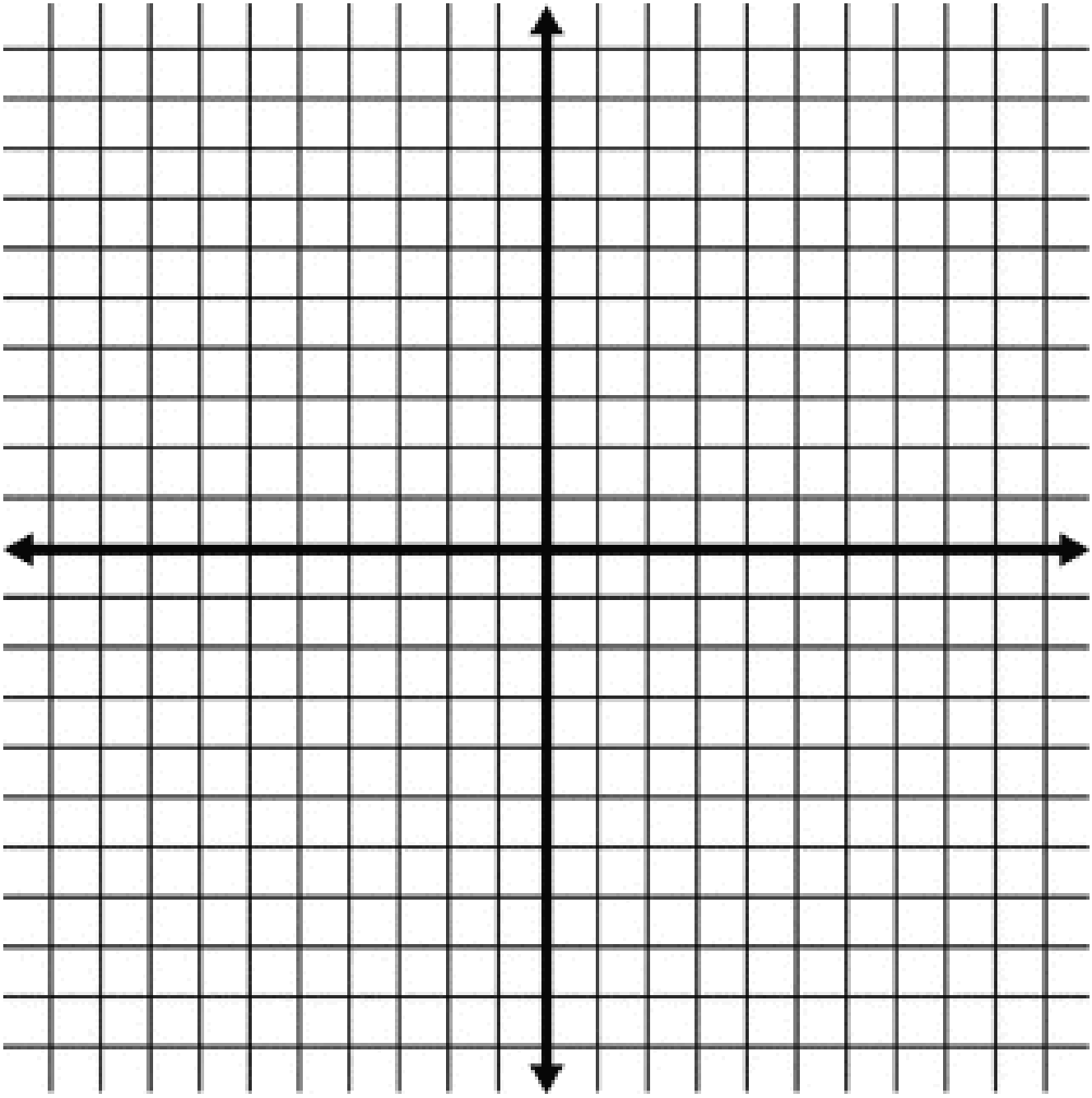
I wonder how much of the box the triangle takes up? Two-thirds maybe? The important thing to understand is that I'm not talking about this *drawing* of a triangle in a box. Nor am I talking about some metal triangle forming part of a girder system for a bridge. There's no ulterior practical purpose here. I'm just *playing*."

So play… how much of the rectangle does the triangle take up? Please share all your thoughts, ideas and designs.

Note To Teacher: If you would like to extend the problem for a student you might ask.
If you inscribed another triangle in the rectangle where the vertex touches the upper length at a different point would your answer to the question change? Why or why not?

Name: _____

Problem # _____



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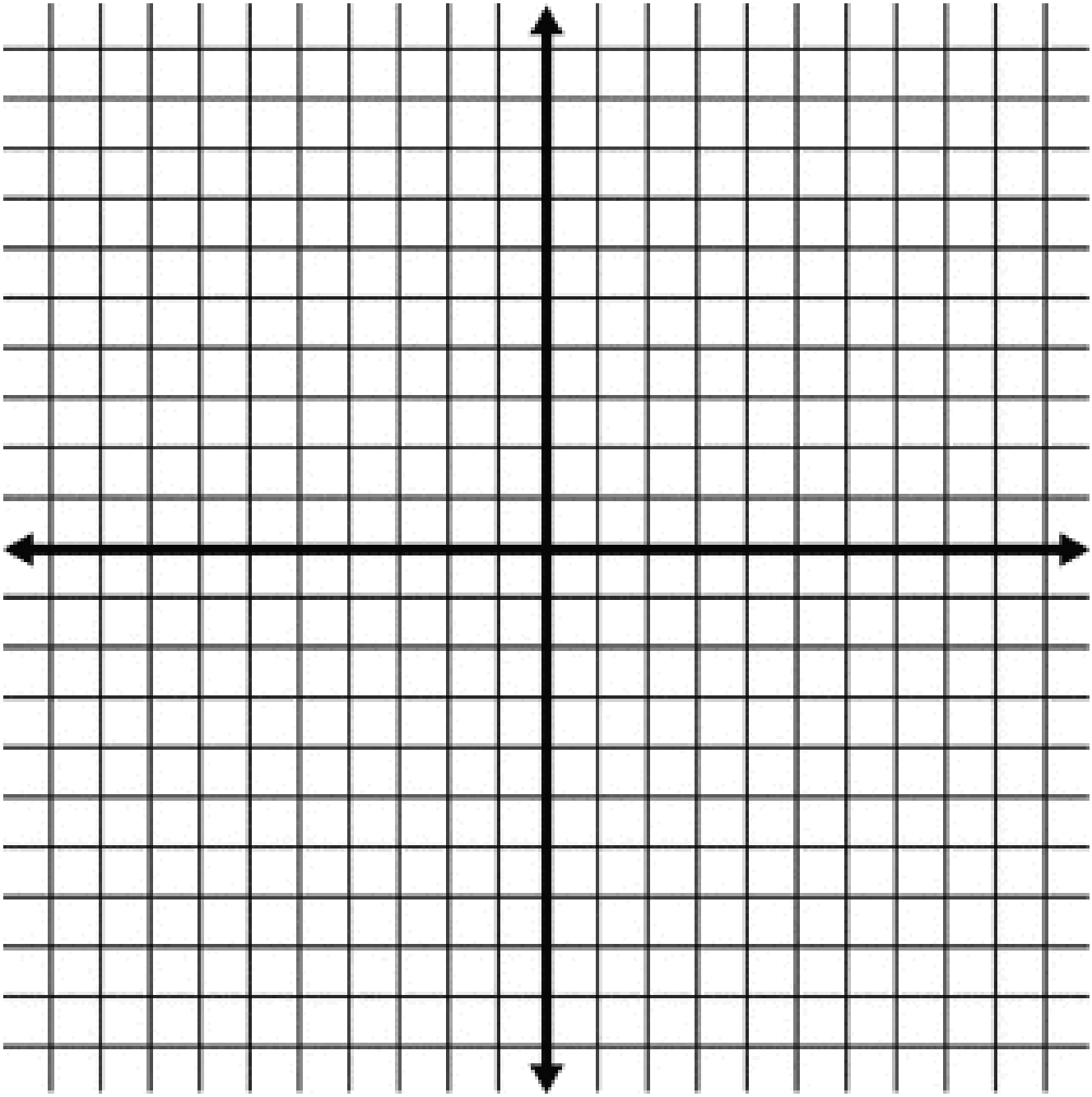

Name: _____

Problem # _____