

Lesson Plans

Name

Instructor


Course

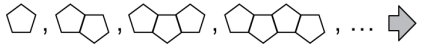
Date


LESSON PLANS






Unit/lesson title: Solving equations using graphical techniques	Lesson duration: 80 minutes	Stage 4	Year 7-8			
Specific teaching target: Solving equations using graphical techniques						
Rationale: Students should develop an understanding of the use of pronumerals as algebraic symbols for numbers of objects rather than for the objects themselves.	Prior knowledge: Students should already be familiar with <ul style="list-style-type: none"> • Equations involving two variables, • The equation of a straight line in slope–intercept form, • Graphing linear functions. 					
Syllabus strand/sub strand: Number patterns and algebraic thinking National Numeracy Learning Progression: NPA7, NPA9 Algebraic relationships <ul style="list-style-type: none"> • Recognises that each point on the graph of a linear relationship represents a solution to a particular linear equation 	Syllabus content description/outcome: <ul style="list-style-type: none"> › Communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM › Recognises and explains mathematical relationships using reasoning MA4-3WM › Creates and displays number patterns; graphs and analyses linear relationships; and performs transformations on the Cartesian plane MA4-11NA <ul style="list-style-type: none"> • Use graphs of linear relationships to solve a corresponding linear equation, with and without the use of digital technologies, e.g., use the graph of $y = 2x + 3$ to find the solution of the equation $2x + 3 = 11$ 🖥️ • Graph two intersecting lines on the same set of axes and read off the point of intersection ▶ explain the significance of the point of intersection of two lines in relation to it representing the only solution that satisfies both equations (Communicating, Reasoning) 🎓 ⚙️ 					
Syllabus elaborations/content	Time	Content/learning experiences	Teaching strategies	Class organisation	Assessment techniques	Resources

Plot coordinates	15 min	<p style="text-align: center;">Step 1: Introduction</p> <p>Teacher teaches students to plot and label points on the Cartesian plane, given coordinates, including those with coordinates that are not whole numbers</p> <p>identify and record the coordinates of given points on the Cartesian plane, including those with coordinates that are not whole numbers</p>	Teacher assists the class to learn about the Cartesian plane, and accurately plot coordinates	Whole-of-class setting	Students' feedback determined their understanding of concepts	A sheet of paper, graph paper, pencil, eraser and ruler per student plus spares
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<p>Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates (ACMMG181)</p>	<p>15 min</p>	<p>Step 2: Finding and sorting out</p> <p>use the notation P' to name the 'image' resulting from a transformation of a point P on the Cartesian plane </p> <p>plot and determine the coordinates for P' resulting from translating P one or more times</p> <p>plot and determine the coordinates for P' resulting from reflecting P in either the x- or y-axis</p> <p>investigate and describe the relationship between the coordinates of P and P' following a reflection in the x- or y-axis, e.g., if P is reflected in the x-axis, P' has the same x-coordinate, and its y-coordinate has the same magnitude but opposite sign (Communicating) **</p> <p>recognise that a translation can produce the same result as a single reflection and vice versa (Reasoning) **</p> <p>plot and determine the coordinates for P' resulting from rotating P by a multiple of 90° about the origin</p> <p>investigate and describe the relationship between the coordinates of P and P' following a rotation of 180° about the origin, e.g., if P is rotated 180° about the origin, the x- and y-coordinates of P' have the same magnitude but opposite sign (Communicating) **</p> <p>recognise that a combination of translations and/or reflections can produce the same result as a single rotation and that a combination of rotations can produce the same result as a single translation and/or reflection (Reasoning)</p>	<p>Teacher demonstrates how to describe translations, reflections, and rotations</p>	<p>Individuals and in pairs</p>	<p>Observing if students can plot coordinates accurately and describe the relationship between the coordinates</p>	<p>A sheet of paper, graph paper, pencil, eraser and ruler per student plus spares</p>
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<p>Plotting and labelling points on the Cartesian plane using given coordinates, including coordinates that are positive and negative. Graphing parabolic relationships of the form $y = a \cdot x^2$ and $y = a \cdot x^2 + b$ without the use of digital technologies.</p>	<p>20 min</p>	<p style="text-align: center;">Step 3: Going further</p> <p>Plot linear relationships on the Cartesian plane, with and without the use of digital technologies (ACMNA193)</p> <p>use objects to build a geometric pattern, record the results in a table of values, describe the pattern in words and algebraic symbols, and represent the relationship on a number grid, e.g.,</p> <p>  </p> <table border="1" data-bbox="969 308 1429 371"> <tr> <td>number of pentagons (p)</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>...</td> </tr> <tr> <td>number of matches (m)</td> <td>5</td> <td>9</td> <td></td> <td>17</td> <td>...</td> </tr> </table> <p>check pattern descriptions by substituting further values (Reasoning) **</p> <p>replace written statements describing patterns with equations written in algebraic symbols, e.g. 'You multiply the number of pentagons by four and add one to get the number of matches' could be replaced with '$m = 4p + 1$' (Communicating, Reasoning) 🎓</p> <p>determine whether a particular pattern can be described using algebraic symbols (Problem Solving) **</p> <p>represent the pattern formed by plotting points from a table and suggest another set of points that might form the same pattern (Communicating, Reasoning) **</p> <p>explain why it is useful to describe the rule for a pattern in terms of the connection between the top row and the bottom row of the table (Communicating, Reasoning) **</p> <p>recognise a given number pattern (including decreasing patterns), complete a table of values, describe the pattern in words and algebraic symbols, and represent the relationship on a number grid 🎓</p> <p>generate a variety of number patterns that increase or decrease and record them in more than one way (Communicating)</p> <p>determine a rule in words to describe the pattern by relating the 'position in the pattern' to the 'value of the term' (Communicating, Problem Solving)</p> <p>explain why a particular relationship or rule for a given pattern is better than another (Communicating, Reasoning) **</p> <p>distinguish between graphs that represent an increasing number pattern and those that represent a decreasing number pattern (Communicating, Reasoning) **</p>	number of pentagons (p)	1	2	3	4	...	number of matches (m)	5	9		17	...	<p>Teacher demonstrates how to plot linear relationships</p>	<p>Individuals and in pairs</p>	<p>Observing if students can accurately plot linear relationships on the Cartesian plane using digital technologies</p>	<p>Computers or tablets with GeoGebra installed</p>
number of pentagons (p)	1	2	3	4	...													
number of matches (m)	5	9		17	...													


		<p>determine whether a particular number pattern forms a linear or non-linear relationship by examining its representation on a number grid (Problem Solving) **</p> <p>use a rule generated from a pattern to calculate the corresponding value for a larger number</p> <p>form a table of values for a linear relationship by substituting a set of appropriate values for either of the pronumerals and graph the number pairs on the Cartesian plane, e.g., given $y = 3x + 1$, form a table of values using $x = 0, 1$ and 2 and then graph the number pairs on the Cartesian plane with an appropriate scale</p> <p>explain why $0, 1$ and 2 are frequently chosen as x-values in a table of values (Communicating, Reasoning) **</p> <p>extend the line joining a set of points on the Cartesian plane to show that there is an infinite number of ordered pairs that satisfy a given linear relationship</p> <p>interpret the meaning of the continuous line joining the points that satisfy a given number pattern (Communicating, Reasoning) **</p> <p>read coordinates from the graph of a linear relationship to demonstrate that there are many points on the line (Communicating) **</p> <p>derive a rule for a set of points that has been graphed on the Cartesian plane</p> <p>graph more than one line on the same set of axes using digital technologies and compare the graphs to determine similarities and differences, e.g., parallel, pass through the same point  **</p>				
Finding features and recognising connections between algebraic and graphic representations of non-linear relationships.	20 min	<p>Step 4: Making Connections</p> <p>Instruct students to identify similarities and differences between groups of linear relationships, e.g.,</p> $y = 3x \quad y = 3x + 2 \quad y = 3x - 2$ $y = x \quad y = 2x \quad y = 3x$ $y = x \quad y = -x \quad \text{(Reasoning) **}$ <p>determine which term of the rule affects the gradient of a graph, making it increase or decrease (Reasoning) **</p>	Direct instruction and teacher led discussion.	Students working individually and in pairs	Determining if students can use reasoning to determine similarities and differences between groups of linear relationships	Computers or tablets with GeoGebra installed

		<p>use digital technologies to graph linear and simple non-linear relationships, such as $y = x^2$ </p> <p>recognise and explain that not all patterns form a linear relationship (Communicating) </p> <p>determine and explain differences between equations that represent linear relationships and those that represent non-linear relationships (Communicating)  </p> <p>Solve linear equations using graphical techniques (ACMNA194)</p> <p>recognise that each point on the graph of a linear relationship represents a solution to a particular linear equation</p> <p>use graphs of linear relationships to solve a corresponding linear equation, with and without the use of digital technologies, e.g., use the graph of $y = 2x + 3$ to find the solution of the equation $2x + 3 = 11$ </p> <p>graph two intersecting lines on the same set of axes and read off the point of intersection</p> <p>explain the significance of the point of intersection of two lines in relation to it representing the only solution that satisfies both equations (Communicating, Reasoning)</p>			, and their effectiveness in using graphical techniques to solve linear equations	
Syllabus elaborations/content	Time	Content/learning experiences	Teaching strategies	Class organisation	Assessment techniques	Resources
	10 min	<p>Conclusion (Presentation/Reflection)</p> <p>Teacher explains the importance of develop an understanding of the use of pronumerals as algebraic symbols for numbers of objects rather than for the objects themselves.</p>	Teacher leads the conclusion on how to apply mathematics in the real-world.	Whole-of-class setting.		Photographs and videos
Planning for Evaluation	Providing guiding questions that assist students in learning course concepts by themselves					
Quality Teaching/Quality Learning Framework						

	<p>Intellectual Quality</p> <ul style="list-style-type: none"> <input type="checkbox"/> Deep knowledge <input checked="" type="checkbox"/> Deep understanding <input type="checkbox"/> Problematic knowledge <input checked="" type="checkbox"/> Higher-order thinking <input type="checkbox"/> Metalanguage <input type="checkbox"/> Substantiative communication 	<p>Quality Learning Environment</p> <ul style="list-style-type: none"> <input type="checkbox"/> Explicit quality criteria <input checked="" type="checkbox"/> Engagement <input checked="" type="checkbox"/> High expectations <input checked="" type="checkbox"/> Social support <input type="checkbox"/> Students' self-regulation <input checked="" type="checkbox"/> Student direction 	<p>Significance</p> <ul style="list-style-type: none"> <input checked="" type="checkbox"/> Background knowledge <input type="checkbox"/> Cultural knowledge <input checked="" type="checkbox"/> Knowledge integration <input type="checkbox"/> Inclusivity <input checked="" type="checkbox"/> Connectedness <input type="checkbox"/> Narrative

Stage 5 Lesson Plan

Unit/lesson title: Graphing parabolas from quadratic equations	Lesson duration: 80 minutes	Stage 5	Year 9-10
Specific teaching target: Graphing parabolas from quadratic equations			
Rationale: Students should use appropriate terminology, diagrams and symbols in mathematical contexts, provides reasoning to support conclusions that are appropriate to the context, and graph simple non-linear relationships	Prior knowledge: Students should already be familiar with <ul style="list-style-type: none"> Graph simple non-linear relations, with and without the use of digital technologies 		
Syllabus strand/sub strand: Non-linear relationships National Numeracy Learning Progression: NPA7 Non-linear relationships Recognises that a relationship between two quantities that is not a linear relationship (i.e., is not a relationship that has a graph that is a straight line) is therefore a non-linear relationship, such as where one quantity varies directly or inversely as the square or cube (or other power) of the other quantity, or where one quantity varies exponentially with the other.	Syllabus content description/outcome: Graph simple non-linear relationships, with and without the use of digital technologies, and solve simple related equations (ACMNA296) <ul style="list-style-type: none"> graph parabolic relationships of the form $y = kx^2$, $y = kx^2 + c$, with and without the use of digital technologies identify parabolic shapes in the environment (Reasoning) describe the effect on the graph of $y = x^2$ of multiplying x^2 by different numbers (including negative numbers) or of adding different numbers (including negative numbers) to x^2 (Communicating, Reasoning) ** determine the equation of a parabola, given a graph of the parabola with the main features clearly indicated (Reasoning) 🎓 ** determine the x-coordinate of a point on a parabola, given the y-coordinate of the point sketch, compare and describe, with and without the use of digital technologies, the key features of simple exponential curves, e.g. sketch and describe similarities and differences of the graphs of $y = 2^x$, $y = -2^x$, $y = 2^{-x}$, $y = -2^{-x}$, $y = 2^x + 1$, $y = 2^x - 1$ 🎓 ** describe exponentials in terms of what happens to the y-values as the x-values become very large or very small, and the y-value for $x = 0$ (Communicating, Reasoning) ** use Pythagoras' theorem to establish the equation of a circle with centre the origin and radius of the circle r recognise and describe equations that represent circles with centre the origin and radius r 🎓 sketch circles of the form $x^2 + y^2 = r^2$ where r is the radius of the circle 		

Syllabus elaborations/content	Time	Content/learning experiences	Teaching strategies	Class organisation	Assessment techniques	Resources
Describe and interpret parabolas	15 min	<p style="text-align: center;">Step 1: Introduction</p> <p>Prior to the lesson students will be encouraged to take photographs of the parabolas they find in their surroundings.</p> <p>Describe, interpret, and sketch parabolas, hyperbolas, circles and exponential functions and their transformations (ACMNA267) find x- and y-intercepts, where appropriate, for the graph of $y = ax^2 + bx + c$, given a, b and c</p> <p>graph a variety of parabolas, including where the equation is given in the form $y = ax^2 + bx + c$, for various values of a, b and c</p> <p>use digital technologies to investigate and describe features of the graphs of parabolas given in the following forms for both positive and negative values of k, b and c eg</p> $y = kx^2 \quad y = kx^2 + c \quad y = (x + b)^2$ $y = (x + b)^2 + c$ <p>(Communicating, Reasoning)  **</p> <p>describe features of a parabola by examining its equation (Communicating) **</p> <p>determine the equation of the axis of symmetry of a parabola using: **</p>	Teacher assists the class to describe and interpret parabolas	Whole-of-class setting	Students' feedback determined their understanding of concepts	Photographs

the midpoint of the interval
joining the points at which
the parabola cuts the x -axis

the formula $x = -\frac{b}{2a}$

find the coordinates of the vertex of a
parabola by: **

using the midpoint of the interval
joining the points at which
the parabola cuts the x -axis
and substituting to obtain the
 y -coordinate of the vertex

using the formula for the axis of
symmetry to obtain the x -
coordinate and substituting
to obtain the y -coordinate of
the vertex

completing the square on x in the
equation of the parabola

identify and use features of parabolas and
their equations to assist in sketching
quadratic relationships, e.g., identify
and use the x - and y -intercepts,
vertex, axis of symmetry and
concavity

<p>Generate quadratic expressions</p>	<p>30 min</p>	<p>Step 2: Finding and sorting out</p> <p>determine quadratic expressions to describe particular number patterns, e.g., generate the equation $y = x^2 + 1$ for the table</p> <table border="1" data-bbox="768 300 1048 373"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>y</td> <td>1</td> <td>2</td> <td>5</td> <td>10</td> <td>17</td> <td>26</td> </tr> </table> <p>graph hyperbolic relationships of the form $y = \frac{k}{x}$ for integer values of k</p> <p>describe the effect on the graph of $y = \frac{1}{x}$ of multiplying $\frac{1}{x}$ by different constants (Communicating) **</p> <p>explain what happens to the y-values of the points on the hyperbola $y = \frac{k}{x}$ as the x-values become very large or closer to zero (Communicating) 🎓 **</p> <p>explain why it may be useful to choose both small and large numbers when constructing a table of values for a hyperbola (Communicating, Reasoning) **</p> <p>graph a variety of hyperbolic curves, including where the equation is given in the form $y = \frac{k}{x} + c$ or $y = \frac{k}{x+b}$ for integer values of k, b and c</p> <p>determine the equations of the asymptotes of a hyperbola in the form $y = \frac{k}{x} + c$ or $y = \frac{k}{x+b}$ (Problem Solving) **</p>	x	0	1	2	3	4	5	y	1	2	5	10	17	26	<p>Teacher demonstrates how to generate quadratic expressions</p>	<p>Individuals and in pairs</p>	<p>Observing if students can graph quadratic expressions</p>	<p>A sheet of paper, graph paper, pencil, eraser and ruler per student plus spares</p>
x	0	1	2	3	4	5														
y	1	2	5	10	17	26														

identify features of hyperbolas from their equations to assist in sketching their graphs, e.g., identify asymptotes, orientation, x - and/or y -intercepts where they exist (Problem Solving, Reasoning) **




describe hyperbolas in terms of what happens to the y -values of the points on the hyperbola as x becomes very large or very small, whether there is a y -value for every x -value, and what occurs near or at $x = 0$ (Communicating, Reasoning) **

recognise and describe equations that represent circles with centre (a, b) and radius r **

establish the equation of the circle with centre (a, b) and radius r , and graph equations of the form $(x - a)^2 + (y - b)^2 = r^2$ (Communicating, Reasoning) **

determine whether a particular point is inside, on, or outside a given circle (Reasoning) **

find the centre and radius of a circle whose equation is in the form $x^2 + y^2 + ax + by + c = 0$ by completing the square (Problem Solving)

<p>Identify different types of graphs from their equations</p>	<p>105min</p>	<p>Step 3: Going further</p> <p>identify and name different types of graphs from their equations, e.g., $(x - 2)^2 + y^2 = 4$, $y = (x - 2)^2 - 4$, $y = 4^x + 2$, $y = x^2 + 2x - 4$, $y = \frac{2}{x-4}$ **</p> <p>determine how to sketch a particular curve by determining its features from its equation (Problem Solving) **</p> <p>identify equations whose graph is symmetrical about the y-axis (Communicating, Reasoning) **</p> <p>determine a possible equation from a given graph and check using digital technologies </p> <p>compare and contrast different types of graphs and determine possible equations from the key features, e.g. $y = 2$, $y = 2 - x$, $y = (x - 2)^2$, $y = 2^x$, $(x - 2)^2 + (y - 2)^2 = 4$, $y = \frac{1}{x-2}$, $y = 2x^2$ (Communicating, Reasoning) ** </p> <p>determine the points of intersection of a line with a parabola, hyperbola, or circle, graphically and algebraically </p> <p>compare methods of finding points of intersection of curves and justify choice of method for a particular pair of curves (Communicating, Reasoning) **</p>	<p>Teacher demonstrates how to demonstrate different types of graphs from their equations</p>	<p>Individuals</p>	<p>Observing if students can accurately sketch a particular curve by determining features from its equation</p>	<p>Computers or tablets with GeoGebra installed</p>
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Describe, interpret and sketch cubics, other curves and their transformations	10 min	<p>Step 4: Making Connections Describe, interpret and sketch cubics, other curves and their transformations graph and compare features of the graphs of cubic equations of the form $y = kx^3$ $y = kx^3 + c$ $y = k(x - a)(x - b)$ describing the effect on the graph of different values of a, b, c and k 🎓</p> <p>graph a variety of equations of the form $y = kx^n$ for n an integer, $n \geq 2$, describing the effect of n being odd or even on the shape of the curve **</p> <p>graph curves of the form $y = kx^n + c$ from curves of the form $y = kx^n$ for n an integer, $n \geq 2$ by using vertical transformations **</p> <p>graph curves of the form $y = k(x - b)^n$ from curves of the form $y = kx^n$ for n an integer, $n \geq 2$ by using horizontal transformations **</p>	Direct instruction and teacher led discussion.	Students working individually and in pairs	Determining if students can describe, interpret and sketch cubics, other curves and their transformations	A sheet of paper, graph paper, pencil, eraser and ruler per student plus spares
Syllabus elaborations/content	Time	Content/learning experiences	Teaching strategies	Class organisation	Assessment techniques	Resources



	10 min	<p>Conclusion (<i>Presentation/Reflection</i>) Teacher explains other learning areas and real-life examples of graphs, e.g., exponential graphs used for population growth in demographics, radioactive decay, town planning, etc.</p>	Teacher leads the conclusion on how to apply parabolas and quadratic equations in real life.	Whole-of-class setting.		Photographs and videos			
Planning for Evaluation		Providing guiding questions that assist students in learning course concepts by themselves							
		<p style="text-align: center;">Quality Teaching/Quality Learning Framework</p> <table style="width: 100%; border: none;"> <tr> <td style="width: 33%; vertical-align: top;"> <p>Intellectual Quality</p> <input type="checkbox"/> Deep knowledge <input checked="" type="checkbox"/> Deep understanding <input type="checkbox"/> Problematic knowledge <input checked="" type="checkbox"/> Higher-order thinking <input type="checkbox"/> Metalanguage <input type="checkbox"/> Substantiative communication </td> <td style="width: 33%; vertical-align: top;"> <p>Quality Learning Environment</p> <input type="checkbox"/> Explicit quality criteria <input checked="" type="checkbox"/> Engagement <input checked="" type="checkbox"/> High expectations <input checked="" type="checkbox"/> Social support <input type="checkbox"/> Students' self-regulation <input checked="" type="checkbox"/> Student direction </td> <td style="width: 33%; vertical-align: top;"> <p>Significance</p> <input checked="" type="checkbox"/> Background knowledge <input type="checkbox"/> Cultural knowledge <input checked="" type="checkbox"/> Knowledge integration <input type="checkbox"/> Inclusivity <input checked="" type="checkbox"/> Connectedness <input type="checkbox"/> Narrative </td> </tr> </table>					<p>Intellectual Quality</p> <input type="checkbox"/> Deep knowledge <input checked="" type="checkbox"/> Deep understanding <input type="checkbox"/> Problematic knowledge <input checked="" type="checkbox"/> Higher-order thinking <input type="checkbox"/> Metalanguage <input type="checkbox"/> Substantiative communication	<p>Quality Learning Environment</p> <input type="checkbox"/> Explicit quality criteria <input checked="" type="checkbox"/> Engagement <input checked="" type="checkbox"/> High expectations <input checked="" type="checkbox"/> Social support <input type="checkbox"/> Students' self-regulation <input checked="" type="checkbox"/> Student direction	<p>Significance</p> <input checked="" type="checkbox"/> Background knowledge <input type="checkbox"/> Cultural knowledge <input checked="" type="checkbox"/> Knowledge integration <input type="checkbox"/> Inclusivity <input checked="" type="checkbox"/> Connectedness <input type="checkbox"/> Narrative
<p>Intellectual Quality</p> <input type="checkbox"/> Deep knowledge <input checked="" type="checkbox"/> Deep understanding <input type="checkbox"/> Problematic knowledge <input checked="" type="checkbox"/> Higher-order thinking <input type="checkbox"/> Metalanguage <input type="checkbox"/> Substantiative communication	<p>Quality Learning Environment</p> <input type="checkbox"/> Explicit quality criteria <input checked="" type="checkbox"/> Engagement <input checked="" type="checkbox"/> High expectations <input checked="" type="checkbox"/> Social support <input type="checkbox"/> Students' self-regulation <input checked="" type="checkbox"/> Student direction	<p>Significance</p> <input checked="" type="checkbox"/> Background knowledge <input type="checkbox"/> Cultural knowledge <input checked="" type="checkbox"/> Knowledge integration <input type="checkbox"/> Inclusivity <input checked="" type="checkbox"/> Connectedness <input type="checkbox"/> Narrative							










Stage 6: Trigonometric Functions

Unit/lesson title: Trigonometric functions		Lesson duration: 80 minutes		Stage 6	Year 11-12	
Specific teaching target: Trigonometric functions						
Rationale: Students should develop an understanding of trigonometric functions			Prior knowledge: Students should already be familiar with <ul style="list-style-type: none"> • Trigonometry and measure of angles 			
Syllabus strand/sub strand: Trigonometric functions National Numeracy Learning Progression: NPA9 Trigonometric functions The principal focus of this subtopic is to use trigonometric identities and reciprocal relationships to simplify expressions, to prove equivalences and to solve equations			Syllabus content description/outcome: <ul style="list-style-type: none"> › Uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-1 › Uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes MA11-3 › Uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities MA11-4 › Uses appropriate technology to investigate, organise, model and interpret information in a range of contexts MA11-8 › Provides reasoning to support conclusions which are appropriate to the context MA11-9 			
Syllabus elaborations/content	Time	Content/learning experiences	Teaching strategies	Class organisation	Assessment techniques	Resources
Trigonometric functions and identities	15 min	<p>Step 1: Introduction</p> <ul style="list-style-type: none"> • Use the sine, cosine and tangent ratios to solve problems involving right-angled triangles where angles are measured in degrees, or degrees and minutes U • Establish and use the sine rule, cosine rule and the area of a triangle formula for solving problems where angles are measured in degrees, or degrees and minutes AAM U • Find angles and sides involving the ambiguous case of the sine rule 	Teacher assists the class to learn about trigonometric functions and identities	Whole-of-class setting	Students' feedback determined their understanding of concepts	A sheet of paper, graph paper, pencil, eraser and ruler per student plus spares

		<p>– Use technology and/or geometric construction to investigate the ambiguous case of the sine rule when finding an angle, and the condition for it to arise 🧩 🖨</p> <ul style="list-style-type: none"> • solve problems involving the use of trigonometry in two and three dimensions <p>AAM 📄</p> <p>– interpret information about a two or three-dimensional context given in diagrammatic or written form and construct diagrams where required</p> <p>solve practical problems involving Pythagoras’ theorem and the trigonometry of triangles, which may involve the ambiguous case, including finding and using angles of elevation and depression and the use of true bearings and compass bearings in navigation AAM 📄 🧩</p>				
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Radians	15 min	<p>Step 2: Finding and sorting out</p> <ul style="list-style-type: none"> • understand the unit circle definition of $\sin \theta$, $\cos \theta$ and $\tan \theta$ and periodicity using degrees (ACMMM029) <ul style="list-style-type: none"> – sketch the trigonometric functions in degrees for $0^\circ \leq x \leq 360^\circ$ • define and use radian measure and understand its relationship with degree measure (ACMMM032) <ul style="list-style-type: none"> – convert between the two measures, using the fact that $360^\circ = 2\pi$ radians – recognise and use the exact values of $\sin \theta$, $\cos \theta$ and $\tan \theta$ in both degrees and radians for integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (ACMMM035) • understand the unit circle definition of $\sin \theta$, $\cos \theta$ and $\tan \theta$ and periodicity using radians (ACMMM034) • solve problems involving trigonometric ratios of angles of any magnitude in both degrees and radians <ul style="list-style-type: none"> • recognise the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ and sketch on extended domains in degrees and radians (ACMMM036) • derive the formula for arc length, $l = r\theta$ and for the area of a sector of a circle, $A = \frac{1}{2}r^2\theta$ 	Teacher demonstrates how to define and use radian measures	Individuals and in pairs	Observing if students can define and use radian measures	A sheet of paper, graph paper, pencil, eraser and ruler per student plus spares
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		<ul style="list-style-type: none"> • solve problems involving sector areas, arc lengths and combinations of either areas or lengths 				
Reciprocal trigonometric functions	20 min	<p>Step 3: Going further</p> <ul style="list-style-type: none"> • define the reciprocal trigonometric functions, $y = \operatorname{cosec} x$, $y = \sec x$ and $y = \cot x$ <ul style="list-style-type: none"> – $\operatorname{cosec} A = \frac{1}{\sin A}$, $\sin A \neq 0$ – $\sec A = \frac{1}{\cos A}$, $\cos A \neq 0$ – $\cot A = \frac{\cos A}{\sin A}$, $\sin A \neq 0$ • sketch the graphs of reciprocal trigonometric functions in both radians and degrees • prove and apply the Pythagorean identities $\cos^2 x + \sin^2 x = 1$, $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \operatorname{cosec}^2 x$ (ACMSM046) <ul style="list-style-type: none"> – know the difference between an equation and an identity • use $\tan x = \frac{\sin x}{\cos x}$ provided that $\cos x \neq 0$ <ul style="list-style-type: none"> • prove trigonometric identities • evaluate trigonometric expressions using angles of any magnitude and complementary angle results • simplify trigonometric expressions and solve trigonometric equations, including those that reduce to quadratic equations   	Teacher demonstrates how to define reciprocal trigonometric functions	Individuals	Observing if students can accurately define trigonometric functions	A sheet of paper, graph paper, pencil, eraser and ruler per student plus spares

<p>Finding features and recognising connections between trigonometric ratios and real-life scenarios</p>	<p>20 min</p>	<p>Step 4: Making Connections</p> <ul style="list-style-type: none"> • review and use the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle AAM  • use technology to investigate the sign of $\sin \sin A$ and $\cos \cos A$ for $0^\circ \leq A \leq 180^\circ$   • determine the area of any triangle, given two sides and an included angle, by using the rule $A = \frac{1}{2}ab \sin C$, and solve related practical problems AAM   • solve problems involving non-right-angled triangles using the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (ambiguous case excluded) AAM  – find the size of an obtuse angle, given that it is obtuse • solve problems involving non-right-angled triangles using the cosine rule, $c^2 = a^2 + b^2 - 2ab \cos C$ AAM  • understand various navigational methods <ul style="list-style-type: none"> – understand the difference between compass and true bearings – investigate navigational methods used by different cultures, including those of Aboriginal and Torres Strait Islander Peoples   • solve practical problems involving Pythagoras' theorem, the 	<p>Direct instruction and teacher led discussion.</p>	<p>Students working individually and in pairs</p>	<p>Determining if students can recognise connections between trigonometric ratios and real-life scenarios</p>	<p>Computers or tablets, a sheet of paper, graph paper, pencil, eraser and ruler per student plus spares</p>
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		<p>trigonometry of right-angled and non-right-angled triangles, angles of elevation and depression and the use of true bearings and compass bearings AAM U ✖</p> <ul style="list-style-type: none"> – work with angles correct to the nearest degree and/or minute <ul style="list-style-type: none"> • construct and interpret compass radial surveys and solve related problems 🖨 ✖ 				
Syllabus elaborations/content	Time	Content/learning experiences	Teaching strategies	Class organisation	Assessment techniques	Resources
	10 min	<p>Conclusion (<i>Presentation/Reflection</i>)</p> <ul style="list-style-type: none"> › Teacher provides reasoning to support conclusions which are appropriate to the context of trigonometrics and real—life events 	Teacher leads the conclusion on how to apply trigonometrics in the real-world.	Whole-of-class setting.		Photographs and videos
Planning for Evaluation		Providing guiding questions that assist students in learning course concepts by themselves				
		Quality Teaching/Quality Learning Framework				
		<p>Intellectual Quality</p> <ul style="list-style-type: none"> <input type="checkbox"/> Deep knowledge <input checked="" type="checkbox"/> Deep understanding <input type="checkbox"/> Problematic knowledge <input checked="" type="checkbox"/> Higher-order thinking <input type="checkbox"/> Metalanguage <input type="checkbox"/> Substantiative communication 	<p>Quality Learning Environment</p> <ul style="list-style-type: none"> <input type="checkbox"/> Explicit quality criteria <input checked="" type="checkbox"/> Engagement <input checked="" type="checkbox"/> High expectations <input checked="" type="checkbox"/> Social support <input type="checkbox"/> Students' self-regulation <input checked="" type="checkbox"/> Student direction 	<p>Significance</p> <ul style="list-style-type: none"> <input checked="" type="checkbox"/> Background knowledge <input type="checkbox"/> Cultural knowledge <input checked="" type="checkbox"/> Knowledge integration <input type="checkbox"/> Inclusivity <input checked="" type="checkbox"/> Connectedness <input type="checkbox"/> Narrative 		

Justification for the Inquiry Approach

Stage 4 Lesson Plan

The lesson plan for stage 4 addresses how to solve equations using graphical techniques. The lesson plan is categorized into four main groups involving introduction, finding and sorting out, going further, and making connections. Lesson prerequisites include being knowledgeable on how to solve equations involving two variables, knowing the equation of a straight line in slope–intercept form, and skills in graphing linear functions. The objective of the lesson plan is to provide students with skills and knowledge needed to solve equations using graphical techniques and using technology to solve linear equations. The introduction section introduces students to the Cartesian plane and teaches them how to plot coordinates on the Cartesian plane. The teacher teaches the class as a whole to provide them with general knowledge about the Coordinates. The teacher determines the students' understanding of concepts based on the feedback they provide. The second aspect of the lesson, which includes finding and sorting out goes a step further as it introduces students to the concepts of translation and reflection. Students are required to transform points on the Cartesian plane, and the teacher should observe if students can plot coordinates accurately and describe the relationship between the coordinates. The concept looks for the students' reasoning and communication skills. Step 3 entails going further into the concept and describing how to plot relationships into the Cartesian plane using digital technologies. The students are shown how to use computers or tablets installed with GeoGebra to plot linear relationships on the Cartesian plane. The fourth step of the lesson involves making connections, where the students are assisted to identify similarities and differences between groups of linear relationships. Students are required to use reasoning to determine which term of the rule affects the gradient of a graph, making it increase or decrease, and to use digital technologies to solve linear equations. In the conclusion, the teacher explains the importance of develop an understanding of the use of pronumerals as algebraic symbols for numbers of objects and explains the use of linear equations in real life.

Stage 5 Lesson Plan

The lesson plan for stage 5 addresses how to graph parabolas from quadratic equations. The lesson plan is categorized into four main groups involving introduction, finding and sorting out, going further, and making connections. Lesson prerequisites include graphing simple non-linear relations, with and without the use of digital technologies. The objective of the lesson plan is to enable students to use appropriate terminology, diagrams and symbols in mathematical contexts, provides reasoning to support conclusions that are appropriate to the context, and graph simple non-linear relationships.

The introduction section introduces students to parabolas. The teacher assists the class to describe and interpret parabolas. The teacher teaches the class as a whole to provide them with general knowledge about the Coordinates. The teacher determines the students' understanding of concepts based on the feedback they provide. The second aspect of the lesson includes generating quadratic expressions. Students are required to work as individuals and in pairs. The teacher should observe if students can graph quadratic expressions. The concept looks for the students' reasoning and communication skills. Step 3 entails going further into the concept and describes how to identify different types of graphs from their equations. The students are shown how to use computers or tablets installed with GeoGebra to plot linear relationships on the Cartesian plane. The fourth step of the lesson involves making connections, where students are required to describe, interpret and sketch cubics, other curves and their transformations. In the conclusion, the teacher explains other learning areas and real-life examples of graphs, e.g., exponential graphs used for population growth in demographics, radioactive decay, town planning, etc.

Stage 6 Lesson Plan

Lesson plan 6 addresses trigonometric functions. Students should have a clear understanding of trigonometry and measure of angles. The lesson plan has five main anticipated outcomes including using algebraic and graphical techniques to solve problems, using the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes, using the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities, using appropriate technology to investigate, organise, model and interpret information in a range of contexts, and providing reasoning to support conclusions which are appropriate to the context.

The introduction section of the lesson plan addresses trigonometric functions and identities. The teacher will assist the whole class to learn about trigonometric functions and identities. The teacher determines the students' understanding of concepts based on the feedback they provide. The second aspect of the lesson includes a lesson about radians. The teacher demonstrates how to define and use radian measures. The third section of the lesson plan involves reciprocal trigonometric functions. The teacher demonstrates how to define reciprocal trigonometric functions. The fourth step of the lesson involves making connections, where students are required to review and use the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle, and to incorporate technology into the trigonometrics. The conclusion reiterates the importance of using trigonometrics to solve real-life challenges.

The three lesson plans reflect the principles and practices of early mathematics teachings. The lesson plans focus on students' engagement. According to Watt, Carmichael, and Callingham (2017), a student's engagement is associated with positive school outcomes. That is why the teacher requires the students should be engaged in the lessons by working individually and in groups. The teaching strategy is effective in promoting the learning of mathematical concepts. As stated by Geiger, Anderson, and Hurrell (2017), powerful pedagogical practices used by teachers provides students with opportunities to learn successfully. The lesson plans have used pedagogical approaches involving active learning and student-to-student engagement. The active learning promotes the students' interest in the lessons and enables the teacher to identify areas in which the students are lagging behind. On the other hand, student-to-student engagement facilitates collaboration in learning, as students are able to learn from their peers.

References

- Geiger, V., Anderson, J., & Hurrell, D. (2017). A case study of effective practice in mathematics teaching and learning informed by Valsiner's zone theory. *Mathematics Education Research Journal*, *29*(2), 143-161.
- Watt, H. M., Carmichael, C., & Callingham, R. (2017). Students' engagement profiles in mathematics according to learning environment dimensions: Developing an evidence base for best practice in mathematics education. *School Psychology International*, *38*(2), 166-183.