

Problem 1: Pierre's Paper

Pierre folded a square piece of paper vertically to make two rectangles. Each rectangle had a perimeter of 39 inches. How long is each side of the original square? What is the area of the original square? What is the area of one of the rectangles?

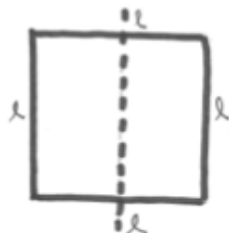
This problem calls on student knowledge of the properties of squares and rectangles as well as their knowledge of area and perimeter. Understanding the relationships between the lengths of the rectangle's sides is the key to solving it.

If students are having difficulty moving forward, the following questions may help them:

- How does knowing that this figure is a square help us know about the dimensions of the rectangle? How are the dimensions of the rectangle related to each other?
- What is the unit we are counting?
- Think of the rectangle's shorter side (or longer side) as 1 unit.

Below, Solution A solves for the longer side of the rectangle and uses a more abstract representation of the thinking, while Solution B solves for the shorter side of the rectangle.

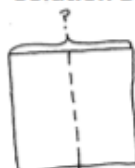
Solution A



$$\begin{aligned}\text{Each side length} &= l \\ l + l + \frac{1}{2}l + \frac{1}{2}l &= 39'' \\ 3l &= 39''\end{aligned}$$

$$\begin{aligned}\text{Each side of the square is } 13''. \\ \text{Square's area is } 13 \times 13 &= 169 \text{ in}^2. \\ \text{The area of the rectangle is} \\ 13 \times 6\frac{1}{2} &= (13 \times 6) + (13 \times \frac{1}{2}) = \\ 78 + 6.5 &= 84.5 \text{ in}^2.\end{aligned}$$

Solution B



$$\text{Rectangle } p = 39 \text{ in}$$

$$\begin{array}{l} \text{Rectangle length } \square \times 2 \\ \text{rectangle width } \square \times 2 \end{array} \left. \vphantom{\begin{array}{l} \square \times 2 \\ \square \times 2 \end{array}} \right\} 39$$

$$\begin{aligned}6 \text{ units} &= 39 \\ 1 \text{ unit} &= 6\frac{1}{2}\end{aligned}$$

$$\begin{array}{l} \text{Square's sides } 6\frac{1}{2} + 6\frac{1}{2} = 13 \\ \text{Square's area } 13 \times 13 = 169 \\ \text{Rectangle area } 169 \div 2 = 84.5 \end{array}$$

The square's sides are 13 inches long.

The area of the square is 169 in².

The area of the rectangle is 84.5 in².

Problem 2: Shopping with Elise

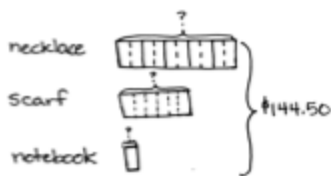
Elise saved \$184. She bought a scarf, a necklace, and a notebook. After her purchases, she still had \$39.50. The scarf cost three-fifths the cost of the necklace, and the notebook was one-sixth as much as the scarf. What was the cost of each item? How much more did the necklace cost than the notebook?

This problem is fairly straightforward mathematically. However, students will need to find a common unit for all three items in order to determine the cost of the notebook. Once this is established, the costs of the other items may be found easily. Students may attempt to find a solution through fraction multiplication. This approach may stall when trying to determine the fraction of the money spent on the necklace. The following may provide scaffolding for students experiencing difficulty:

- Which item's tape should be the longest? The shortest?
- How can we make these units the same size?
- Begin with the notebook as 1 unit. If the notebook is 1 sixth the cost of the scarf, then how many times as much is the scarf's cost to the cost of the notebook?

Both solutions below begin by finding the amount spent on the three items. While both use the cost of the notebook as 1 unit, Solution A begins with the necklace and uses the fraction information to subdivide the other tapes. Solution B uses a multiplicative approach thinking of the scarf's cost as 6 times as much as the cost of the notebook.

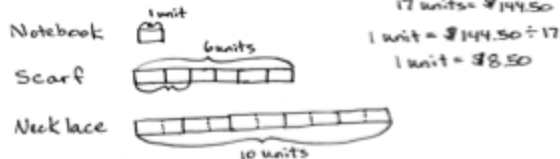
Solution A



$$\begin{aligned} 17 \text{ units} &= 144.50 \\ 1 \text{ unit} &= 8.50 \\ \text{necklace } \$8.50 \times 10 &= \$85 \\ \text{scarf } \$8.50 \times 6 &= \$51 \\ \text{notebook } \$8.50 \times 1 &= \$8.50 \end{aligned}$$

The necklace cost \$85, The scarf costs \$51, The notebook costs \$8.50

Solution B



$$\begin{aligned} \text{Notebook costs } &\$8.50 \\ \text{The scarf costs } &\$8.50 \times 6 = \$51.00 \\ \text{The necklace costs } &\$8.50 \times 10 = \$85.00 \text{ or } \\ &\$51.00 \div 3 = \$17.00 \quad \$17.00 \times 5 = \$85.00 \\ \$85 + \$51 + \$8.50 &= \$144.50 \\ \$144.50 + 39.50 &= \$184.00 \end{aligned}$$

Problem 3: The Hewitt's Carpet

The Hewitt family is buying carpet for two rooms. The dining room is a square that measures 12 feet on each side. The den is 9 yards by 5 yards. Mrs. Hewitt has budgeted \$2,650 for carpeting both rooms. The green carpet she is considering costs \$42.75 per square yard, and the brown carpet's price is \$4.95 per square foot. What are the ways she can carpet the rooms and stay within her budget?

While the calculations for solving this problem are simple multiplication and addition, the path to finding the appropriate numbers on which to operate requires a high degree of organization. Students must attend not only to finding the various combinations that are possible, but they must also attend to the units in which the areas and prices are given.

Students may choose to use only one unit of measure for the areas and prices, or they may use a combination. The following scaffolds may support struggling students:

- Are the areas expressed in the same unit? Can we use them as they are or must we convert?
- How might we organize the information so that we can keep track of our thinking?
- What are the combinations of carpet that Mrs. Hewitt can choose? Predict which combination will be the most expensive? Which the least expensive? How do you know? How can that prediction help you to move forward?
- Consider the prices per square yard and per square foot. Which of these carpets is the more expensive? How do you know? How might this information help you to organize your thoughts?

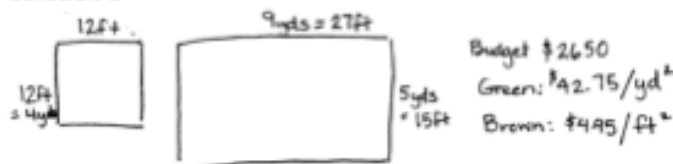
Both of the solutions to the right show good organization of the calculations used to solve. Solution A converts the carpet prices to match the area units of the rooms. Solution B converts the dimensions of the rooms to match the units of the prices.

Solution A

Green \$42.75/yd ² 4.75/ft ²	Brown \$44.55/yd ² \$4.95/ft ²	
Dining 144 ft ² x \$4.75 = \$684. Green	Den 45 yd ² x \$42.75 = \$1,923.75 Green	Total \$2,607.75
Green \$684.00	Brown 45 yd ² x \$44.55 = \$2,004.75	\$2,688.75
Brown 144 ft ² x \$4.95 = \$712.80	Green \$1,923.75	\$2,636.55
Brown \$712.80	Brown 2,004.75	\$2,717.55

Mrs. Hewitt has 2 choices: both rooms in green, or the den in green and the dining room in brown.

Solution B



$$\text{DR Green} \rightarrow 16\text{yd}^2 \times \$42.75 = \$684.00$$

$$\text{Brown} \rightarrow 144\text{ft}^2 \times \$4.95 = \$712.80$$

$$\text{Den Green} \rightarrow 45\text{yd}^2 \times \$42.75 = \$1,923.75$$

$$\text{Brown} \rightarrow 405\text{ft}^2 \times \$4.95 = \$2,004.75$$

$$\text{Den } \boxed{A} \$1,923.75 \quad \boxed{B} \$1,923.75 \quad \boxed{C} \$2,004.75 \quad \boxed{D} \$2,004.75$$

$$\text{Dining Rm } \boxed{A} \$684.00 \quad \boxed{B} \$712.80 \quad \boxed{C} \$712.80 \quad \boxed{D} \$684.00$$

Mrs. Hewitt can have both rooms green or the den green + the dining room brown.

Problem 4: AAA Taxi

AAA Taxi charges \$1.75 for the first mile and \$1.05 for each additional mile. How far could Mrs. Leslie travel for \$20 if she tips the cab driver \$2.50?

Students encounter a part-part-whole problem with varying unit size in the AAA Taxi Problem. They must first consider the cost of the first mile and tip, and then determine how many groups of \$1.05 can be made from the remaining \$15.75.

To scaffold, consider the following:

- Will all of the \$20 be used to pay for the mileage? Why not?
- Do all the miles cost the same? How do we account for that in our model?
- How would you solve this if all the miles cost the same? What if the tip was the same as the cost for the miles?

Solution A begins by counting on from the first mile. Solution B chooses to represent the problem with a tape diagram and divides to find how many units with a value of \$1.05 there are once the sum of the tip and first mile are subtracted from the \$20.

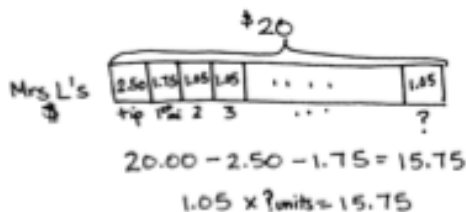
Solution A

mile

- 1 \$1.75 + 2.50 (tip) = \$4.25
- 2 \$4.25 + 1.05 = \$5.30
- 3 \$5.30 + 1.05 = \$6.35
- 6 \$6.35 + 3.15 = \$9.50
- 9 \$9.50 + 3.15 = \$12.65
- 12 \$12.65 + 3.15 = \$15.80
- 15 \$15.80 + 3.15 = \$18.95
- 16 \$18.95 + 1.05 = \$20.00

Mrs. Leslie can travel 16 miles for \$20.

Solution B



$$\begin{array}{r} 15 \\ 105 \overline{) 1575} \\ \underline{-105} \\ 525 \\ \underline{-525} \\ 0 \end{array}$$

Mrs. Leslie can go 16 miles.

Problem 5: Pumpkins and Squash

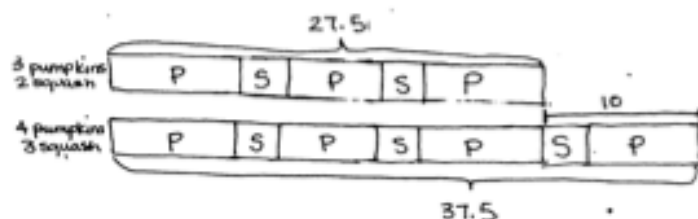
Three pumpkins and two squash weigh 27.5 pounds. Four pumpkins and three squash weigh 37.5 pounds. Each pumpkin weighs the same as the other pumpkins, and each squash weighs the same as the other squash. How much does each pumpkin weigh? How much does each squash weigh?

This problem is a departure from the routine problems in most of Grade 5 in that students must unitize two different variables (1 pumpkin and 1 squash) as a single unit. Once the difference is found between the quantities, students have several avenues for finding the weights of the individual pumpkin and squash.

- Draw the tapes to represent the weights for the two situations. Which tape is longer? How much longer?
- How many more pumpkins are in the second tape? How many more squash?
- Outline the difference with a red pen. Can you find this same combination in the rest of the tape? How many can you find?

Both solutions below use tape diagrams to show that the difference between the two known facts is a combination of one pumpkin and one squash. Next, they reason that the sum of the weights of a pumpkin and squash is 10 pounds. From there, they can see two of those pumpkin and squash units in relationship to the 27.5 pound group. It is clear then that the weight of the pumpkin has to be 7.5 pounds.

Solution A



$$1 \text{ unit} = \boxed{P} \boxed{S} = 10 \text{ lbs}$$

$$2 \text{ units} + \boxed{P} = 27.5 \text{ lbs}$$

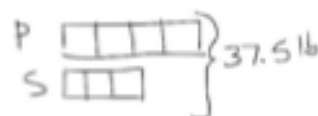
$$20 + \boxed{P} = 27.5 \text{ lbs}$$

$$1 \text{ pumpkin} = 7.5 \text{ lbs}$$

$$10 - 7.5 = 2.5$$

One pumpkin weighs
7.5 pounds +
one squash weighs
2.5 pounds.

Solution B



Problem 6: Toy Cars and Trucks

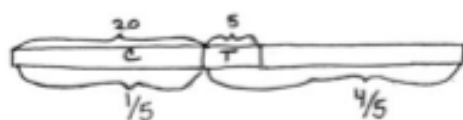
Henry had 20 convertibles and 5 trucks in his miniature car collection. After Henry's aunt bought him some more miniature trucks, Henry found that one-fifth of his collection consisted of convertibles. How many trucks did his aunt buy?

This problem requires students to process a before-and-after scenario. The larger quantity in the *before* situation becomes the smaller quantity in the *after* situation. This change in fractional relationship may be depicted in various ways. Students should be careful to model only 5 fifths in the *after* model—1 fifth for the convertibles and 4 fifths for the trucks. Use the following to scaffold student understanding:

- Draw Henry's convertibles and trucks before his aunt gave him more trucks. Draw the convertibles and trucks after his aunt gave him more.
- What amount stayed the same?
- Which is more, the cars or trucks? (Ask for both before and after. Have students simply draw the bars longer and shorter.)
- Refer to the convertibles tape in the after model. Ask, "If this is 1 fifth, what is the whole?"

Solution A combines the before and after models into one tape. The numbering on the top represents the *before* while the numbering below represents the *after*. Solution B also uses fraction division to determine the whole. Solution C uses a unit approach, with the number of trucks in the beginning as 1 unit.

Solution A



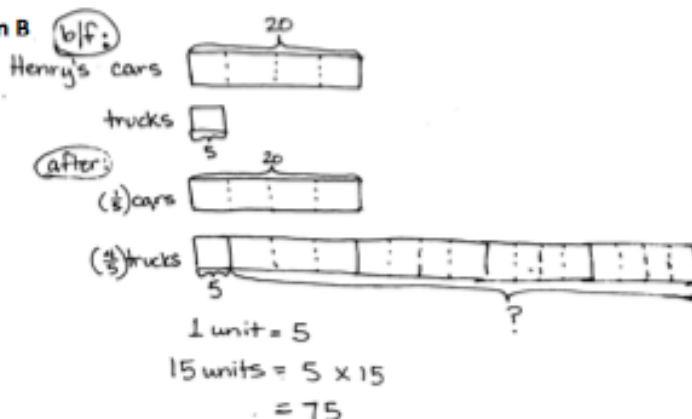
$$\frac{1}{5} \times y = 20$$

$$20 \div \frac{1}{5} = y$$

$$y = 100$$

20 is $\frac{1}{5}$ of 100.
 $100 - 20$ convertibles = 80
 $80 - 5$ trucks he already had = 75
 His aunt bought 75 trucks.

Solution B



Henry's aunt bought him 75 trucks.

Solution C



$3 \times 20 + 15 = 75$
 His aunt bought 75 trucks.
 That's a lot of trucks!

Problem 7: Pairs of Scouts:

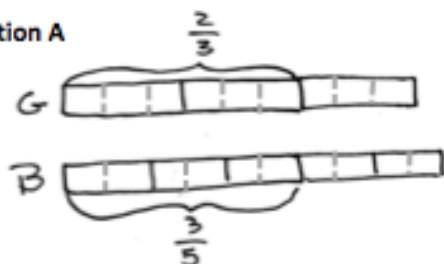
Some girls in a Girl Scout troop are pairing up with some boys in a Boy Scout troop to practice square dancing. Two-thirds of the girls are paired with three-fifths of the boys. What fraction of the scouts is square dancing?

This problem challenges students to consider what they know about fraction equivalence. The key to this problem lies in recognizing the need for equal numbers of units. That is, equal numerators must be found! Once students can visualize that 6 of the girls' units are the same as 6 of the boys' units, a fraction of the total number of units can be found. Scaffold with the following:

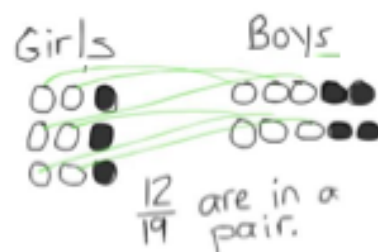
- We know the same number of girls as boys are dancing. Are these units the same size? How can we make them the same size?
- How can 2 units be the same amount as 3 units? Only if one unit is larger than the other. For example, 2 yards equals 6 feet if we consider 1 larger unit and a smaller unit.
- Make sure that once students make 6 units in each tape for the dancing scouts, they also subdivide the remaining units in each bar. This will create the 19 total units.

Solution A uses a tape diagram to model the equal amounts and then decompose to make the boy and girls units equal. Solution B uses an array approach to match up girls and boys.

Solution A



Solution B



I know: 2 girl units = 3 boy units

I can make these units the same size! $\frac{2}{3} = \frac{6}{9}$ and $\frac{3}{5} = \frac{6}{10}$

So, now... 6 girl units = 6 boy units

There are 9 of these units for the girls and 10 of these units for the boys.

$$\begin{aligned}\text{Fraction dancing} &= \frac{\# \text{ units dancing}}{\text{total units}} \\ &= \frac{6+6}{9+10} \\ &= \frac{12}{19} \text{ dancing}\end{aligned}$$

$\frac{12}{19}$ of the scouts are dancing.

Problem 8: Sandra's Measuring Cups

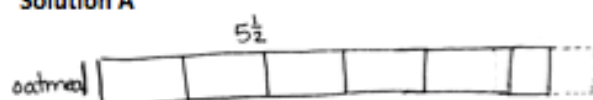
Sandra is making cookies that require $5\frac{1}{2}$ cups of oatmeal. She has only two measuring cups: a one-half cup and a three-fourths cup. What is the smallest number of scoops that she could make in order to get $5\frac{1}{2}$ cups?

Recognizing that using a larger unit will require fewer scoops is the beginning of understanding this problem. Students may try to name the total using all halves or all fourths, but will find that neither measure can be used exclusively. Using the larger measure first to scoop as much as possible, then moving to scoop the remainder with the smaller cup is the more efficient method of solving. To scaffold, ask the following questions:

- Which measuring cup is larger? How does knowing which is larger help you?
- Predict which measuring cup will do the job more quickly? How do you know?
- How many scoops will it take using just the half-cup measure? How many if only the larger cup is used? Is it possible to scoop all the oatmeal and fill the three-fourths cup every time?

All three solutions pictured below use the strategy of beginning with the larger cup measure. However, Solution A uses a unitary approach, decomposing the fourths into a multiple of 3 and a multiple of 2. Solution B counts on by three-fourths and then by halves. Solution C works at the numerical level to guess and check.

Solution A



11 halves in $5\frac{1}{2}$

22 fourths in $5\frac{1}{2}$

$\times$ all $\frac{1}{2}$ c = 11 scoops

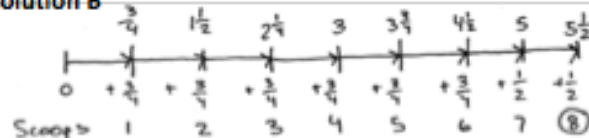
$\times$ all $\frac{3}{4}$ c = 7 scoops w/ $\frac{1}{4}$ left

22 fourths = 18 fourths + 4 fourths

$(6 \times 3 \text{ fourths}) + (2 \times 2 \text{ fourths})$

8 scoops (6 with $\frac{3}{4}$ c and 2 with $\frac{1}{2}$ c) will be the fewest.

Solution B



The least number of scoops is 8.

Solution C

$$\frac{3}{4} > \frac{1}{2}$$

$$5\frac{1}{2} = \frac{11}{2} = \frac{22}{4}$$

$$7 \times \frac{3}{4} = \frac{21}{4} \text{ but only } \frac{1}{4} \text{ left}$$

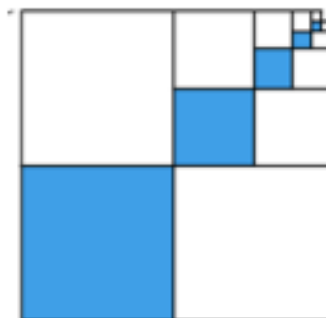
$$6 \times \frac{3}{4} = \frac{18}{4} \text{ with } \frac{4}{4} \text{ or } \frac{2}{2} \text{ left}$$

So 6 $\frac{3}{4}$ -scoops and 2 $\frac{1}{2}$ -scoops

8 total scoops minimum

Problem 9: Blue Squares

The dimensions of each successive blue square pictured to the right are half that of the previous blue square. The lower left blue square measures 6 inches by 6 inches.



- Find the area of the shaded part.
- Find the total area of the shaded and unshaded parts.
- What fraction of the figure is shaded?

There are multiple ways to visualize this graphic, each leading to a different approach to solving. Students may see that there are 3 identical sets of graduated squares. Out of these 3 identical sets, only 1 set is shaded.

Students may also do the work to find the fraction of the whole that the smallest shaded square represents and use an additive approach to finding the shaded area. The shaded area might then be used to find the total area. In contrast, the fraction that is shaded might be used in conjunction with the total area to name the area of the shaded parts. Scaffolds could include the following:

- Can you find the shaded area of just the first three squares (or L's)?
- Cut the graphic apart into separate L's or separate squares. What can you say about the fraction that is shaded in each one?
- How long is the side of each shaded square?
- What if the little square wasn't missing? What would be the area of the whole square? What part of that whole is missing?

Solution A uses the additive approach mentioned above to find the shaded area, which is multiplied by 3 to find the total. Solution B works backwards to name the fraction that is shaded, then finds the total area by using subtraction from a 12 by 12 square's area. These two pieces of information are then used to find the area of the shaded region in square inches.

a) Solution A

$$\begin{aligned} \text{Shaded Area: } & (6 \times 6) + (3 \times 3) + (1\frac{1}{2} \times 1\frac{1}{2}) + (\frac{3}{4} \times \frac{3}{4}) + (\frac{3}{8} \times \frac{3}{8}) \\ & = 36 + 9 + 2\frac{1}{4} + \frac{9}{16} + \frac{9}{64} \\ & = 47 + \frac{16}{64} + \frac{36}{64} + \frac{9}{64} \\ & = 47 + \frac{61}{64} \\ & = 47\frac{61}{64} \end{aligned}$$

$$\begin{aligned} \text{b) Total Area: } & 47\frac{61}{64} \times 3 \\ & = 141\frac{183}{64} \\ & = 141 + 2\frac{55}{64} \\ & = 143\frac{55}{64} \end{aligned}$$

$$\begin{aligned} \text{c) Fraction shaded: } & 1 \text{ out of } 3 = \frac{1}{3} \\ & \text{sets of squares} \\ \text{Shaded area is } & 47\frac{61}{64} \text{ in}^2 \\ \text{Total area is } & 143\frac{55}{64} \text{ in}^2 \\ \text{Fraction shaded is } & \frac{1}{3}. \end{aligned}$$

Solution B

This is easier if you do the fraction first. There are 3 sets of graduated squares. 1 out of 3 sets is shaded.

$$\text{c) Fraction shaded} = \frac{1}{3}$$

$$\begin{aligned} \text{b) Total area} & = (12 \text{ in} \times 12 \text{ in}) - (\frac{3}{8} \text{ in} \times \frac{3}{8} \text{ in}) \\ & = 144 \text{ in}^2 - \frac{9}{64} \text{ in}^2 \\ & = 143\frac{55}{64} \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \text{a) area shaded} & = \frac{1}{3} \times 143\frac{55}{64} \text{ in}^2 \\ & = 47\frac{2}{3} \text{ in}^2 + \frac{55}{192} \text{ in}^2 \\ & = 47\frac{128}{192} \text{ in}^2 + \frac{55}{192} \text{ in}^2 \\ & = 47\frac{183}{192} \text{ in}^2 \\ & = 47\frac{61}{64} \text{ in}^2 \end{aligned}$$