UNIT 4 PROPAGATION OF MECHANICAL WAVES

Key unit competence: To evaluate the propagation of mechanical waves.

INTRODUCTORY ACTIVITY

Photo of a boy, girl, window with wire mesh and a wall painted white acting as a screen

The above picture is a set that was done by year two student teachers to study the behavior of light as it passes through a narrow opening.

Study it carefully and answer the following questions.

a) The student A behind the window torched the torch and light passed through the wire mesh that is fixed on the window. What do you think is the nature of images observed on the screen by student B?

b) Explain what causes the nature of the images observed on the screen.

c) Assuming Student A used two torches giving light of different colors of light, would images on the screen be identical as observed in (a) above.

d) Now, the wire mesh in the window is replaced with another one with big holes (slits are widened). Explain what this change will have on the images formed on the screen.

e) Explain how this experiment is significant in real life.

4.1 Interference of waves.

ACTIVITY 4.1

Among other examples of waves, water waves (ripples) can be analyzed in real life by applying any disturbance to surface of water. In fact, we usually realize these waves due to different disturbances on water bodies and they include, wind, aquatic animals in water, boats that sail on top of water and others.

Basing on the observation made above,

a. Fill the basin with water and leave it to settle.
b. Apply a single disturbance at the center and note down your observations.
c. Apply disturbances on different points on the surface and note down your observations.
d. Distinguish the two cases (When there was a single disturbance and multiple disturbance)
e. Explain the scientific phenomenon that explain what happened in (c) above.
f. Discuss other scenarios where similar phenomenon stated in (e) above occur.

4.1.1 Coherent sources

Coherent sources are those which emit light waves of the same wavelength or frequency which are always in phase with each other or have a constant phase difference. Two coherent and monochromatic sources can together produce the phenomenon of interference.

When light passes through a slit with a size that is close to the light's wavelength, the light will **diffract**, or **spread out** in waves.

Interference is a phenomenon in which two waves superpose(meet) to form a resultant wave of greater, lower, or the same amplitude

Young's method for producing two coherent light sources involves illuminating a pair of slits with a single source. Another arrangement for producing an interference pattern with a single light source is known as ***Lloyd's mirror***.

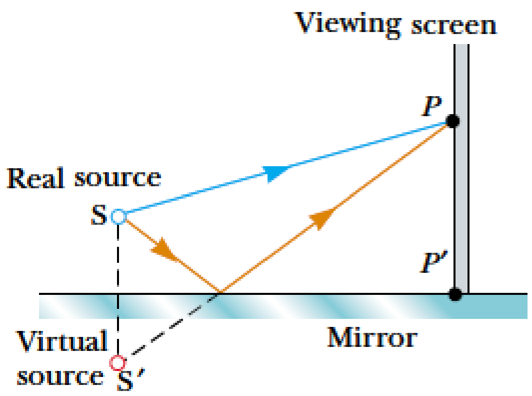


Fig.4. 1 Lloyd's mirror.

A point light source is placed at point S close to a mirror, and a viewing screen is positioned some distance away and perpendicular to the mirror. Light waves can reach point P on the screen either directly from S to P or by the path involving reflection from the mirror.

An interference pattern is produced at point P on the screen as a result of the combination of the direct ray (blue) and the reflected ray (brown). The reflected ray undergoes a phase change of 180°.

In order to observe interference in light waves, the following conditions must be met:

The sources must be coherent—that is, they must maintain a constant phase with respect to each other.
The sources should be monochromatic—that is, of a single wavelength.
The interfering waves Must Obey the Principal of superposition.

As an example, single-frequency sound waves emitted by two side-by-side loudspeakers driven by a single amplifier can interfere with each other because the two speakers are coherent—that is, they respond to the amplifier in the same way at the same time.

If two light bulbs are placed side by side, no interference effects are observed because the light waves from one bulb are emitted independently of those from the other bulb. The emissions from the two light bulbs do not maintain a constant phase relationship with each other over time. Light waves from an ordinary source such as a light bulb undergo random phase changes in time intervals less than a nanosecond. Such light sources are said to be incoherent.

When light passes through two or slits, the waves from one slit will **interfere** with the waves from the other:

Constructive interference occurs when two crests or two troughs meet forming a wave with a larger crest or lower trough.
Destructive interference occurs when a crest meets a trough cancelling each other to produce a smaller wave or no wave at all.

4.1.2 Principle of superposition

The Principle Of Superposition states that when two or more waves meet at a point, the resultant displacement at that point is the vector sum of the individual displacement of each wave

Consider the displacement $y_1 = a\sin(\omega t - \Phi)$ of a progressive sinusoidal wave at time t and at a distance x from the origin and moving to right.
Consider also the displacement y_2 of an identical wave travelling in opposite direction given by

$$y_2 = a\sin(\omega t + \Phi)$$

By principal of superposition, the resultant Y is got from $Y = y_1 + y_2$

$$Y = a\sin(\omega t - \Phi) + a\sin(\omega t + \Phi)$$

$$Y = 2a\sin(\omega t)\cos\Phi$$

$$but\ \Phi = kx$$

$$Y = 2a\sin(\omega t)\cos kx$$

The only variable part of equation is $\sin\omega t$ This means that the amplitude of the resultant amplitude A is given by equation $A = 2a\cos kx$

4.1.3 Young's double-slit experiment

In Young's experiment, two very narrow parallel slits, separated by a distance d, are cut into a thin sheet of metal. Monochromatic light, from a distant light source, passes through the slits and eventually hits a screen a comparatively large distance D from the slits. The experimental setup is sketched in Fig.4.2

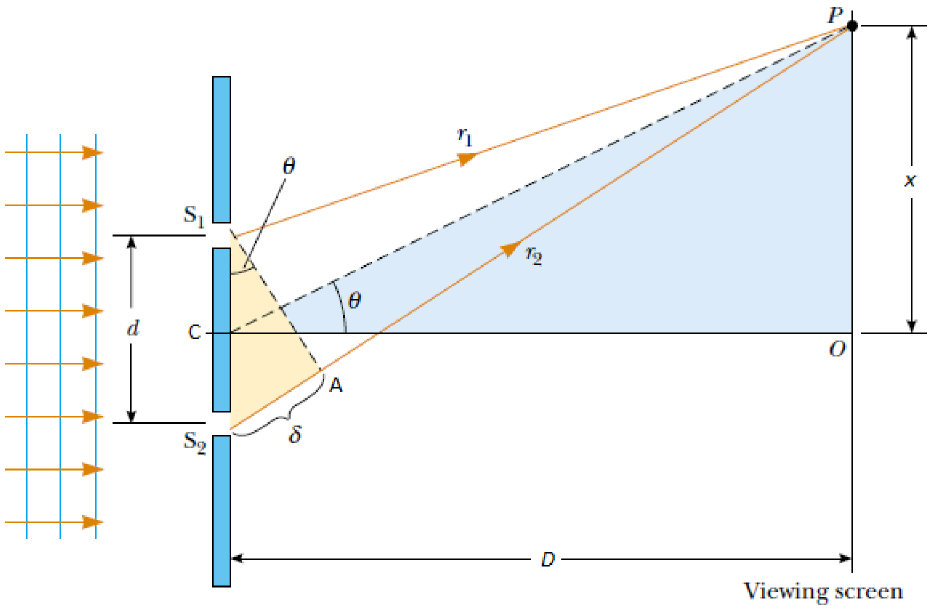


Fig.4. 2 Measurement Ray of wavelength from Young's experiment

According to Huygens 'principle, each slit radiates spherical light waves. The light waves emanating from each slit are superposed on the screen. If the waves are 180^0 out of phase then **destructive interference** occurs, resulting in a dark patch on the screen. On the other hand, if the waves are completely in phase then **constructive interference** occurs, resulting in a light patch on the screen.

The light from S_1 and S_2 produces on a viewing screen a visible pattern of bright and dark parallel bands called **fringes**. hen the light from S_1 and that from S_2 both arrive at a point on the screen such that constructive interference occurs at that location, a bright fringe appears. When the light from the two slits combines destructively at any location on the screen, a dark fringe result.

For any given point on the other side of the incident wave, we have the following geometry:

If we assume that r_1 and r_2 are parallel, which is approximately true if L is much greater than d, then the difference in length δ of the two rays from the slits to their point of intersection is

$$S_2P - S_1P = \lambda = d\sin\theta$$

If we know the distance between adjacent slit centers or grating space (d) and measure θ , λ can be calculated. In fig.4.3 S_1 and S_2 are the two slits; O is the position of the central bright band. The path difference between waves reaching O from S_1 and S_2 is zero, i.e. $S_1O = S_2O$, they therefore arrive in phase and so there is a bright fringe at O. P is the position of the first bright band and C is the midpoint of S_1S_2. S_2A must be one wavelength longer than S_1P. If S_1A is drawn perpendicular to S_2P then S_2A will be approximately equal to wave length.

If P is near O then S_1P and S_2P are approximately parallel to CP so the triangles S_1S_2P and CAO will be similar.

From triangle S_1S_2P: $\sin\theta = \frac{S_1P}{S_2S_1} = \frac{n\lambda}{d}$

From triangle OCA: $\tan\theta = \frac{AO}{CO} = \frac{x_n}{D}$ where x_n is the distance of the n^{th} fringe from the centre O.

Now the angle θ is very small so $\tan\theta = \sin\theta$ from trigonometry.

Then $\frac{n\lambda}{d} = \frac{x_n}{D} \Leftrightarrow x_n = \frac{n\lambda D}{d}$

For (n-1) Th before n fringe $x_n = \frac{(n-1)\lambda D}{d}$

To find the distance between two fringes (separation of fringe): $x_n - x_{n-1} = \frac{\lambda D}{d}$

Assigning the fringe separation, the letter ***x***,

$$x = \frac{\lambda D}{d} ie$$
$$\lambda = \frac{dx}{D}$$

Or in words, wavelength $\lambda = \frac{slit\ separation x\ bright\ band\ separation}{dis\tan ce\ of\ slits\ from\ screen}$

Exampl 4.1 e

1. A viewing screen is separated from a double-slit source by 1.2 m. The distance between the two slits is 0.030 mm. The second-order bright fringe (*m* = 2) is 4.5 cm from the center line.

(A) Determine the wavelength of the light.
(B) Calculate the distance between adjacent bright fringes.

Answer

a) From equation: $x_n = \frac{n\lambda D}{d}$ with n = 2, $x_{bright} = 4.5\times10^{-2}m$, $L = 1.2m$ and $d = 3.0\times10^{-5}$

We find $\lambda = \frac{\lambda_{bright} d}{nD} = 5.6x10^{-7} m$

b) From equation: $x_{n-1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d} = \frac{5.6x10^{-7} x1.2}{3.0x10^{-5}} = 2.2cm$

Predicting the location of interference fringes

We can derive quite simply the most important characteristics of the Fraunhofer diffraction pattern from a single slit. First consider two narrow strips, one just below the top edge of the drawing of the slit and one at its center, shown in end view in Fig. 4.4. The difference in path length to point P is $\frac{a}{2}\sin\theta$ where a is the slit width and θ is the angle between the perpendicular to the slit and a line from the center of the slit to P.

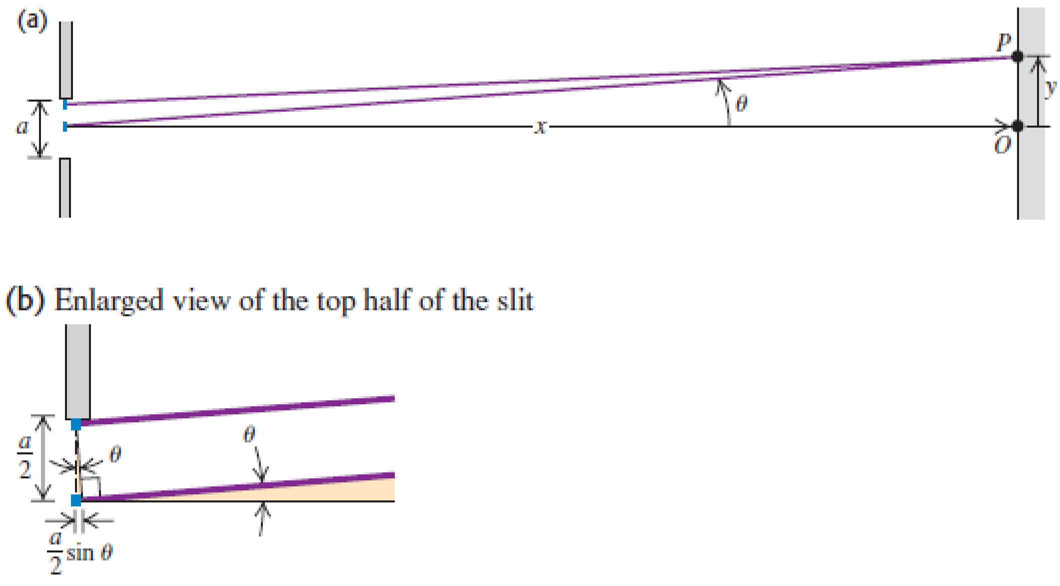


Fig.4. 3Side view of a horizontal slit.(a) the path difference to P is $\Delta x = \frac{a}{2}\sin\theta$ **When the distance to the screen is much greater than the slit width the rays from a distance apart may be considered parallel.(b)** θ **is usually very small, so we can use the approximations** $\sin\theta = \theta$ **then the condition for dark band is** $y_m = \frac{mx\lambda}{a}$

The **condition for bright fringes** (constructive interference) is given by the following:

$$d\sin\theta = m\lambda$$

In this equation, m is the order number of the fringe. The central bright fringe at $\theta = 0$ is called the zeroth-order maximum, or the central maximum, the first maximum on either side of the central maximum, which occurs when $m = \pm 1$ is called the first order maximum, and so forth.

Similarly, when the path difference is an odd multiple of $\frac{\lambda}{2}$, the two waves arriving at screen are 180º out of phase, giving rise to destructive interference, the **condition for dark fringes**, or destructive interference, is given by the following equation:

$$d\sin\theta = (m+\frac{1}{2})\lambda$$

If m = 0 in this equation, the path difference is $d\sin\theta = \frac{\lambda}{2}$ which is the condition under which the first dark fringe forms on either side of the bright central maximum. Likewise, if $m = \pm 1$, the path difference is $\frac{3\lambda}{2}$, which is the condition for the second dark fringe on each side of central maximum, and so forth.

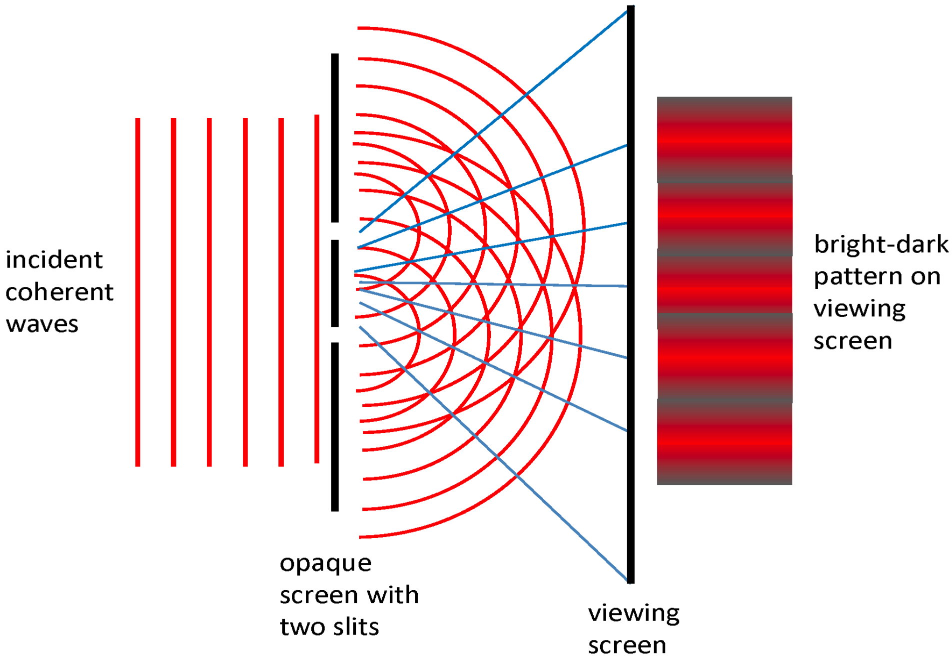


Fig.4. 4 showing interference fringes leading to dark and bright fringes.

**Exampl 4.2
e**

1. A light source emits visible light of two wavelengths: $\lambda = 430mm$. and. $\lambda = 510mm$ .The source is used in a double-slit interference experiment in which $D = 1.5m$ and $d = 0.0250mm$. Find the separation distance between the third-order bright fringes.

Answer

From equation: $x_n = \frac{n\lambda D}{d} = \frac{430x10^{-9}x1.50}{0.0250x10^{-3}} = 7.74x10^{-2}m$

$x'_n = \frac{510x10^{-9}x1.50}{0.0250x10^{-3}} = 9.18x10^{-2}m$

Hence, the separation distance between the two fringes is

$\Delta x = x'_n - x_n = 1.44x10^{-2}m$

2. Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit. Find the positions of the first dark fringes and the width of the central bright fringe.

Answer

The two dark fringes that flank the central bright fringe correspond to $m = \pm 1$ in Equation

$$\sin\theta = m\frac{\lambda}{d}$$

$$\sin\theta = m\frac{\lambda}{d}$$

Hence, we find that $\sin\theta = \pm\frac{580x10^{-9}}{0.300x10^{-3}} = \pm 1.93x10^{-3}$

From the triangle in Figure 4.3, note that $\tan\theta = \frac{x_1}{D}$

Because θ is very small, we can use the approximation $\sin\theta = \tan\theta = \frac{x_1}{D}$

Therefore, the positions of the first minima measured from the central axis are given by

$$x_1 = \pm D\sin\theta = \pm D\frac{\lambda}{d} = \pm 2.00x1.93x10^{-3} = 3.87x10^{-3}m$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe. Hence, the width of the central bright fringe is equal to

$$2x_1 = 2x3.87x10^{-3} = 7.74x10^{-3}m$$

Note that this value is much greater than the width of the slit.

APPLICATIONS OF INTERFERENCE OF WAVES IN REAL LIFE.

Below are summarized applications of interference in real life

a) Signal processing: reference signal is modulated by a sinusoidal waveform, and dynamic displacement is derived from the envelope curves of the interference signal.
b) In laser production. In laser processing a beam is made to interfere within a cavity leading to amplification of light.
c) Light Amplification. Light can be amplified by making light to interfere constructively.

APPLICATION ACTIVITY 4.1

1. What is the necessary condition on the path length difference between two waves that interfere (a) constructively and (b) destructively?

2. If Young's double-slit experiment were performed under water, how would the observed interference pattern be affected?

3. In Young's double-slit experiment, why do we use monochromatic light? If white light is used, how would the pattern change?

4. The distance between the two slits is 0.030 mm. the second-order bright fringe (m = 2) is measured on a viewing screen at an angle of 2.15^0 from the central maximum. Determine the wavelength of the light.

5. A 2-slit experiment is set up in which the slits are 0.03 m apart. A bright fringe is observed at an angle 10° from the normal. What is wavelength of electromagnetic radiation being used?

6. In Young's double slit experiment the separation between the 1st and 5th bright fringes is $2.5\ mm$. When the wavelength used is $6.2\times10^{-17}m$. The distance from the slits to screen is 0.8 m. Calculate the separation of the slits.

4.2 Stationary or standing waves

ACTIVITY 4.2

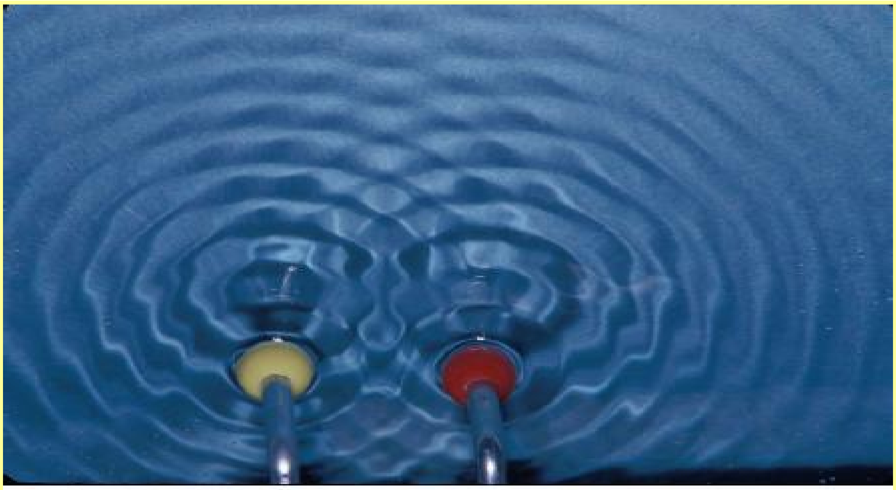


Figure 15.2 The interference pattern obtained using two point sources to produce circular water waves in a ripple tank

The figure above shows ripples of water waves interact after meeting.

a) Account for the causes of different shapes shown on the surface of water
b) Explain the appearance of surface of water if one circular dipper was used instead of two.
c) Comment on the energy and displacement of resultant wave when the different waves meet..

4.2.1 Concept of stationary wave

Standing wave also known as a **stationary wave**, *is wave pattern that results when two waves of the same frequency; wavelength and amplitude travelling in opposite directions in the same medium interfere or meet.*

The point at which the two waves cancel are called **node**. There is no motion in the string at the nodes, but midway between two adjacent nodes, the string vibrates with the largest amplitude. These points are called **antinodes**. At points between successive nodes the vibrations are in **phase**.

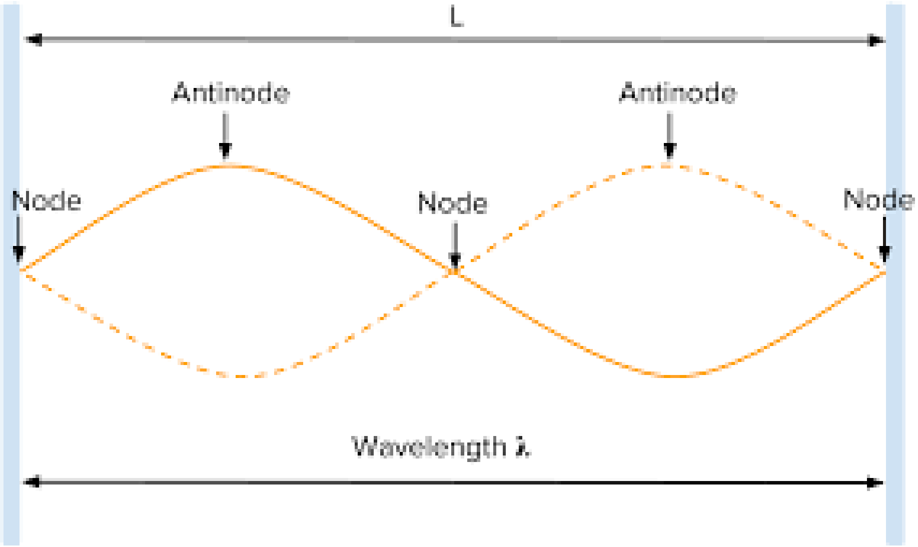


Fig.4. 5 Standing wave with 3 nodes

Standing-wave produced at various times by two waves of equal amplitude traveling in opposite directions. For the resultant wave y, *the nodes points of zero displacement, and the antinodes are points of maximum displacement.*

The standing wave has 3 nodes: One at either end and one in the middle, in that case there are two loops, corresponding to a crest and trough. Thus, this standing wave has a wavelength equal to the string length. i.e.

$$\lambda = L$$

A single loop corresponds to either a crest or trough alone, while two loops correspond to a crest and trough together, or one wave length.

Because a single loop corresponds to either a crest or trough alone, this standing wave corresponds to one half of a wavelength. Thus, the wavelength in this case is equal to twice the string length i.e.

$$L = \frac{\lambda}{2}$$
$$\lambda = 2L$$

Stationary waves are present in the vibrating strings of musical instruments. A violin string, for instance, when bowed or plucked, vibrates as a whole, with nodes at the ends, and also vibrates in halves, with a node at the center, in thirds, with two equally spaced nodes, and in various other fractions, all simultaneously. The vibration as a whole produces the fundamental tone, and the other vibrations produce the various harmonics.

4.2.2 Stationary wave equations

The extreme is free

A stationary wave can be considered as a produced by superposition of two progressive waves, of the same amplitude and frequency, travelling in opposite directions.

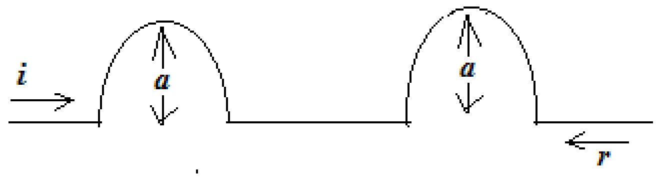


Fig.4. 6

Suppose $y_1 = A\sin 2\pi(\frac{t}{T} - \frac{x}{\lambda})$ is a plane progressive wave travelling in one direction along x axis.

Then $y_2 = A\sin 2\pi(\frac{t}{T} + \frac{x}{\lambda})$ represents a wave of the same amplitude and frequency travelling in opposite direction so the resultant displacement, y, is given by $\sin a + \sin b = 2\sin(\frac{a+b}{2})\cos(\frac{a+b}{2})$

$$y_1 + y_2 = A\sin 2\pi(\frac{t}{T} - \frac{x}{\lambda}) + A\sin 2\pi(\frac{t}{T} + \frac{x}{\lambda})$$

$$y_1 + y_2 = A[\sin 2\pi(\frac{t}{T} - \frac{x}{\lambda}) + \sin 2\pi(\frac{t}{T} + \frac{x}{\lambda})]$$

Let $let\ y_1 + y_2$ be y

$$Y = 2A\sin\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda}$$

$$Y = 2A\cos kx \sin \omega t$$

This equation represents the wave function of a standing wave where $k = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi}{T}$ **Position of antinodes**

The element with the *greatest* possible displacement from equilibrium has an amplitude of 2*A*, and we define this as the amplitude of the standing wave. The positions in the medium at which this maximum displacement occurs are called **antinodes**. The antinodes are located at positions for which the coordinate *x* satisfies the condition

$$2A\cos kx = \pm 2A$$
$$\cos kx = \pm 1$$
$$kx = n\pi$$

$$\frac{2\pi}{\lambda}x = n\pi$$
$$x = \frac{n\lambda}{2}$$

The distance between 2 successive antinodes is $\frac{\lambda}{2}$

The distance between a node and an adjacent antinode is $\frac{\lambda}{4}$

Position of nodes

The maximum amplitude of an element of the medium has a minimum value of zero when x satisfies the condition:

$$2A\cos kx = 0 \Leftrightarrow \cos kx = 0$$
$$kx = \frac{2n+1}{2}\pi$$

$$\frac{2\pi}{\lambda}\lambda = \frac{2n+1}{2}\pi$$
$$x = \frac{2n+1}{4}\lambda$$

These points of zero amplitude are called nodes.

The extreme is fixed

The reflected wave is opposite phase with the incident wave

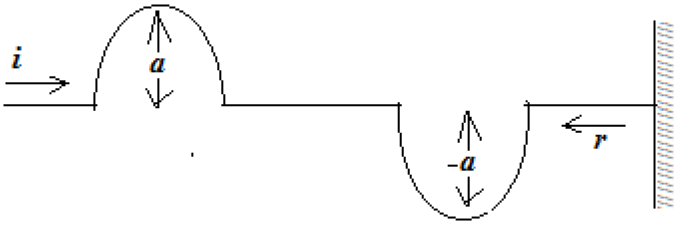


Fig.4. 7

$$y_1 = a\sin(\omega t - kx) \quad and \quad y_2 = a\sin(\omega t + kx)$$

$$y_1 + y_2 = 2a\sin kx\cos\omega t$$

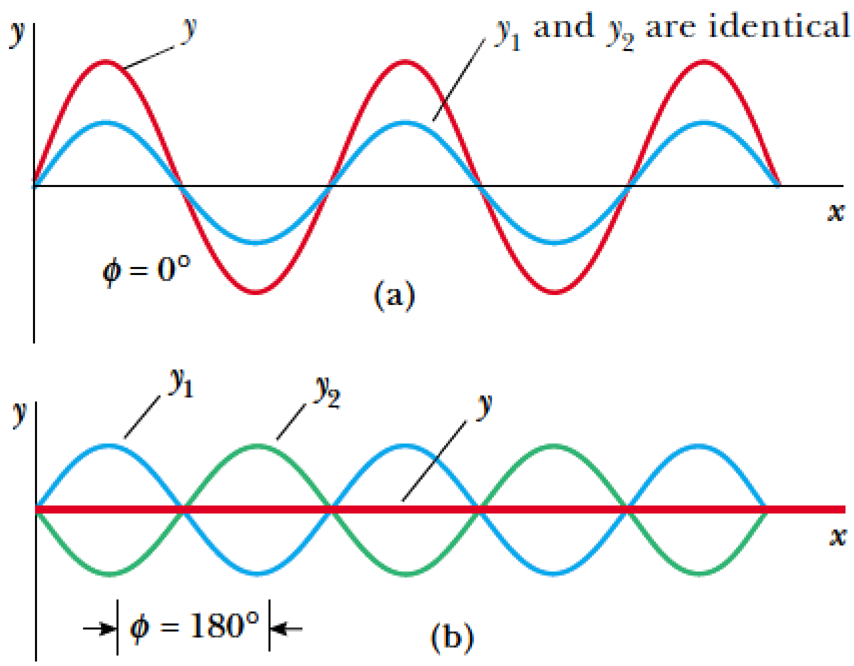


Fig.4. 8*The superposition of two identical waves* **y***1 and* **y**$_2$ *(blue and green) to yield a resultant wave (red). (a) When* **y**$_1$ *and* **y**$_2$ *are in phase, the result is constructive interference. (b) When* **y**$_1$ *and* **y**$_2$ *are* π *rad* *out of phase, the result is destructive interference.*

DIFFERENCE BETWEEN STATIONARY WAVES AND PROGRESSIVE WAVES

STATIONARY WAVES/STANDING WAVE	TRAVELLING WAVE/PROGRESSIVE WAVE
• The wave shape does move	• The wave shape progresses
• Neighboring points have different amplitudes	• Neighboring points have the same amplitude
• Neighboring points have the same phase	• Neighboring points have different phase
• It store energy	• It transmits energy

APPLICATIONS OF STATIONARY WAVES.

Stationary/Standing waves are applied in the following

i) In musical instruments (strings and pipes) in production of musical notes

ii) In the manufacture of laser beam.

iii) In production of sound from the vocal cord

Exampl 4.3 e

1. 1.Two waves traveling in opposite directions produce a standing wave. The individual wave functions are $y_1 = 4.0\sin(3.0x - 2.0t)$ *and* $y_2 = 4.0\sin(3.0x + 2.0t)$ where *x* and *y* are measured in centimeters.

(A) Find the amplitude of the simple harmonic motion of the element of the medium located at $x = 2.3$ cm

Answer

The standing wave is described by Equation $y = 2A\cos kx \sin \omega t \ where\ k = \frac{2\pi}{\lambda}, \omega = \frac{2\pi}{T}$ in this problem, we have $A = 2.0$ cm, $k = 3.0$ rad/cm, and $\omega = 2.0\, rad/s$.

Thus, $y = 2A\cos kx\omega t = 8.0\cos 3.0x \sin 2.0t$

Thus, we obtain the amplitude of the simple harmonic motion of the element at the position $x = 2.3\,cm$ by evaluating the coefficient of the cosine function at this position:

$$y_m = 8.0\cos(3.0x2.3)rad = 6.5cm$$

(B) Find the positions of the nodes and antinodes if one end of the string is at $x = 0$.

Answer

With $k = \frac{2\pi}{\lambda} = 3rad/s$, we see that the wavelength is $\lambda = \frac{2\pi}{3}$.

Therefore, from Equation $x = \frac{2n+1}{4}\lambda$ we find that the nodes are located at $x = \frac{(2n+1)}{4}\lambda$

it follows that $x = \frac{(2n+1)\pi}{6}$

and from Equation $x = \frac{n\lambda}{2}$ we find that the antinodes are located at $x = \frac{n\lambda}{2} = \frac{n\pi}{3}$

(C) What is the maximum value of the position in the simple harmonic motion of an element located at an antinode?

Answer

The maximum position of an element at an antinode is the amplitude of the standing wave, which is twice the amplitude of the individual traveling waves:

$$Y_{\max} = 2A\cos kx = 8.0\cos 3.0x = 8.0x(\pm 1) = \pm 8.0cm$$

where we have used the fact that the maximum value of $\cos kx = \pm 1$

APPLICATION ACTIVITY 4.2

1.Explain the meaning of displacement node and antinodes.
2.Discus at least 5 the characteristics of stationary waves.
3. Show how the standard travelling wave equation can be varied by substitution to get:

a) $y = A\sin 2\pi(\frac{t}{T} - \frac{x}{\lambda})$.

b) $y = A\sin 2\pi(\omega t - kx)$, $f = \frac{1}{T}, V = \lambda f$ and $k = \frac{2\pi}{\lambda}$ may be useful.

4. A travelling wave is represented by the equation $y = 8\sin(40\pi t - 0.8\pi x)$ where y is in cm. State a wave that would superimpose to the one given to give rise to a stationary wave. Hence calculate the amplitude and velocity of the stationary wave.

Skills Lab 4

In this activity you will construct a ripple tank. A ripple tank is a device that is majorly made of glass, wood (timber), source of light like a torch, stroboscope, source of electricity, motor, White sheet of paper to act as a screen and others. It is used to study all properties of water waves.

Make a research about its working and then using the described materials, construct a functioning ripple tank. Your structure may appear as one shown below.

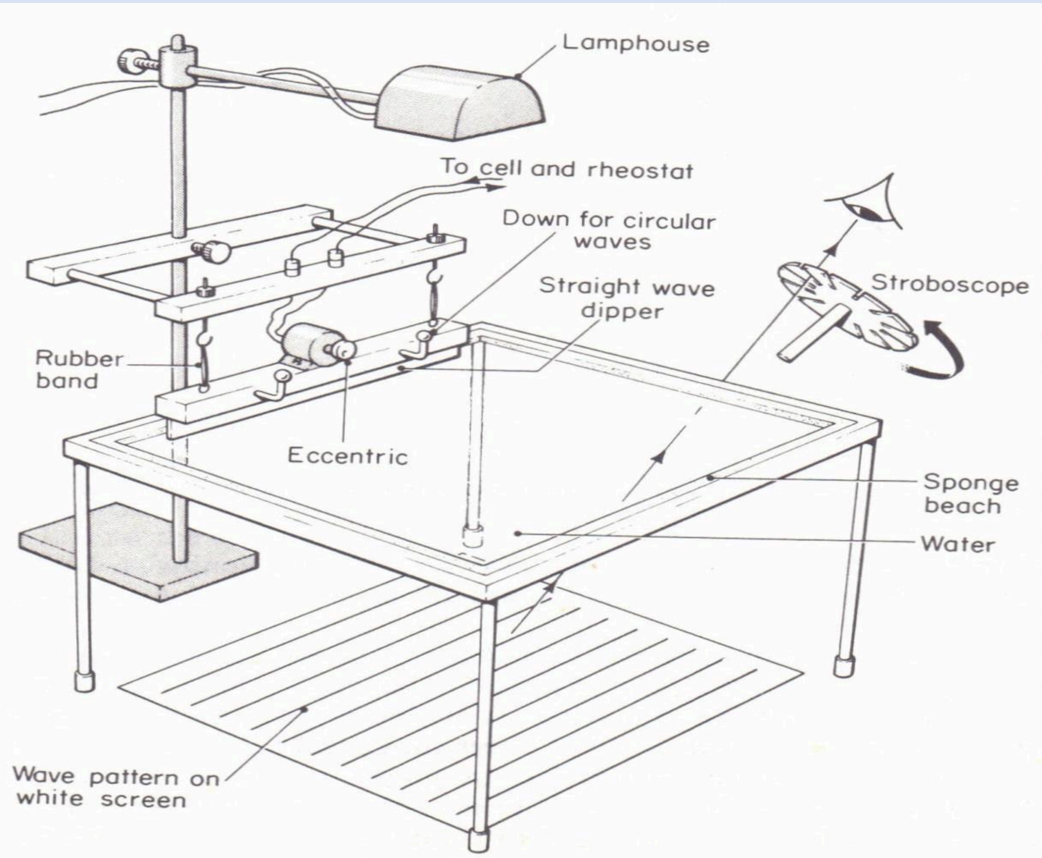
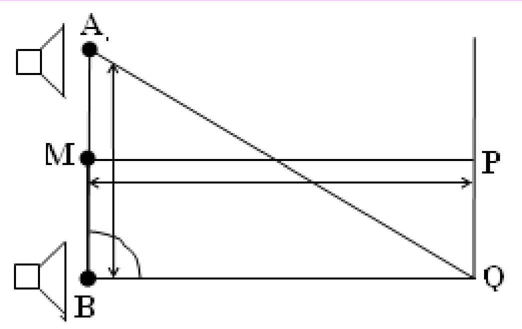


Fig. 26.3. The ripple tank

End Unit Assessment 4

1. Two small loudspeakers A and B 1.00 m apart, are connected to the same oscillator so that both emit sound waves of frequency 1700 Hz in phase. A sensitive detector, moving parallel to the line AB along PQ 2.40 m away as shown in Fig. below, detects a maximum wave at P on the perpendicular bisector MP of AB and another maximum wave when it first reaches a point Q directly opposite to B. Calculate the speed c of the sound waves in air from these measurements.



2. White light passes through two slits 0.5 mm apart and an interference pattern is observed on screen 2.5 m away. The first- order fringe resembles a rainbow with violet and red light at either end. The violet light falls about 2.0 mm and the red 3.5 mm from the center o the central white fringe. Estimate the wavelength of the violet and red lights.

3. In young's double-slit experiment the distance between the slits and the screen is 1.60 m, using light of wavelength 5.89×10^{-7} m the distance between the centre of the interference pattern and the forth bright fringe on either side is 16 mm. what is the slit separation?

4. Light, with a wavelength of 500 nm, is shone through 2 slits, which are 0.05 m apart. What are the angles to the normal of the first three dark fringes?

5. Does the vertical speed of a segment of a horizontal taut string, through which a wave is traveling, depend on the wave speed?