

CHAPTER 3

DAY 6 – ADDITION, DIFFERENCE OF MATRICES, MULTIPLICATION OF MATRIX BY A SCALAR

Addition of Matrices

Let A and B two matrices of same order. Then, by addition of the matrices A and B , we mean the addition of corresponding elements of A and B .

☐ Only matrices of same order can be added.

☐ E.g. : let $A = \begin{bmatrix} 1 & 3 & -6 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 & 5 & 3 \end{bmatrix}$.

$$\text{Then, } A + B = \begin{bmatrix} 1 & 3 & -6 & 7 \end{bmatrix} + \begin{bmatrix} 4 & -2 & 5 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 1 & -1 & 11 \end{bmatrix}$$

☐ Properties

- **Commutative Law** - Let A and B two matrices of same order. Then

$$A + B = B + A$$

- **Associative Law** - Let A , B and C be three matrices of same order. Then

$$A + (B + C) = (A + B) + C$$

- **Existence of additive identity** – Let A be any $m \times n$ matrix and O be the $m \times n$ zero matrix. Then $A + O = O + A = A$. That is, O is the additive identity of matrix addition.

- **Existence of additive inverse** - Let A be any $m \times n$ matrix. Then, there exists another matrix $-A$ of the same order such that $A + (-A) = (-A) + A = O$. That is, $-A$ is the additive inverse of matrix addition.

Difference of Matrices

Let A and B two matrices of same order. Then, the difference of the matrices A and B , is defined as $A - B = A + (-B)$

☐ Only matrices of same order can be subtracted.

☐ E.g. : let $A = \begin{bmatrix} 1 & 3 & -6 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 & 5 & 3 \end{bmatrix}$.

$$\text{Then, } A - B = \begin{bmatrix} 1 & 3 & -6 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -2 & 5 & 3 \end{bmatrix} = \begin{bmatrix} -3 & 5 & -11 & 4 \end{bmatrix}$$

Multiplication of matrix by a scalar

Let $A = [a_{ij}]$ be any matrix and k be any scalar. Then, the multiplication of matrix by a scalar is defined as $kA = k[a_{ij}] = [ka_{ij}]$.

□ E.g. Let $A = \begin{bmatrix} 1 & 3 & -6 & 7 \end{bmatrix}$, $k = 2$.

$$\text{Then, } kA = 2\begin{bmatrix} 1 & 3 & -6 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 6 & -12 & 14 \end{bmatrix}$$

□ Properties

Let A and B be two matrices of same order and k and l be any two scalars. Then,

- $k(A + B) = kA + kB$.
- $(k + l)A = kA + lA$.

Questions

1. Let $A = \begin{bmatrix} 2 & 4 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 & 3 & 4 \end{bmatrix}$. Find each of the following
 - a. $A + B$
 - b. $A - B$
 - c. $3A - C$
 - d. $2B + 3C$
 - e. Show that $A + (B + C) = (A + B) + C$

2. Simplify

$$\cos \cos x \begin{bmatrix} \cos \cos x & \sin \sin x & -\sin \sin x & \cos \cos x \end{bmatrix} + \sin \sin x \begin{bmatrix} \sin \sin x & -\cos \cos x & \cos \cos x & \sin \sin x \end{bmatrix}$$

3. Find x, y if $2\begin{bmatrix} 1 & 3 & 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 1 & 8 \end{bmatrix}$
4. Find the values of x, y, z and t such that $2\begin{bmatrix} x & y & z & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 & 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 & 4 & 6 \end{bmatrix}$
5. If $x\begin{bmatrix} 2 & 3 \end{bmatrix} + y\begin{bmatrix} -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 10 \end{bmatrix}$, find the values of x, y .
6. Find values matrices X and Y if
 - a. $X + Y = \begin{bmatrix} 7 & 0 & 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 & 0 & 3 \end{bmatrix}$
 - b. $2X + 3Y = \begin{bmatrix} 2 & 3 & 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 & -1 & 5 \end{bmatrix}$
 - c. $Y = \begin{bmatrix} 3 & 2 & 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 & -3 & 2 \end{bmatrix}$

More questions from this section can be practiced!